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Estimation of the Eddy Diffusion Coefficient Using Total Electron Content Data

Ankit Goel, Aaron Ridley, and Dennis S. Bernstein

***Abstract*— Large-scale physics models typically involve unknown parameters that are either inaccessible or representative of complex, unknown dynamics. The focus of this paper is on the estimation of the eddy diffusion coefficient (EDC), which represents the dissipative effect of turbulent mixing in the lower atmosphere. Retrospective cost parameter estimation is used to estimate EDC using measurements of the total electron content. The novel element of the paper is concurrent optimization of the filter that defines the retrospective cost function. Since this optimization problem is biquadratic, an alternating convex search algorithm is used.**

I. Introduction

In many applications of science and engineering, a mathematical description of a system is available from first principles, but the values of the parameters describing the system are unknown or uncertain. This may be due to practical limitations of measurement. For example, estimation of a friction coefficient must rely on a dynamic model due to the fact that direct measurements of the friction force are not available. In such cases, we say that the parameter is *inaccessible*.

In addition to inaccessibility, a parameter may be unknown due to its *representational* nature. For example, lift and drag coefficients are used as an alternative to integrating the pressure field over the surface of an airfoil.

The present paper is concerned with estimation of such uncertain parameters arising in a model of the upper atmosphere, which extends from about 60 km to 1000 km. The basis of this model is the Navier-Stokes equations coupled with electrodynamics equations. The source terms driving the ionosphere, such as the solar flux and the Earth's magnetic field, as well as coupling effects such as viscosity and diffusion, are constructed statistically [1] or empirically to match the predicted output of the model with measurements from various ground stations and satellites in orbit.

It is well known that turbulent diffusion affects energy deposition and transport of chemical species in the upper atmosphere. Eddy diffusion coefficient (EDC) is used to model this mixing process. However, no measurement device can directly measure EDC, and no first-principles physics model is available to determine its value. Nevertheless, models of the upper atmosphere routinely simulate the turbulent mixing at various altitudes by using shape functions parameterized by EDC [2].

In this paper, we apply retrospective cost parameter estimation (RCPE) to the problem of estimating EDC. RCPE is a specialization of retrospective cost model refinement (RCMR), which can be used to identify the dynamics of an inaccessible subsystem [3], [4]. RCPE was applied to atmospheric models in [5]–[7].

The estimates of EDC obtained in the present paper are based on measurements of the total electron content (TEC). TEC, defined as the total number of electrons integrated along a vertical column of one meter squared cross section, is a widely used quantity to describe the ionosphere. TEC is measured in TEC unit (TECU), where 1 TECU $= 10^{16}$ electrons/m^2. The free electrons in the ionosphere cause delay in the propagation of radio waves in the atmosphere. Ionospheric irregularities also cause random amplitude and phase fluctuations in the signals [8]. TEC measurements are thus used to correct positioning errors and improve the accuracy of the Global Navigation Satellite Systems [9]. Consequently, the physics of TEC are widely studied [10]–[13]. TEC is routinely used to estimate the state of the ionosphere and thermosphere [14]–[16]. The use of TEC measurements to estimate EDC is a novel element of the present paper.

The present paper extends RCPE by simultaneously optimizing the filter $G_{\rm f}$, which is used to define the retrospective cost function [17], [18]. Within the context of adaptive control, $G_{\rm f}$ is chosen to capture knowledge of the leading sign, relative degree, and nonminimum-phase zeros. For parameter estimation within the context of a high-order nonlinear model such as the atmospheric model considered in this paper, there are no clear guidelines for constructing $G_{\rm f}$. Consequently, the present paper updates $G_{\rm f}$ online through a combined optimization procedure. A related optimization technique within the context of adaptive control was considered in [19], [20].

The optimization problem involving both the unknown parameters and the filter $G_{\rm f}$ turns out to be biquadratic. Although this biquadratic function is strictly convex in each variable separately, it is highly nonconvex as a joint function of its arguments. Unfortunately, numerical algorithms that converge globally to the global minimizer of biquadratic functions are not available [21], and this poses a technical challenge within the context of parameter estimation for large-scale physics models. In the present paper we apply an alternating convex search method that is guaranteed to converge to a local minimizer.

Ankit Goel is with the Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109, USA, ankgoel@umich.edu.

