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SHORT COMMUNICATION

Comments on 'Output feedback adaptive command following and disturbance rejection for nonminimum phase uncertain dynamical systems'

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SUMMARY

We provide numerical examples and analysis to show that the adaptive controller given by Theorem 3.1 of Yucelen *et al.* [1] may fail to stabilize plants under the stated conditions. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: adaptive control; convergence; reference model

1. INTRODUCTION

Theorem 3.1 of Yucelen *et al.* [1] provides an adaptive controller for a model reference adaptive control problem. This result implies that the error $e(t) \triangleq x_f(t) - x_m(t)$ converges to zero, where $x_f(t)$ is the state of a nonminimal-state-space realization of the plant and $x_m(t)$ is the state of the reference model. In addition, Theorem 3.1 implies that the state $x_p(t)$ of the plant in its minimal-state-space realization is bounded. In [1], Theorem 3.1 is illustrated by several numerical examples. For each example, the error $e(t)$ is shown to converge to zero, and the plant output $y(t)$ is shown to follow the command.

Theorem 3.1 is of interest because the controller does not require (1) knowledge of the sign of the high-frequency gain of the plant; (2) any assumptions on the locations of the poles or zeros (e.g., the plant need not be minimum phase); or (3) knowledge of any poles or zeros of the plant. Stabilization with this limited level of modeling information is shown in [2] to be possible if the order of a stabilizing controller is known. However, to our knowledge, Yucelen *et al.* [1] are the first to provide an explicit controller. As a point of comparison, the controller in [3] does not require knowledge of the sign of the high-frequency gain, but is limited to minimum-phase systems.

In this note, we modify Example 5.3 of Yucelen *et al.* [1] in several ways, and we show that the plant output $y(t)$ may be unbounded, and thus the state $x_p(t)$ may be unbounded. We trace this observation to an error in the proof of Theorem 3.1.

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2. NUMERICAL EXAMPLE

Example 5.3 of Yucelen *et al.* [1] is the plant

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -20 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad C = [-1 \quad 0], \quad x(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}. \tag{3}$$

In addition, the command $r(t)$ driving the modified reference model (47) of Yucelen *et al.* [1] is identically zero. The poles of A are given by $0.25 \pm 4.4651\jmath$. Figure 1 verifies the numerical results shown in Figure 5 of Yucelen *et al.* [1]. In particular, the plant output $y(t)$ converges to zero.

We now consider two variations of (3). For both examples and unless indicated otherwise, we use exactly the same reference model, adaptive parameters, initial conditions, and command that are used for Example 5.3 in [1].

