Colorful Curvature

Nonlinearities provide a never-ending challenge to systems and control. We savor linear systems since we have a good understanding of how they behave, and there is much we can do to analyze and predict their performance. As we move away from linearity, we need to scrutinize each nonlinearity that we encounter to assess its benefit or detriment in terms of what we wish to achieve.

The first issue to consider is the way in which the nonlinearity is interconnected with the linear dynamics. For example, consider a cascade interconnection. If the nonlinearity precedes the linear system, then the system is Hammerstein; if it occurs after the linear system, then the system is Wiener. Nonlinear actuation makes a system Hammerstein, while nonlinear sensing makes it Wiener. In both cases the intermediate signal is assumed to be unknown. It seems that Wiener systems are more difficult to deal with than Hammerstein systems for the simple reason that knowing the input to the nonlinearity is more useful than knowing its output. Of course, all systems

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Xiaohua Xia (center) with his son Fuxun and wife Xiaolan on a trip to Shangri-LaYunnan.

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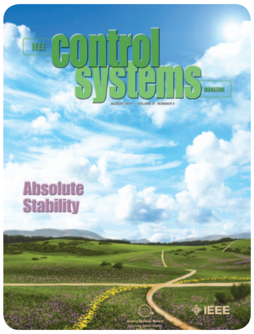
have Hammerstein and Wiener characteristics in practice since all sensors and all actuators have nonlinear behavior, for example, due to saturation.

Although connecting a nonlinearity to a linear system in cascade presents challenges, things get a lot more interesting when the nonlinearity is connected in feedback. It's a standard trick in systems theory to extract a component of a system and represent it as if it were in a feedback loop. The systems and control community is adept at this practice since one or our main interests is in feedback control, but the same technique can be useful in virtually all modeling venues.



The shape of a nonlinearity determines the potential degradation of system performance. For a saturation nonlinearity, the main impact is due to the loss of range, that is, the fact that the nonlinearity is not onto. By operating the system so that the sensor and actuator range limits are never reached, the system can operate in a "linear" manner. For command-following problems, this may be achievable by using a reference governor to modify the command on the fly, effectively by restricting the allowable commands. For problems with disturbances, however, it may be impossible to avoid amplitude and rate limits. In these cases, the actuator does not provide the desired input to the system, which can result in integrator windup and instability.

A saturation nonlinearity is typically increasing and, aside from the saturation regions, is one-to-one and odd, which are properties that it shares with linear functions. Most importantly, a one-to-one function is invertible over its range. For other nonlinearities, we're not so lucky. A deadzone nonlinearity, which may arise due to stiction, provides zero output during operation near the equilibrium and thus is not one-to-one. Like a function with constant regions, a quadratic nonlinearity is not one-to-one, but, more inadequately, is not onto since it cannot produce negative inputs. Physically, an electromagnetic has a quadratic nonlinearity since the force it applies is proportional to the


Hartmut Logemann enjoying a pint of real ale at the Star Inn in Bath, United Kingdom.


Bayu Jayawardhana with his daughter Felicia on the island of Schiermonnikoog in The Netherlands.


Jin Yan visiting Stone Mountain in Georgia.


Jin Yan with her parents and (far right) her aunt.


Rick and Rebecca Hindman with their daughters Jamie (left) and Cate and their son Cooper.

square of the current; changing the sign of the current reverses the poles of the electromagnet and thus allows negative inputs when the target is another magnet, but the quadratic nonlinearity remains.

The shape of a nonlinearity determines the potential degradation of system performance.

Dual to the case of actuator nonlinearities is the issue of sensor nonlinearities. In this case, the loss of onto-ness is not critical. Instead, the crucial property is one-to-one-ness, which determines the ability to distinguish outputs of the dynamical system. In many applications, such as an oxygen sensor in an engine, the sensor provides only binary signals, that is, zeros and ones, which represent whether the sensed quantity is below or above a given threshold. In these cases, it is not possible to invert the nonlinearity to determine the output of the linear system.

Beyond issues of structure (Hammerstein, Wiener, or feedback) and shape (one-to-one-ness and onto-ness), a nonlinearity may be uncertain. For example, we may not know the precise slope (that is, the scale factor) and range of a saturation nonlinearity, and we might not know the width of the dead zone of a deadzone nonlinearity. In fact, all of these characteristics may change with operating conditions, which indicates the presence of yet deeper nonlinearities. Consequently, when we analyze a system with a nonlinearity, it is useful to focus on the key characteristics of the nonlinearity rather than its precise shape.

The article by Bayu Jayawardhana, Hartmut Logemann, and Eugene Ryan delves into these problems by revisiting one of the most fundamental streams of systems theory, namely, absolute stability theory. In absolute stability theory, a static, possibly time-varying, nonlinearity is assumed to be interconnected by feedback with a linear system. The nonlinearity is assumed to be unknown other than certain features, such as the sector to which it belongs and whether or not it is known to be increasing. A sector is the region bounded by two lines of different slope passing through the origin. A nonlinearity belongs to the sector if each of its values lies between the lines that bound the sector. The path of flowers on the cover is reminiscent of this property.

In the early days of modern control theory, a problem of great interest—known as the Lur'e problem—was to determine conditions under which the feedback interconnection consisting of a linear dynamic system and a nonlinearity has a stable equilibrium. The Aizerman conjecture expressed the hope that a sufficient condition for stability for all nonlinearities residing in a specified sector is that stability holds for all *linear* functions inside that sector. Unfortunately, counterexamples showed that this conjecture is false. Later, the Kalman conjecture weakened the conjecture by restricting the nonlinearities to have bounded slope. This conjecture also proved to be false. However, the later development of the circle and Popov criteria provided valid sufficient conditions under which the Aizerman conjecture is true. This subject is now known as absolute stability theory, which in turn provides the foundation for modern robust control theory.

