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The diversity of life is amazing. Between quasi-life (viruses) and us, we share the Earth with some pretty exotic living things. To bring organization to this diversity, biologists have devised a systematic classification that neatly arranges all of these entities—including pineapples, periwinkles, and porcupines—into a logical hierarchy based on form and function.

Perhaps we need to do the same thing in systems and control theory to get a better handle on nonlinear systems. One approach might be to list some properties of linear systems and then ask whether or not these properties carry over to nonlinear systems. By producing examples of nonlinear systems for which these cherished properties fail, we can begin to appreciate the diversity of nonlinear systems. We could do the same thing for non-elephants by looking for animals that do not have tusks or trunks.

Existence is the first property to fail, since it's easy to construct nonlinear differential equations whose solutions escape to infinity on a finite interval. Easier to defeat is superposition; in fact, superposition would be surprising if it were to hold even for a small set of initial conditions and inputs. Stability also shows stark differences. In a linear system, the set of equilibria is always a subspace; in the most extreme case, only the origin. In a nonlinear system, however, the set of equilibria may consist of a finite number of isolated points. In a linear system, one equilibrium is Lyapunov stable if and only if every equilibrium is Lyapunov stable, while if

Digital Object Identifier 10.1109/MCS.2009.934590

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an equilibrium is locally asymptotically stable then it is globally asymptotically stable and thus the only equilibrium. In contrast, a nonlinear system can have multiple isolated equilibria with different types of stability. In fact, continuity can require that multiple equilibria be present, as shown by attempting to comb the fuzz on a tennis ball.

The speed of convergence is also a critical difference. The trajectories of a linear system converge exponentially, whereas the trajectories of a nonlinear system can converge either more quickly or more slowly than exponentially. It's the "slower than exponential" case that causes problems when nonlinear systems are interconnected.

Furthermore, every asymptotically stable linear system is bounded-input, bounded-output (BIBO) stable, whereas a nonlinear system with a globally asymptotically stable equilibrium need not be BIBO stable. Hence, while an asymptotically stable linear system has the wonderful property that a bounded periodic input produces an output that "converges" to a periodic output, no such statement can be made about nonlinear systems.

Beyond the stability of equilibria, a nonlinear system can have an entirely different type of behavior, namely, a limit cycle. In addition, nonlinear systems can exhibit chaos and strange attractors, for which linear systems have no analogue.

Instead of searching for qualitative distinctions, another way to examine nonlinear systems is to build them up from simple blocks, LEGO style. For example, we can view $\dot{x} = f(x) + g(x)h(u)$ as an integrator in feedback with the nonlinear function f and with the input u entering through nonlinear operations. This approach allows us to classify nonlinear systems in terms of the properties of the static maps f, g, and h. For example, the effect of the input u is weakened when the function g is too small; likewise, the effect of u becomes problematic when $h(u)$ cannot produce all values that we might like, the most common case occurring when h is a saturation function. Lifetimes of research have been devoted to this important nonlinearity.

Whereas saturation manifests itself only when u is large, other nonlinearities are more local in nature. The deadzone nonlinearity is a typical example, giving rise to backlash and hysteresis. Like saturation, the deadzone has been studied by generations of researchers.

Dennis Bernstein and Anthony D'Amato visiting Musée du Mur de l'Atlantique in Ouistreham, France.

But saturation and deadzone are child's play compared to the convective operator in the Navier-Stokes equation, which, together with the viscosity term, makes turbulence possible. Again, lifetimes devoted to a critical nonlinearity.

A taxonomy of nonlinearities would help us assess the rich landscape of control systems that are not linear. Saturation, deadzone, and the convective operator remind us that the nonlinear kingdom has no shortage of non-elephants worthy of yet more generations of control researchers and practitioners. Although sensors, actuators, and computers will continue to get faster and better, saturation, deadzone, and turbulence will be with us forever.

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Positive Resistance

Still, Black had no easy time convincing others at Bell Labs of the utility of his idea. He recalled that Jewett supported him, but that the director of research, Harold Arnold, refused to accept a negative-feedback amplifier and directed Black to design conventional amplifiers instead. Black had similar difficulties with the U.S. Patent Office. His application for a "Wave Translation System," originally filed in 1928 was not granted until 1937. To a generation of engineers who had struggled to make the vacuum tube amplify at all, throwing away the hard-won gain seemed absurd.

More important, no one could understand how an amplifier's output could be fed back to its input without a progressive, divergent series of oscillations. Bell engineers at the time found it difficult to make a high-gain amplifier *without* feedback. Subtle, uncontrolled feedback paths would arise through unintentional effects such as stray capacitance between wires, or even between elements within the tube itself, and cause the amplifier to go into "parasitic oscillation" or "singing" (much like the whistling in a poorly tuned public address system). In 1924, for example, two BTL engineers, H.T. Friis and A.G. Jensen, studied what they called "feed-back or regeneration" as it occurred through a tube, noting that "it makes the total amplification vary irregularly in a very undesirable manner and also makes the set "sing" at certain frequencies. Black's work went against the grain for experienced amplifier designers: they sought to eliminate feedback, not to incorporate it.

—David A. Mindell, "Opening Black's Box, Rethinking Feedback's Myth of Origin," *Technology and Culture*, vol. 41, pp. 419–420, July 2000.