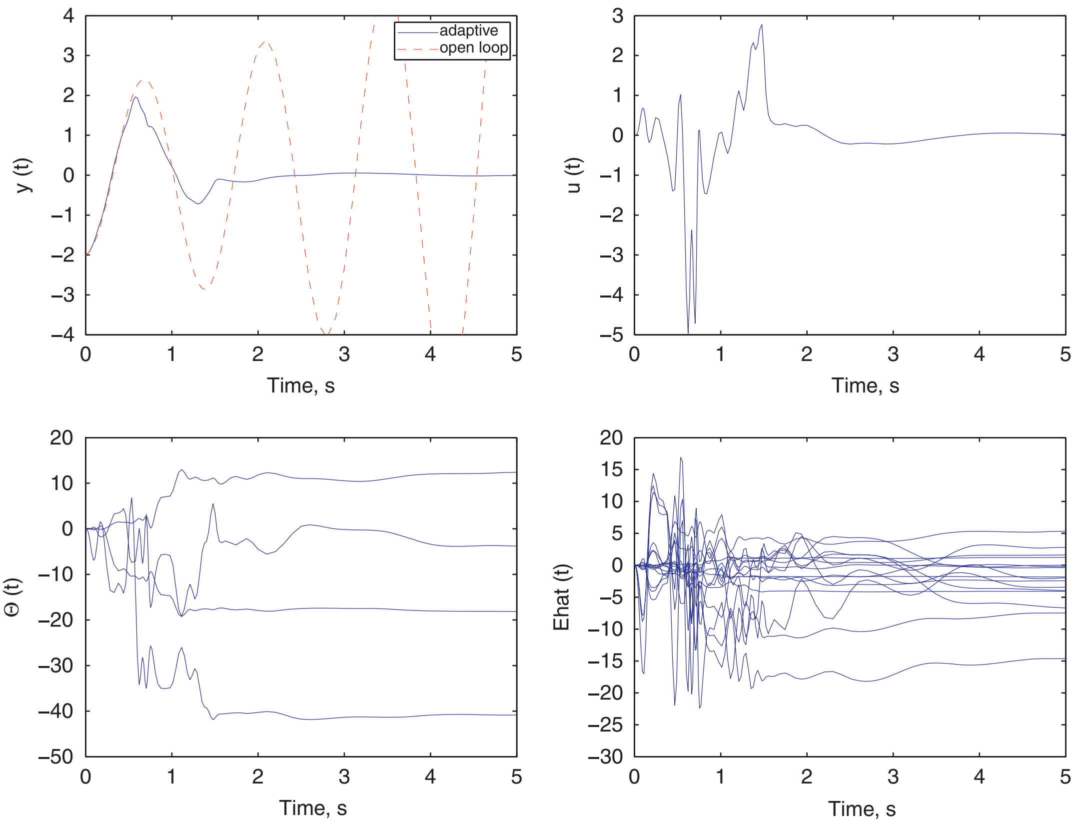

Figure 1. Closed-loop response for Example 5.3 of Yucelen *et al.* [1] using the data given by (3). These plots verify the numerical results shown in Figure 5 of Yucelen *et al.* [1].

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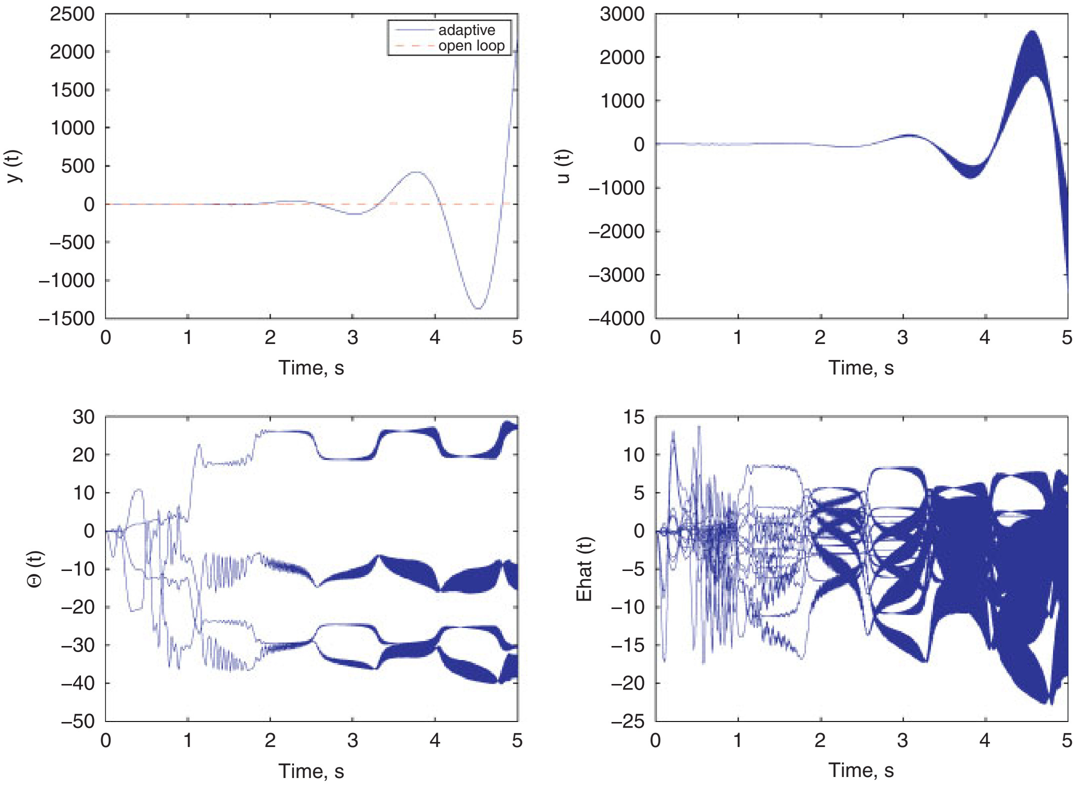

Figure 2. Closed-loop response for the adaptive controller presented in Example 5.3 of Yucelen *et al.* [1] with B replaced by $-B$, which changes the sign of the high-frequency gain. This information is not required by the assumptions of Theorem 3.1 of Yucelen *et al.* [1]. This figure indicates that $y(t)$ is unbounded.