Aaron Ridley is with the Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI 48109, USA, ridley@umich.edu.

Dennis S. Bernstein is with the Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109, USA, dsbaero@umich.edu.

The paper is organized as follows. In Section II, we describe GITM and the role played by EDC. In Section III, we formulate the problem of estimating an unknown parameter in a dynamic system, in Section IV, we present the RCPE algorithm. In section V, an algorithm to optimize the biquadratic retrospective cost function is presented. A low-dimensional system illustrating the application of RCPE is presented in Section VI. Estimation of EDC using TEC is presented in VII. Finally, in Section VIII, we summarize the results of the paper and discuss future directions.

II. Global Ionosphere-Thermosphere Model

GITM is a computational code that models the thermosphere and the ionosphere of the Earth as well as that of various planets and moons by solving coupled continuity, momentum, and energy equations [22]. By propagating the governing equations, GITM computes neutral, ion, and electron temperatures, neutral-wind and plasma velocities, and mass and number densities of neutrals, ions, and electrons. GITM uses a uniform grid in latitude with width $\frac{2\pi}{n_{\rm lat}}$ rad, where $n_{\rm lat}$ is the number of grid points. In longitude and altitude, GITM uses a stretched grid to account for temperature and density variations.

GITM is implemented in parallel, where the computational domain (the atmosphere from 100 km to 600 km) is divided into blocks. Ghost cells border the physical blocks to exchange information. GITM can be run in one-dimensional mode, where horizontal transport is ignored, or in global three-dimensional mode. Furthermore, GITM can be run at either a constant or a variable time step, which is calculated by GITM based on the physical state and the user-defined CFL number in order to maintain numerical stability. To initialize GITM, neutral and ion densities and temperatures for a chosen time are set using the Mass Spectrometer and Incoherent Scatter radar (MSIS) model [23] and International Reference Ionosphere (IRI) [24].

The model inputs for GITM are 10.7 cm solar radio flux (F10.7 index), hemispheric power index (HPI), interplanetary magnetic field (IMF), solar wind plasma (SWP), and solar irradiance, all of which are read from a text file containing the time of measurements and the measured values. These signals are available from various terrestrial sensor platforms.

The present paper focuses on estimation of the eddy diffusion coefficient (EDC), which plays a key role in GITM and many computational codes simulating the upper atmosphere. Specifically, EDC models turbulent mixing in the upper atmosphere [25]. According to mass continuity and the momentum equations, the altitude profile of the neutral constituents changes from full mixing at lower altitudes (below 150 km), where turbulent mixing prevails, to molecular diffusion at higher altitudes (above 150 km). The value of EDC represents the intensity of the turbulent mixing, which is a key factor in determining the free-electron density in the ionosphere, usually measured by total electron content (TEC). In the present paper, EDC is estimated by using measurements of TEC.

III. Parameter-Estimation Problem

Consider the multi-input, multi-output (MIMO) discrete-time physical system model

$$x(k+1) = f\left(x(k), u(k), \nu\right) + w_1(k), \tag{1}$$
$$y(k) = h\left(x(k), u(k), \nu\right) + w_2(k), \tag{2}$$

where $x \in \mathbb{R}^{l_x}$ is the state, $u \in \mathbb{R}^{l_u}$ is the input, $y \in \mathbb{R}^{l_y}$ is the measured output, $w_1 \in \mathbb{R}^{l_x}, w_2 \in \mathbb{R}^{l_y}$ are the process and measurement noise, and $\nu \in \mathbb{R}^{l_\nu}$ is the unknown parameter.

Next, we consider the estimation model

$$\hat{x}(k+1) = f\left(\hat{x}(k), u(k), \hat{\nu}(k)\right), \tag{3}$$
$$\hat{y}(k) = h\left(\hat{x}(k), u(k), \hat{\nu}(k)\right), \tag{4}$$

where $\hat{x}(k)$ is the computed state, $\hat{y}(k)$ is the output of the estimation model, and $\hat{\nu}(k)$ is the output of the parameter estimator at step k. The parameter estimator is updated by minimizing a cost function based on the performance variable

$$z(k) \stackrel{\triangle}{=} \hat{y}(k) - y(k) \in \mathbb{R}^{l_y}. \tag{5}$$

The problem objective is to estimate ν using measurements of u and y. The parameter-estimation problem is represented by the block diagram in Figure 1.

