The Frequency Domain

by Dennis S. Bernstein

This article is dedicated to Edwin Howard Armstrong, virtuoso of the frequency domain and true hero to the engineering profession.

"Any tune. Any key."
— Milt Bernstein, jazz musician

In Search of Poles

When I was young there were many newsstand hobby magazines that published monthly plans for building some kind of electronics project. My favorite was *Radio-TV Experimenter* which, sadly, disappeared and was replaced by the modernized *Popular Electronics*. Unfortunately, the plans in *Popular Electronics* were much too sophisticated for me. A typical project such as a do-it-yourself electronic organ required a mere 200 components, exotic ICs, and printed circuit boards—a rather daunting project for a 12-year-old.

Nevertheless, I was fascinated by the jargon in *Popular Electronics*, although I had virtually no idea what any of it meant. One mysterious word that appeared from time to time was *pole*. As far as I could tell, filters had poles, and the more poles a filter had, the better and more expensive it was. I knew that filters somehow separated radio stations that crowded each other, so it made sense that better filters with more poles ought to cost more. But what on earth was a "pole," anyway? I had absolutely no idea.

Only much later did I learn what a pole is, but I had to go to college to find out. In this article, I'd like to give you some idea of what a pole is and why it is a significant concept. To do this, I will give you a little tour of the mysterious world that control engineers call the *frequency domain*. This article is intended as a conceptual preview for undergraduate students and assumes minimal technical background; I hope that instructors will recommend this article to their introductory classes.

The frequency domain is a kind of hidden companion to our everyday world of time. We describe what happens in the time domain as *temporal* and in the frequency domain as *spectral*. Roughly speaking, in the time domain we measure how long something takes, whereas in the frequency domain we measure how fast or slow it is. If you think these sound like the same thing you're essentially right, since these are two ways of viewing the same thing. So why do control engineers like the frequency domain so much? In a nutshell, the reason is this: Most signals and processes involve both fast and slow components happening at the same time. Frequency domain analysis separates these components and helps to keep track of them.

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Vibrations and Musical Scales

Let's start by talking about something you've heard a lot about but may not have thought much about: sound and music. First, it's helpful to remember that each key on a piano causes a string (and as many as three identical strings for the higher sounds) to vibrate. The low-sounding strings, which are longer, thicker, and less taut, vibrate more slowly than the high-sounding strings, which are shorter, thinner, and more taut. Each string vibrates at a *pitch*, or *frequency*, determined by its length, mass per unit length, and tension. The frequency of vibration of a string, which measures how fast the string vibrates, is simply the number of times it undergoes a cycle of motion in one second. The central pitch on the piano, middle C, has a frequency of $f_C = 261.625$ cps (*cycles per second*), or Hz (*Hertz*), named after Heinrich Hertz (1857-1894). This frequency is set by tradition and varies slightly among musical groups.

An *octave* is a frequency interval that extends from one frequency to twice that frequency. Pitches that are an octave apart have frequencies that are related by a factor of 2. Going up one octave from middle C yields the pitch C′, which has the frequency $f_{C'} = 2f_C = 523.25$ Hz. Likewise, two octaves comprise a frequency interval of four, three octaves comprise a frequency interval of eight, and so on. It's interesting that the ear perceives two pitches an octave apart as essentially the same pitch. For a vocalist, an octave is a fairly large interval, and it tends to have a dramatic effect. In the song "Over the Rainbow" from *The Wizard of Oz*, the two notes in the first word "Somewhere" span an octave, as do the first two notes in the word "wherever" in the theme song "My Heart Will Go On" from the movie *Titanic*.

The factor-of-2 interval is so fundamental to hearing that most Western music (from classical to jazz to pop) is based on it. More precisely, this music is based on the *diatonic scale*, which partitions the octave into eight notes (counting both the first and the last), which explains the "oct" in octave. To confirm this, you can see that the key pattern on a

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piano repeats over and over, and each complete pattern has eight white keys. The frequency range of a piano extends from 27.5 to 4186.0 Hz, which corresponds to a frequency interval of $4186.0/27.5 \approx 152.22$. This interval spans more than seven octaves (a frequency interval of 128) but fewer than eight octaves (a frequency interval of 256). Although humans can hear sounds that are either higher or lower than the range of a piano, it's difficult to identify the pitches of these sounds. Below the range of human hearing are the *infrasonic* frequencies used by whales and elephants, and above it are the *ultrasonic* frequencies useful in medical applications and for calling dogs.