Example 1
Let

$$A=\begin{bmatrix}0 & 1\\ -20 & 0.5\end{bmatrix},\quad B=\begin{bmatrix}2\\ -1\end{bmatrix},\quad C=[-1\quad 0],\quad x(0)=\begin{bmatrix}2\\ -1\end{bmatrix}. \tag{4}$$

This example is identical to (3) except that B is replaced by $-B$, which changes the sign of the high-frequency gain. Application of the adaptive controller of Theorem 3.1 of Yucelen *et al.* [1] requires no modification in this case. The numerical results shown in Figure 2 indicate that $y(t)$ is unbounded.

Example 2
Let

$$A=\begin{bmatrix}0 & 1\\ -20+\alpha & 0.5\end{bmatrix},\quad B=\begin{bmatrix}-2\\ 1\end{bmatrix},\quad C=[-1\quad 0],\quad x(0)=\begin{bmatrix}2\\ -1\end{bmatrix}, \tag{5}$$

where α is a nonnegative constant. This example is identical to (3) except that the parameter α modifies the locations of the poles so that their real part is constant but their imaginary parts decrease in magnitude as α increases. In particular, both poles become real and equal for $\alpha=19.9375$. Figure 3 indicates that $y(t)$ is unbounded for $\alpha=15$. Finally, Figure 4 shows the upper bound for the values of the tuning parameter Γ_{E} for which $y(t)$ is bounded. As shown in Figure 4, this range decreases as α increases. In particular, for $\alpha\geqslant 15.6$, there are no values of Γ_{E} for which $y(t)$ converges to zero.