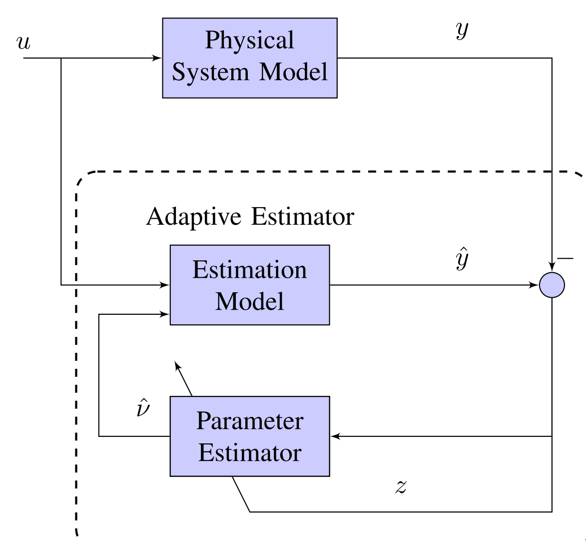

Fig. 1: Parameter-estimation problem. The physical system modeled by the physical system model (1), (2) is driven by u and produces measurements y. The adaptive estimator consists of the estimation model (3), (4), which is driven by measurements of u and where the parameter estimate $\hat{\nu}$ is updated by the parameter estimator, which minimizes the error signal z.

IV. Retrospective Cost Parameter Estimation

In this section, we present the RCPE algorithm used to compute the estimate $\hat{\nu}$ of the unknown parameter ν.

A. Parameter Estimator

We consider a parameter estimator represented by an ARMA model with a built-in integrator. The parameter estimate $\hat{\nu}$ is thus given by

$$\hat{\nu}(k) = \sum_{i=1}^{n_\mathrm{c}} P_i(k)\hat{\nu}(k-i) + \sum_{i=1}^{n_\mathrm{c}} Q_i(k)z(k-i) + R(k)g(k), \tag{6}$$

where

$$g(k) = g(k-1) + z(k-1), \tag{7}$$

and $P_i(k) \in \mathbb{R}^{l_\nu \times l_\nu}$, $Q_i(k), R(k) \in \mathbb{R}^{l_\nu \times l_y}$ are the coefficient matrices, which are updated by the RCPE algorithm. The integrator is embedded in the estimator to ensure that $z(k) \to 0$ as $k \to \infty$ and thus, assuming identifiability and data persistency, that $\hat{\nu}(k) \to \nu$ as $k \to \infty$.

We rewrite (6) as

$$\hat{\nu}(k) = \Phi(k)\theta(k), \tag{8}$$

where the regressor matrix $\Phi(k)$ is defined by

$$\Phi(k) \triangleq I_{l_\nu} \otimes \phi^\mathrm{T}(k) \in \mathbb{R}^{l_\nu \times l_\theta},$$

where

$$\phi(k) \triangleq \begin{bmatrix} \hat{\nu}(k-1) \\ \vdots \\ \hat{\nu}(k-n_\mathrm{c}) \\ z(k-1) \\ \vdots \\ z(k-n_\mathrm{c}) \\ g(k) \end{bmatrix}, \tag{9}$$

$$\theta(k) \triangleq \mathrm{vec}\left[\ P_1(k)\cdots P_{n_\mathrm{c}}(k)\ Q_1(k)\cdots Q_{n_\mathrm{c}}(k)\ R(k)\ \right] \in \mathbb{R}^{l_\theta}, \tag{10}$$

$l_\theta \triangleq l_\nu^2 n_\mathrm{c} + l_\nu l_y(n_\mathrm{c}+1)$, "$\otimes$" is the Kronecker product, and "vec" is the column-stacking operator.