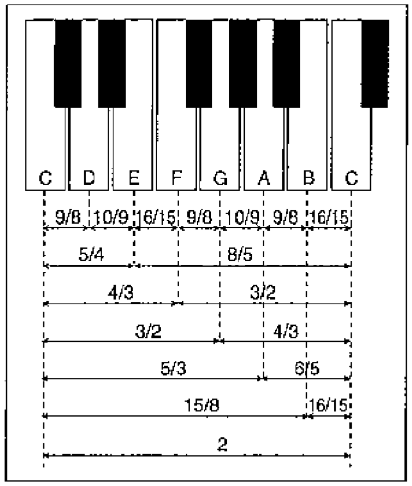

Figure 1. *Frequency intervals for just piano tuning.*

The eight notes of the diatonic scale are partly determined by seven aesthetically pleasing frequency ratios as defined by the Ancient Greeks. From smallest to largest, these "true" frequency intervals are 6/5 (minor third), 5/4 (major third), 4/3 (fourth), 3/2 (fifth), 8/5 (minor sixth), 5/3 (major sixth), and, of course, 2 (octave). Fig. 1 shows the span of each of these intervals in the context of one octave on the piano keyboard. It can be seen that these frequency intervals divide the octave in four different ways, namely,

$$\begin{aligned}(6/5)(5/3)&=(5/4)(8/5)\\&=(4/3)(3/2)\\&=(3/2)(4/3)\\&=2.\end{aligned}$$

These divisions fix four notes of the diatonic scale between C and C′, namely, E, F, G, and A, whose pitches are

$$f_E=(5/4)f_C,\quad f_F=(4/3)f_C,$$
$$f_G=(3/2)f_C,\quad f_A=(5/3)f_C.$$

Two pitches, namely D and B, remain to be fixed. To fix D, start with C, go up two fifths and then down one octave to obtain

$$f_D=(2)^{-1}(3/2)(3/2)f_C=(9/8)f_C$$

and, to fix B, start with C, go up one fifth and then up one major third to obtain

$$f_B=(5/4)(3/2)f_C=(15/8)f_C.$$

The intervals 9/8 and 15/8 are the *second* and *seventh*, respectively.

Putting all these ratios together, we see that the diatonic scale CDEFGABC′ is a partition of the octave into seven intervals whose product is precisely two, that is,

$$(9/8)(10/9)(16/15)(9/8)\cdot(10/9)(9/8)(16/15)=2.$$

All of this appears to be a bit arbitrary, but it is a neat and tidy construction in the frequency domain involving only the frequency intervals 9/8, 10/9, and 16/15.

Just and Equal Temperament Tuning

So far so good, but now the plot thickens a little. Unfortunately, when these intervals are combined successively, problems arise. For example, suppose that you go up by four fifths and then down by two octaves and, finally, down by a major third. Your resulting frequency ratio is then $(5/4)^{-1}\,(2)^{-2}\,(3/2)^4 = 81/80 = 1.0125$. Although this frequency ratio is close to unity, it differs by enough to produce an unpleasant dissonance. Another example is: go up eight fifths, go up one major third, and, finally, go down five octaves, yielding a frequency ratio of $(2)^{-5}\,(5/4)\,(3/2)^8 = 32{,}805/32{,}768 \approx 1.0011$. Thus a few aesthetically pleasing frequency intervals quickly give rise to lots of unpleasant ones.

One solution to this problem is to somehow approximate by a uniform interval the frequency intervals 9/8, 10/9, and 16/15 that comprise the diatonic scale. To do this, you need to first notice that $10/9 \approx 1.1111$ and $9/8 = 1.125$ are very close, and both of these are also close to the square of the ratio $16/15 \approx 1.0667$, which is $(16/15)^2 = 256/225 \approx 1.1378$. Therefore, we can approximate 10/9, 9/8, and 16/15 by using a single, uniform frequency ratio, the *semitone*. This approximation uses one semitone for each interval of size 16/15 and two semitones for each interval of size 10/9 or 9/8. Hence, altogether there are 12 semitones in one octave, which implies that a semitone is equal to $2^{1/12} \approx 1.0595$. In addition to the eight pitches in one octave of the diatonic scale, the semitone gives rise to five additional pitches, or *accidentals*, which account for the five black keys on the piano. An accidental is represented by a sharp (♯) or a flat (♭), which indicates an increase or decrease, respectively, of the base frequency by a semitone. The five accidentals arising from the use of the semitone are C♯ = D♭, D♯ = E♭, F♯ = G♭, G♯ = A♭, and A♯ = B♭.