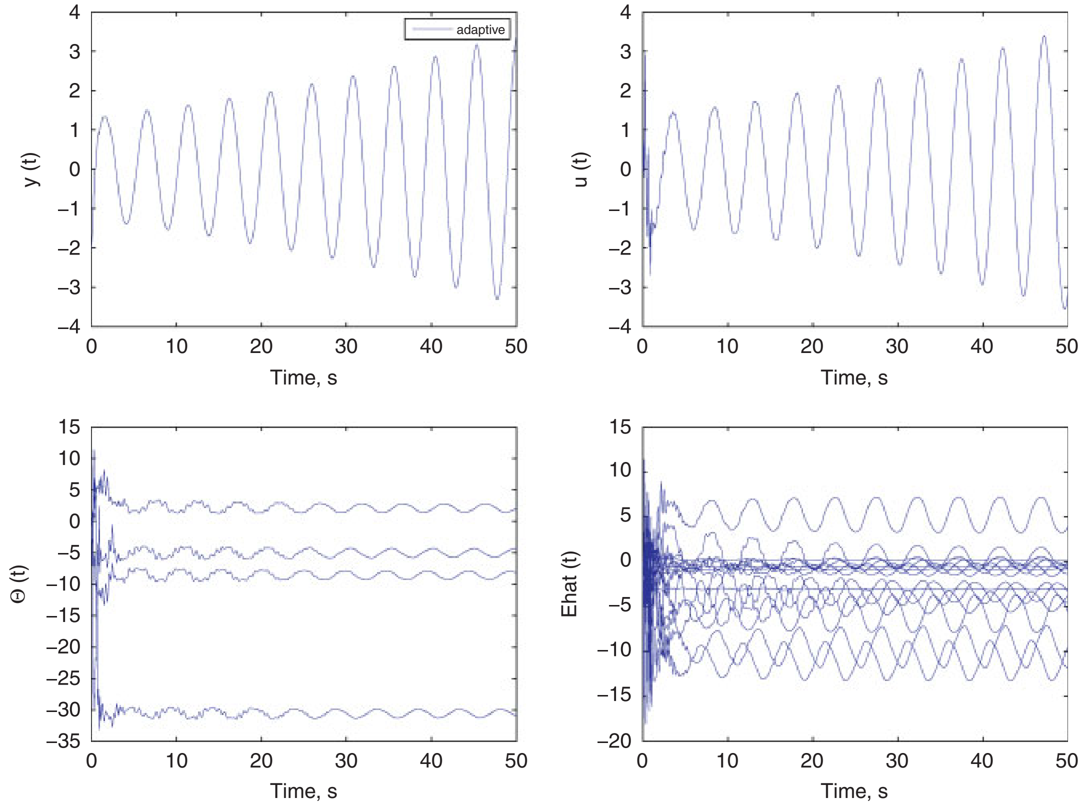

Figure 3. Closed-loop response for the adaptive controller presented in Example 5.3 of Yucelen *et al.* [1], where the imaginary parts of the eigenvalues of A are decreased from ± 4.47 to ± 2.22. These plots indicate that $y(t)$ is unbounded.

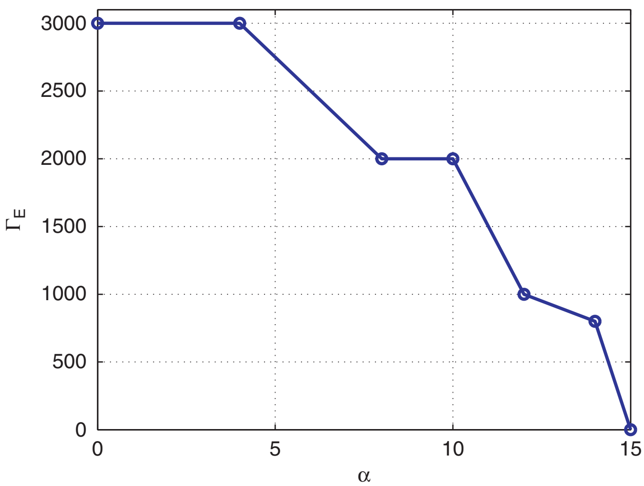

Figure 4. The range of the tuning parameter Γ_E for which the plant output $y(t)$ is bounded as a function of α. Note that Γ_E decreases as α increases. Furthermore, for $\alpha \geqslant 15.6$, there are no values of Γ_E for which $y(t)$ converges to zero.

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3. DISCUSSION

The model reference adaptive control problem in [1] introduces a reference model in Equation (44). However, the error signal $e(t)$ used by the adaptive controller of Theorem 3.1 refers to the modified reference model (47), which differs from (44) by the feedback injection term $\eta(t) \triangleq -\hat{E}(t)x_f(t)$, where $\hat{E}(t)$ is given by (48) of Yucelen *et al.* [1]. The last sentence of Theorem 3.1 states: 'Furthermore, $x_p(t)$, $t \geqslant 0$, satisfying (1) is bounded for all $x_p(0) \in \mathbb{R}^n$.' This statement is discussed in the previous paragraph of the proof of Theorem 3.1, which refers to the asymptotic stability of (47). However, there is no proof that any equilibrium of (47) is asymptotically stable or that its state $x_m(t)$ is bounded. More precisely, the assumption that the dynamics matrix A_m of the modified reference model (47) is asymptotically stable is not sufficient to conclude that $x_m(t)$ is bounded. This discrepancy is due to the feedback injection term $\eta(t)$, which is included in the modified reference model (47). Consequently, $x_m(t)$ may be unbounded, and thus, the state $x_f(t)$ of the nonminimal state-space realization may follow the unbounded reference model state $x_m(t)$, while the error $e(t)$ is guaranteed to converge to zero.

4. CONCLUSIONS

Modifications of Example 5.3 of Yucelen *et al.* [1] show that the output $y(t)$ of the controlled plant may be unbounded. Theorem 3.1 of Yucelen *et al.* [1] guarantees that the error defined in terms of a modified reference model converges to zero. However, this modified reference model may have unbounded behavior, thus explaining why the modifications of Example 5.3 exhibit unbounded response. Consequently, the statement in Theorem 3.1 of Yucelen *et al.* [1] that the state $x_p(t)$ is bounded is false.

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