B. Retrospective Performance Variable

We define the retrospective performance variable

$$\hat{z}(k) = z(k) + \hat{G}_\mathrm{f}(\mathbf{q})(\Phi(k)\hat{\theta} - \hat{\nu}(k)), \tag{11}$$

where $\mathbf{q}$ is the forward-shift operator, $\hat{\theta} \in \mathbb{R}^{l_\theta}$ contains the parameter estimator coefficients to be optimized, and $\hat{G}_\mathrm{f}$ is an FIR filter of order n_f to be optimized, given by

$$\hat{G}_\mathrm{f}(\mathbf{q}) = \sum_{i=1}^{n_\mathrm{f}} \frac{\hat{N}_i}{\mathbf{q}^i}, \quad \hat{N}_i \in \mathbb{R}^{l_y \times l_\nu}. \tag{12}$$

We rewrite (11) as

$$\hat{z}(k) = z(k) + \hat{N}\Phi_\mathrm{b}(k)\hat{\theta} - \hat{N}U_\mathrm{b}(k), \tag{13}$$

where

$$\hat{N} \triangleq \left[\ \hat{N}_1 \quad \cdots \quad \hat{N}_{n_\mathrm{f}}\ \right] \in \mathbb{R}^{l_y \times n_\mathrm{f} l_\nu},$$

$$\Phi_\mathrm{b}(k) \triangleq \begin{bmatrix} \Phi(k-1) \\ \vdots \\ \Phi(k-n_\mathrm{f}) \end{bmatrix} \in \mathbb{R}^{l_\nu n_\mathrm{f} \times l_\theta},$$

$$U_\mathrm{b}(k) \triangleq \begin{bmatrix} \hat{\nu}(k-1) \\ \vdots \\ \hat{\nu}(k-n_\mathrm{f}) \end{bmatrix} \in \mathbb{R}^{l_\nu n_\mathrm{f}}.$$

C. Retrospective Cost Function

Using the retrospective performance variable $\hat{z}(k)$, we define the retrospective cost function

$$J(k, \hat{\theta}, \hat{N}) \triangleq \sum_{i=1}^{k} \hat{z}^\mathrm{T}(i) R_z \hat{z}(i) + \hat{\theta}^\mathrm{T} R_\theta \hat{\theta}, \tag{14}$$

where R_z and R_θ are positive definite. Note that the retrospective cost $J(k, \hat{\theta}, \hat{N})$ is a biquadratic function, that is, $J(k, \hat{\theta}, \hat{N})$ is a quadratic function of $\hat{\theta}$ for fixed $\hat{N}$, and a quadratic function of $\hat{N}$ for fixed $\hat{\theta}$. We use alternating convex search (ACS) described in [21] to optimize the retrospective cost with respect to the parameter estimator coefficients and the filter. The parameter estimate is given by

$$\hat{\nu}(k) = \Phi(k)\theta(k), \tag{15}$$

where $\theta(k)$ is the minimizer of $J(k, \hat{\theta}, \hat{N})$ at the k^th step.

V. Biquadratic Retrospective Cost Optimization

In this section we show that $J(k, \hat{\theta}, \hat{N})$ defined by (14) is biquadratic as a joint function of the arguments $\hat{\theta}$ and $\hat{N}$ and strictly convex in $\hat{\theta}$ and $\hat{N}$ separately. This property suggests an optimization algorithm in which $J(k, \hat{\theta}, \hat{N})$ is minimized alternately with respect to $\hat{\theta}$ and $\hat{N}$

A. Retrospective cost as a quadratic function of $\hat{\theta}$

We write the retrospective cost function (14) as

$$\begin{aligned} J(k, \hat{\theta}, \hat{N}) = \hat{\theta}^\mathrm{T} A_\theta(k, \hat{N})\hat{\theta} + \hat{\theta}^\mathrm{T} b_\theta(k, \hat{N}) \\ + b_\theta(k, \hat{N})^\mathrm{T}\hat{\theta} + c_\theta(k, \hat{N}), \end{aligned} \tag{16}$$

where

$$A_\theta(k, \hat{N}) \triangleq \sum_{i=1}^{k} \Phi_\mathrm{b}(i)^\mathrm{T} \hat{N}^\mathrm{T} \hat{N} \Phi_\mathrm{b}(i) + R_\theta \in \mathbb{R}^{l_\theta \times l_\theta}, \tag{17}$$

$$b_\theta(k, \hat{N}) \triangleq \sum_{i=1}^{k} \left(\hat{N}\Phi_\mathrm{b}(i)\right)^\mathrm{T} \left(z(i) - \hat{N}U_\mathrm{b}(i)\right) \in \mathbb{R}^{l_\theta}, \tag{18}$$

$$\begin{aligned} c_\theta(k, \hat{N}) \triangleq \sum_{i=1}^{k} U_\mathrm{b}(i)^\mathrm{T} \hat{N}^\mathrm{T} \hat{N} U_\mathrm{b}(i) + z(i)^\mathrm{T} z(i) \\ + U_\mathrm{b}(i)^\mathrm{T} \hat{N}^\mathrm{T} z(i) \in \mathbb{R}. \end{aligned} \tag{19}$$