True ratios constitute *just tuning* for the diatonic scale, whereas the use of the semitone as the uniform frequency interval constitutes *equal-temperament tuning*. As I already suggested, just tuning is not practical since it gives rise to additional pitches that are not present in the diatonic scale. On the other hand, equal-temperament tuning solves many of the problems associated with just tuning, but it is imperfect since it makes some intervals too large and others too small. For example, the major third 5/4 is approximated by four semitones, yielding $(2^{1/12})^4 \approx 1.2599$ (too large), whereas the fifth 3/2 is approximated by seven semitones yielding $(2^{1/12})^7 \approx 1.4983$ (too small). With equal-temperament tuning, note that seven octaves are exactly the same frequency interval as 12 equal-temperament fifths since $2^7 = (2^{7/12})^{12} = 128$. On the other hand, 12 true fifths yield a frequency interval of $(3/2)^{12} = 129.7463$. Hence stretching some fifths to be true requires that others be shrunk even more.

Musicians who play instruments that have infinitely variable tuning, such as fretless string instruments and the trombone, as well as vocalists, have the freedom to stretch or shrink intervals to attain just tuning or any other tuning scheme. However, a pianist is forced to play with the one temperament set by the piano tuner.

There are smaller intervals than the semitone, but the piano can't play them unless it is deliberately tuned that way. The American composer Charles Ives wrote for a piano tuned in quarter tones, so that two notes could be as close as $2^{1/24} \approx 1.0293$. Quarter tones are common in Egyptian music, which isn't confined to the occidental accidentals.

The ultimate advantage of equal- (or approximately equal-) temperament tuning over just tuning is the ability to shift the diatonic scale up or down so that it can begin on any semitone. There are thus 12 such shiftings of the diatonic scale, or *keys*. For example, the key of F is based on the diatonic scale FGAB♭CDEF′, whereas the key of D is based on the diatonic scale DEF♯GABC♯D′. The key of C is the only diatonic scale that has no accidentals.

Since a given melody can be played in any of the 12 keys, it is useful to be able to refer to the notes independently of the key. This is done by means of *solfège*, which uses the syllables do, re, mi, fa, sol, la, ti, do (pronounced doe, ray, me, fah, sole, lah, tea, doe, which may be familiar from *The Sound of Music*). To illustrate solfège, the melody of the theme song from the *Wizard of Oz* can be described as: do, do, ti, sol, la, ti, do, do, la, sol, la, fa, mi, do, re, mi, fa, re, ti, do, re, mi, do. Note that these syllables do not refer to absolute pitches, since "do" can be moved to any of the 12 distinct pitches in an octave.

With the diatonic scale structure based on the semitone interval, composers have the flexibility to *transpose* or shift the music into any one of the 12 keys, and transitions from one key to another are commonplace within a single composition. Because of the sizing of musical instruments, however, different keys tend to retain a distinctive character. For certain kinds of music, such as jazz, the choice of key is largely irrelevant, and the musician is more concerned with the relative pitch values determined by the melody and chord structure rather than the absolute pitch values associated with any particular key. For this purpose, solfège with movable "do" allows a musician to focus on the melody and harmony independently of the key, just as, in statics and dynamics, vectors can be defined independently of a coordinate frame.

Harmonics

Music is both a temporal and spectral art, since the composer is in control of the frequency content and how it changes from instant to instant. This kind of art is extremely rich, since the ear can hear many frequencies at the same time, making it possible to hear low instruments and high instruments simultaneously.