The article on absolute stability theory in this issue of *IEEE Control Systems Magazine* (*CSM*) revisits this topic by introducing various elements that are not found in the classical theory. For instance, the Aizerman conjecture is revisited within the context of differential equations cast in terms of complex variables. Within this context, the conjecture is true. Although surprising at first, the distinction between real analysis and its much richer counterpart of complex analysis arises in small-gain theory and the distinction between real and complex parameters. In particular, a "complex parameter" possesses a nontrivial phase angle (that is, neither zero nor 180°) and thus can be viewed as capturing both the magnitude and phase of the frequency response of a dynamical system. As another extension, the authors consider set-valued maps, which greatly enrich the power of the theory, while facilitating the treatment of discontinuous nonlinearities, quantization, and hysteresis maps. Finally, the article also considers the effect of exogenous signals, such as commands and disturbances. All together, this article provides an updated and re-energized treatment of a classical topic that is central to systems and control theory.

The second feature article of this issue, by Jin Yan, Jesse Hoagg, Richard Hindman, and Dennis Bernstein, focuses on a well-known feature of longitudinal aircraft dynamics. Specifically, the article considers the effect of an elevator step deflection on the altitude change of the aircraft. The traditional textbook explanation is that the elevator step deflection provides a pitch moment to the aircraft, which rotates the aircraft and subsequently causes an increase in lift due to the increased angle of attack of the wings of the aircraft. However, the elevator itself is a wing, and the elevator step deflection decreases the lift of the elevator wing, which causes the aircraft to rotate and in turn causes certain points on the aircraft to drop initially in altitude. From the point of view of control, the dynamical characteristic of initially moving in a direction opposite to the asymptotic direction constitutes initial undershoot, which is known to be a result of an odd number of positive zeros, that is, real nonminimum-phase (NMP) zeros A tutorial on NMP zeros is given in an article by Jesse Hoagg and Dennis Bernstein in the June 2008 *CSM*.

The role of NMP zeros in the initial undershoot in longitudinal aircraft dynamics is a staple of undergraduate control textbooks, where it provides an illustrative example of the effect of NMP zeros. What these treatments lack is a full description of the aircraft motion during initial undershoot, including both vertical and horizontal motion. To complete this picture, the article in this issue shows that initial undershoot does not apply uniformly to the entire aircraft; instead, the aircraft has a center of rotation about which the aircraft pivots. In effect, the aft portion of the aircraft initially decreases in altitude, whereas the nose initially increases in altitude. This analysis requires the distinction between the instantaneous velocity center of rotation and the instantaneous acceleration center of rotation, which are possibly different points on the aircraft. A distinction is also made with the concept of center of percussion. Finally, the article analyzes the step response of the linearized system and its initial slope and curvature to an elevator step deflection. The main objective is to relate the step response of the aircraft from an elevator step deflection to the vertical and horizontal velocities at points along the aircraft including the instantaneous acceleration center of rotation. In particular, the dynamics of the aircraft at the instantaneous acceleration center of rotation are shown to be related to vanishing zeros of the relevant transfer functions. The article thus brings aircraft kinematics and dynamics together with classical linear systems analysis to provide an analysis of aircraft motion that owes it features to zeros.

This issue of *CSM* also brings the usual columns, including "25 Years Ago," "CSS News," "Conference Reports," and "People in Control," with interviews with Herb Rauch and Rolf Isermann. It is with great sadness that we report that Herb Rauch passed away during the time that this issue was in press. Herb was a former editor-in-chief (EIC) of this publication; in fact, he was the second EIC, succeeding Mo Jamshidi. We are extremely grateful to Marta Rauch, Herb's daughter, for facilitating this interview. An obituary for Herb will be published in a later issue of *CSM*.

If you look at "Random Inputs" in this issue you will see a timeline retrospective of some of the CDCs leading up to this year's 50th anniversary CDC. This exciting event is only a few months off. Now's the time to start booking flights and hotel. It's a unique event that you won't want to miss.

Dennis S. Bernstein

Proto Garmin

Histories of new machines tend to focus on the process of invention and to suggest that the market is driven by research and development. This is usually not so, even in the case of inventions that in retrospect clearly were fundamental to contemporary society: the telegraph, the telephone, the phonograph, the personal computer. When such things first appear, creating demand is more difficult than creating supply. At first, Samuel Morse had trouble convincing anyone to invest in his telegraph. He spent five years "lecturing, lobbying, and negotiating" before he convinced the US Congress to pay for the construction of the first substantial telegraph line, which ran from Washington to Baltimore. Even after it was operating, he had difficulty finding customers interested in using it. Likewise, Alexander Graham Bell could not find an investor to buy his patent on the telephone, and so he reluctantly decided to market it himself. Thomas Edison found few commercial applications for his phonograph, despite the sensational publicity surrounding its discovery. He and his assistants had the following commercial ideas a month after the phonograph was first shown to the world: to make a speaking doll and other toys, to manufacture speaking "clocks…to call the hour etc., for advertisements, for calling out directions automatically, delivering lectures, explaining the way," and, almost as an afterthought at the end of the list, "as a musical instrument." In the mid 1970s, a prototype personal computer, when first shown to a group of MIT professors, seemed rather uninteresting to them. They could think of few uses for it, and they suggested that perhaps it would be most useful to shut-ins.

—David E. Nye, *Technology Matters, Questions to Live With,* The MIT Press, Cambridge, Massachusetts, pp. 40-41.