For fixed $\hat{N}$, the global minimizer $\theta^*(\hat{N})$ of (14) is given by

$$\theta^*(\hat{N}) = -A_\theta(k, \hat{N})^{-1} b_\theta(k, \hat{N}). \tag{20}$$

B. Retrospective cost as a quadratic function of $\hat{N}$

We write the retrospective cost function (14) as

$$J(k,\hat{\theta},\hat{N}) \triangleq \operatorname{tr}\big(\hat{N}A_N(k,\hat{\theta})\hat{N}^{\mathrm{T}} + \hat{N}B_N(k,\hat{\theta})^{\mathrm{T}} + B_N(k,\hat{\theta})\hat{N}^{\mathrm{T}} + C_N(k,\hat{\theta})\big), \tag{21}$$

where

$$A_N(k,\hat{\theta}) \triangleq \sum_{i=1}^{k}\big(\Phi_{\mathrm{b}}(i)\hat{\theta} - U_{\mathrm{b}}(i)\big)\big(\Phi_{\mathrm{b}}(i)\hat{\theta} - U_{\mathrm{b}}(i)\big)^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{f}}l_\nu \times n_{\mathrm{f}}l_\nu}, \tag{22}$$

$$B_N(k,\hat{\theta}) \triangleq \sum_{i=1}^{k} z(i)\big(\Phi_{\mathrm{b}}(i)\hat{\theta} - U_{\mathrm{b}}(i)\big)^{\mathrm{T}} \in \mathbb{R}^{l_y \times n_{\mathrm{f}}l_\nu}, \tag{23}$$

$$C_N(k,\hat{\theta}) \triangleq \sum_{i=1}^{k} z(i)z(i)^{\mathrm{T}} \in \mathbb{R}^{l_y \times l_y}. \tag{24}$$

For fixed $\hat{\theta}$, the global minimizer $N^*(\hat{\theta})$ of (14) is given by

$$N^*(\hat{\theta}) \triangleq -B_N(k,\hat{\theta})A_N(k,\hat{\theta})^{-\mathrm{T}} \in \mathbb{R}^{l_y \times n_{\mathrm{f}}l_\nu}. \tag{25}$$

C. The structure of G_{f}

The following result shows that the estimate $\hat{\nu}(k)$ of ν is constrained to lie in a subspace determined by the coefficients of G_{f}.

Theorem 5.1: Let $\hat{G}_{\mathrm{f}}(\mathbf{q}) = \dfrac{\hat{N}_1}{\mathbf{q}}$, $R_\theta = \beta I_{l_\theta}$, and $l_y = 1$. Then,

$$\hat{\nu}(k) = \alpha(k)\hat{N}_1^{\mathrm{T}}, \tag{26}$$

where

$$\alpha(k) \triangleq -\frac{1}{\beta}\sum_{i=1}^{k}\phi(k)^{\mathrm{T}}\phi(i-1)[z(i) - \hat{N}_1\hat{\nu}(i-1) + \hat{N}_1\Phi(i-1)\theta(k)]. \tag{27}$$

Theorem 5.1 shows that the estimate $\hat{\nu}(k)$ produced by RCPE with a first-order FIR filter G_{f} is constrained to lie in the range of $\hat{N}_1^{\mathrm{T}} \in \mathbb{R}^{l_\nu \times l_y}$. If ν is scalar and $\hat{N}_1 \neq 0$, then $\mathcal{R}(\hat{N}_1^{\mathrm{T}}) = \mathbb{R}$. We therefore use a first-order FIR filter to estimate a scalar parameter ν.