As mentioned earlier, middle C on a piano vibrates at $f_C = 261.625$ Hz. Actually, the sounds of musical instruments are more complicated than that. When a musical instrument such as the piano or guitar sounds middle C, you hear not only that frequency, that is, the *fundamental* at 261.625 Hz, but you also hear many other frequencies at the same time. Usually these frequencies are *harmonics*, or integer multiples, of the fundamental. For example, the harmonics of C are $2f_C$ (an octave above C), $3f_C$ (an octave and a fifth above C), $4f_C$ (two octaves above C), and so on. Although you can't hear the individual harmonics, their relative strength blends together in such a way that a piano sounds like a piano and a trumpet sounds like a trumpet.

Harmonics are present because the piano strings (not to mention the rest of the piano) are actually vibrating at numerous frequencies at the same time. These frequencies are the imaginary parts of the eigenvalues of a matrix approximation of the string dynamics. The motion of the string is a linear combination of the matrix eigenvectors, which correspond to mode shapes of the string vibrating simultaneously, each at its assigned frequency. If you change the relative strengths of the harmonics slightly, the piano might sound like another instrument, such as a trumpet or a flute, where the expansion and compression of air along a duct of variable length replaces the vibration of a string. Electronic synthesizers can imitate virtually any musical instrument by producing signals with prescribed harmonic content.

Most musical instruments are based on strings and ducts so that the spectral content of the sounds is harmonic, at least ideally. No real string is ideal, however, and thus the small amount of bending stiffness in a real piano string contributes to its sound character. In fact, for piano tuning, the octaves are stretched slightly to avoid dissonance with the slightly raised "first harmonic" of the string tuned to a pitch one octave below. A musical instrument based solely on a bending beam would be nonharmonic, but few instruments are designed this way. An exception is the xylophone, but even in this case the wooden blocks are shaped to render their response more harmonic.

If you want to determine the spectral content of a *periodic signal*, which repeats over and over, you can decompose the signal into a *Fourier series*, which is a sum of harmonically related sinusoids. If the signal is not periodic, however, you can think of it as a periodic signal with an infinite period, and the Fourier series becomes the *Fourier transform*. Both the Fourier series and the Fourier transform give you information about the spectral content of the signal.

In any case, what you hear is the response of the musical instrument to some kind of forcing or excitation. Sometimes this forcing is short lived, as in the case of a piano where the string is struck, or it may be persistent, as in the bowing of a violin string. The spectral content of the response that you hear depends on the *transfer function* of the instrument as well as the spectral content of the excitation. An *impulsive input* can excite vibrations at all frequencies, as does a *white noise* input, which is a persistent signal that has equal spectral content at all frequencies. White noise is a pleasant sound generated by flowing water, rain, and wind.

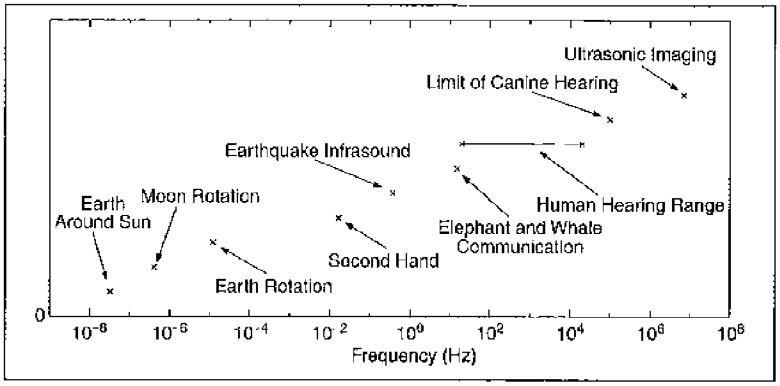

Figure 2. *Mechanical spectrum.*

Rotational Motion and Units

Vibrational motion is closely related to rotational motion, which can also be measured in terms of frequency. The second hand of a clock has a rotational frequency of $1/60 \approx 0.01667$ Hz, but this is better expressed as 1 rpm (*revolution per minute*), the unit preferred by mechanical engineers. Similarly, the 7-inch-diameter vinyl records with the large hole in the center spin at 45 rpm, the 12-inch-diameter vinyl records with the small hole in the center spin at 33-1/3 rpm, and audio CDs spin at 200 to 500 rpm depending on which track is being played. Similarly, a dentist's drill turns at 400,000 rpm, a 2-foot-diameter tire on a car traveling at 60 mph spins at 840 rpm, and the earth rotates at 0.00069 rpm. Although the rotation rate of the earth seems slow, it corresponds to approximately 1000 miles per hour at the equator, where you would weigh 0.35% less than you would at the poles.

On the other hand, control engineers prefer *radians per second* over rpm. Hence 1 Hz, which is exactly 60 rpm, is $2\pi \approx 6.28$ rad/sec. Just remember that of these three units, Hz is the largest, rpm is the smallest (one-sixtieth of a Hz), and rad/sec lies in between (about a sixth of a Hz and about 10 rpm). All these units have the dimensions 1/time because cycle, radian, and revolution are dimensionless ratios of arc length to radius.

Since the unit rad/sec is so widely used in control engineering, we really should have a special name for it. Actually, the name *Steinmetz* has been mentioned in *IEEE Spectrum* from time to time. Its abbreviation Sz has a nice symmetry with Hz and restores to Charles Proteus Steinmetz (1865-1923) the honor that was his before the cps was renamed Hz.

Returning to frequency intervals, although the semitone, third, fifth, octave, and others are useful in music, the only musical interval that engineers use is the octave, mostly for acoustic applications. For larger frequency ranges, control engineers use the *decade*, which is an interval of ten. Note that a decade is a little more than three octaves. Fig. 2 gives the frequencies and frequency ranges for various mechanical motions. Notice that human hearing spans about three decades, or about ten octaves.

Electromagnetic Spectral Content

All the frequency domain examples I have discussed so far concern mechanical motion; however, electromagnetic effects can be described in the frequency domain as well. Electromagnetic waves travel at the speed of light c, which depends on the medium. The speed of propagation is fastest in a vacuum and slower in other media, such as air, water, and glass. The frequency of vibration f depends on the wavelength λ of the electromagnetic phenomenon and the speed of propagation c of the medium according to $f = c/\lambda$.

Long-wavelength electromagnetic waves are radio waves, and their frequencies can range from a few kHz (*kilohertz*) to 300 GHz (1 GHz, or *gigahertz*, is 10^9 Hz). Higher frequencies are emitted by thermal motion, which we call *infrared* radiation, whereas the frequencies of visible light range from 440 THz (red light) to 730 THz (violet light), where 1 THz, or *terahertz*, is 10^{12} Hz. Thus the spectrum of visible light spans slightly less than one octave.

Humans perceive different frequencies within the visible light spectrum as different colors. Unlike the ear, the eye has a nonlinear response to combinations of frequencies. For example, the combination of green and yellow is perceived as blue, even though no blue frequencies are present. If all frequencies are present, then no colors at all are perceived and the image appears white. Fig. 3 gives the frequencies and frequency ranges for various electromagnetic phenomena.

Poles and Roll-Off

Now back to poles. Roughly speaking, a *pole* is a complex number that gives an indication of how a linear dynamical system, which can be described by a linear differential equation, reacts to inputs at various frequencies. Why a complex

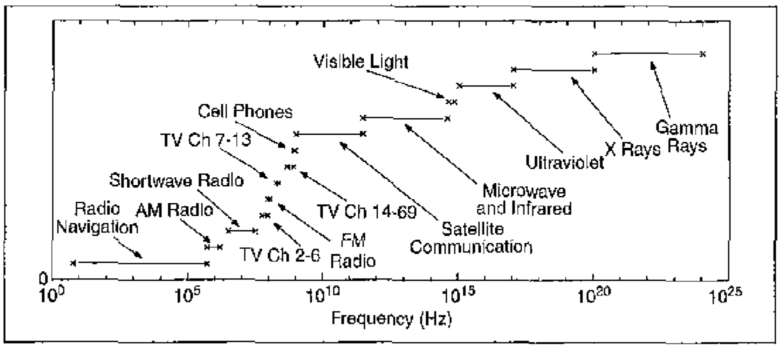

Figure 3. *Electromagnetic spectrum.*

number? First of all, the output response of a stable system to a sinusoidal, or *tonal*, input is eventually (after a transient) sinusoidal. In general, this tonal output will have an amplitude and phase that are different from the amplitude and phase of the input tone. Complex numbers keep track of these differences. The *magnitude* of the complex number determines the ratio of the tonal amplitudes (asymptotic output amplitude divided by input amplitude), whereas the *angle* of the complex number determines the tonal phase difference (phase of the asymptotic output relative to the phase of the sinusoidal input).

The complex numbers that relate the amplitude and phase of the sinusoidal input and output of a linear dynamical system are determined by the *transfer function* of the system, which comes from the *Laplace transform* of the differential equation of the system. At a given tonal input frequency, the magnitude of the transfer function is the *gain* of the transfer function and its phase angle is the *phase shift* of the transfer function. (The Laplace transform and the Fourier transform are closely related. The Laplace transform is used to represent general signals and transfer functions, whereas the Fourier transform is used to determine the response to tonal or random inputs.)

The transfer function of a dynamical system may have both poles and *zeros*. Specifically, each derivative of the state corresponds to one pole, and each derivative of the input corresponds to one zero. Loosely speaking, poles are related to integration and zeros correspond to differentiation.

A dynamical system acts like a *filter* since it reacts differently to tones at different frequencies. The simplest filter, which has one integrator and thus one pole, passes, and possibly amplifies, low-frequency tones but is less responsive to higher frequency tones, which are attenuated. For a periodic or nonperiodic input signal with spectral content at various frequencies, the low-frequency components are left largely alone, whereas the amplitudes of the high-frequency components are attenuated. A one-pole filter *rolls off* at high frequency, and thus it is *low pass*. More elaborate filters can be designed by using more integrators and thus more poles, in which case the high-frequency roll-off rate is proportional to the number of poles. A low-pass filter is useful for separating a signal from noise when the signal has mostly low-frequency components and the noise occurs at high frequency. This is how a filter filters.

High-pass filters can be designed to filter out low-frequency noise. In this case, the filter must have small gain at low frequencies, but must *roll up* to higher gains at higher frequencies. Roll-up is achieved with zeros. A *bandpass* filter, which allows an intermediate range of frequencies to pass, can be designed by combining low-pass and high-pass filters.

A bandpass filter is useful for separating one radio station from another on a crowded dial. For example, to broadcast its signal, each AM station in the United States is allocated a frequency range of bandwidth 10 kHz within the electromagnetic spectrum 535 to 1605 kHz. For amplitude modulation, the radio station modulates the amplitude of a carrier frequency at the center of its 10-kHz frequency range. For example, radio station WAAM in Ann Arbor, MI, modulates the carrier frequency 1600 kHz, which is located at the center of its frequency range of 1595 to 1605 kHz. Since amplitude modulation is essentially a linear process, this electromagnetic bandwidth allows transmission of acoustic frequencies up to 5 kHz. Consequently, AM radio stations cannot broadcast high-fidelity music.

On the other hand, FM radio stations are allocated 150 kHz of electromagnetic spectrum bandwidth. This larger allocation is possible because the electromagnetic spectrum devoted to FM radio is 87.925 to 108.075 MHz (*megahertz* or 10^6 Hz), which is almost 20 times as much bandwidth as the AM radio spectrum. Frequency modulation is a highly nonlinear process that uses the allocated 150-kHz bandwidth to transmit acoustic frequencies up to 15 kHz. Although amplitude modulation would use only 30 kHz to transmit the same acoustic frequency range, frequency modulation is able to use this wider bandwidth advantageously to suppress the effects of noise.

To be useful, a receiver must be sure to allow only the signal from one station to pass through the circuit (whether it's an AM or FM signal). A bandpass filter is used to pass the frequencies of the desired station while attenuating the signals from all other stations, which may be transmitted at higher or lower frequencies than the signal of the desired station. The more poles and zeros a filter has, the sharper the roll-up and roll-off are, and thus the better the ability of the radio to capture one station and reject the others.

The roll-off of a single-pole filter is proportional to the reciprocal of frequency at asymptotically high frequency.

Therefore, at high frequency, the gain of a single-pole filter decreases by a factor of 2 per octave and a factor of 10 per decade. For convenience, filter roll-off is expressed in dB (*decibels*) per frequency interval, which is calculated as 20 times the base-10 logarithm of the ratio of gains of the filter transfer function at the endpoints of the frequency interval. At asymptotically high frequencies, one pole thus rolls off at $20\log 2 \approx 6.0206 \approx 6$ dB per octave or $20\log 10 = 20$ dB per decade. A filter with two poles rolls off asymptotically at 40 dB per decade or 12 dB per octave. Not to slight the other musical intervals, a single-pole filter rolls off asymptotically at $20\log(2^{1/12}) \approx 0.5017 \approx 1/2$ dB per semitone or $20\log(3/2) \approx 3.522$ dB per fifth.

If these roll-off rates are hard to remember, you might prefer to work with a frequency interval that is chosen so that a single-pole filter rolls off asymptotically by exactly 1 dB over the span of the interval. The span of such an interval is $10^{1/20} \approx 1.1220$, which is slightly less than a true musical second $9/8 = 1.125$ or two semitones $(2^{1/12})^2 = 2^{1/6} \approx 1.1225$. An appropriate name for the frequency interval $10^{1/20}$ is the *Armstrong*, after Edwin H. Armstrong (1890-1954), who invented a positive-feedback oscillator that was critical to the development of radio. Armstrong later invented the superheterodyne radio circuit, which survived the transition from tubes to transistors, a remarkable testament to an innovative idea.

To compute asymptotic (high-frequency) roll-off rates, simply note that one pole rolls off at exactly 20 dB per decade or approximately 6 dB per octave or exactly 1 dB per Armstrong. For more poles, simply multiply these numbers by the number of poles. Finally, to compute the interval $I(f_1, f_2)$ of the frequency range f_1 to f_2 in these various dimensions, you can use the formulas

$$\begin{aligned} I(f_1, f_2) &= \log_{10}\frac{f_2}{f_1} \text{ decades} \\ &= \log_2\frac{f_2}{f_1} \text{ octaves} \\ &= 20\log_{10}\frac{f_2}{f_1} \text{ Armstrongs.} \end{aligned}$$

For example, the FM broadcast band, which occupies 20.150 MHz of the electromagnetic spectrum, represents an interval of $20\log(108.075/87.925) \approx 1.79$ Armstrongs. Hence a four-pole filter asymptotically rolls off 7.17 dB over a frequency interval as wide as the FM broadcast band.

Armstrong was also the inventor of high-fidelity FM radio. In fact, the U.S. Federal Communications Commission (FCC) established the frequency range 42 to 50 MHz for FM radio reception, and Armstrong designed receivers and developed FM radio stations in the 1940s. Unfortunately, his work was so threatening to the AM radio industry that, in 1945, the FCC was persuaded to reassign the FM frequency allocation to the fledgling television industry. (However, no television channel was ever established there, which is why there is no Channel 1. Check any TV to confirm this.) Consequently, the half million owners of FM radios that Armstrong designed had nothing to listen to, and the 50 FM radio stations in operation had no one to broadcast to. To add insult to injury, RCA, the leading developer of TV (and owner of many AM radio stations), infringed on Armstrong's FM patents as it adopted FM for the sound transmission system for television. Armstrong was vindicated posthumously in litigation that continued until 1967.

Control Engineering and the Frequency Domain

The frequency domain provides a powerful setting for analyzing the stability and performance of feedback control systems. A feedback controller is essentially a filter whose gain and phase are chosen to modify the response of the controlled system. Large gains are desirable to reduce sensitivity to system uncertainty and to help the system reject disturbances. But there is a danger in the use of feedback, namely, instability, which is the fault of excessive gain or incorrect controller phase. Instability occurs when at least one pole of the closed-loop system is in the wrong place and the closed-loop system has, in effect, infinite gain. A common example of instability is the squeal that occurs when a microphone is placed near a speaker. The acoustic path from speaker to microphone closes a feedback loop that involves a delay, or *phase shift*, which destabilizes the closed-loop system.

Conclusion

When you hear music and see color, you are experiencing the frequency domain. It is all around you, just like the time domain. With this guide to the frequency domain, I hope you will find it a less mysterious and more exciting place.

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Author's Note

As control engineers, we have a special stake in the frequency domain. It is hard to imagine control system analysis confined to the time domain. In this article, I have attempted to show that the frequency domain plays an important role in numerous areas of science and technology, from music to radio to vision. I came from a musical family, and I spent a lot of time learning to play a musical instrument (the clarinet). But my fascination with mathematics and technology and the desire to know *how* things work led me to a career in engineering. Like musicians, engineers enhance the quality of life in innumerable ways, and it is unfortunate that engineering is taken for granted and remains invisible to society as a whole. This is something to think about the next time you listen to jazz on FM.