D. Alternating Convex Search algorithm

The ACS algorithm consists of using (25) and (20) alternately to converge to a stationary point of (14). At step k, ACS consists of the following rules:

1) Set $i = 0$ and choose nonzero N_0.
2) Use (20) with N_i to compute θ_{i+1}.
3) Use (25) with θ_{i+1} to compute N_{i+1}.
4) Compute $J_{i+1}(k,\theta_{i+1},N_{i+1})$ using either (16) or (21).
5) For $i > 2$, if $J_{i+1} - J_i \le \varepsilon$, then stop, where $\varepsilon > 0$ is the user-defined stopping criteria, and set $\theta(k) = \theta_{i+1}$ and $N(k) = N_{i+1}$. Otherwise, replace i by $i+1$ and go to 2).

The parameter estimate is given by

$$\hat{\nu}(k) = \Phi(k)\theta(k). \tag{28}$$

VI. Illustrative Example

In this section RCPE is used to estimate an unknown scalar parameter ν that appears nonlinearly in a linear system realization. Consider the LTI physical system model

$$x(k+1) = A(\nu)x(k) + B(\nu)u(k) + w_1(k), \tag{29}$$

$$y(k) = C(\nu)x(k) + w_2(k), \tag{30}$$

where

$$A(\nu) = \begin{bmatrix} e^{-\nu} & 1-\nu \\ \nu^2 & \log(1+\nu^2) \end{bmatrix}, \tag{31}$$

$$B(\nu) = \begin{bmatrix} \sin\nu \\ 1+\cos\nu \end{bmatrix}, \tag{32}$$

$$C(\nu) = \begin{bmatrix} 1+\nu & \nu^2 \end{bmatrix}. \tag{33}$$

The true value of ν is 0.8. The estimation model is thus

$$\hat{x}(k+1) = A(\hat{\nu}(k))\hat{x}(k) + B(\hat{\nu}(k))u(k), \tag{34}$$

$$\hat{y}(k) = E(\hat{\nu}(k))\hat{x}(k), \tag{35}$$

where $\hat{\nu}(k)$ is the output of the parameter estimator updated by RCPE.

The measurement $y(k)$ is generated using the input $u(k) = 2 + \sin\left(\frac{2\pi}{40}k\right) + \sin\left(\frac{2\pi}{80}k - 0.3\right) + \sin\left(\frac{2\pi}{160}k - 0.5\right)$, the initial state $x(0) = [10\ \ 10]^{\mathrm{T}}$, each component of the process noise w_1 is $\mathcal{N}(0, 10^{-6})$, and the measurement noise w_2 is $\mathcal{N}(0, 10^{-6})$. To reflect the absence of additional information, the initial state $\hat{x}(0)$ of the estimation model and the initial estimate $\hat{\nu}(0)$ of the unknown parameter ν are set to zero. We use RCPE to estimate the unknown parameter ν in the linear system (29), (30) with the nonlinear parameter dependence (31)-(33). We set $n_{\mathrm{c}} = 1$, $n_{\mathrm{f}} = 1$, $R_z = 1$, and $R_\theta = 10^6 I_{l_\theta}$. At each step, ACS is initialized with $N_0 = -1$. Figure 2 shows the estimate $\hat{\nu}(k)$ of ν.

Next, we investigate the effect of the ACS filter initialization N_0 and the weight R_θ on the performance of RCPE. We set $n_{\mathrm{c}} = 1$, $n_{\mathrm{f}} = 1$, and $R_z = 1$. Figure 3 shows the estimate $\hat{\nu}(k)$ of ν and the filter coefficient $N(k)$ for various initializations and weights. Note that RCPE successfully estimates the unknown parameter ν for ACS filter initialization choices ranging several orders of magnitude, thus indicating that ACS filter initialization choice is not critical.

VII. Estimation of EDC in GITM

Finally, we consider the problem of estimating the EDC in the global ionosphere thermosphere model (GITM) using measurements of TEC at a fixed ground station on Earth.

To generate the measurements of TEC, we simulate the upper atmosphere of Earth using GITM for the period starting at 00:00:00, 21-Nov-2002 and ending at 00:00:00, 8-Dec-2002 with EDC = 1750. TEC is computed at every minute at a fictitious ground station located at 1 deg North, 45 deg East. The initial state of the upper atmosphere, comprising neutral and ion densities and temperature, is set using MSIS and IRI for the chosen start time. The inputs to GITM, such as F10.7 index, IMF data, SWP data, and HPI data, are read from text files.

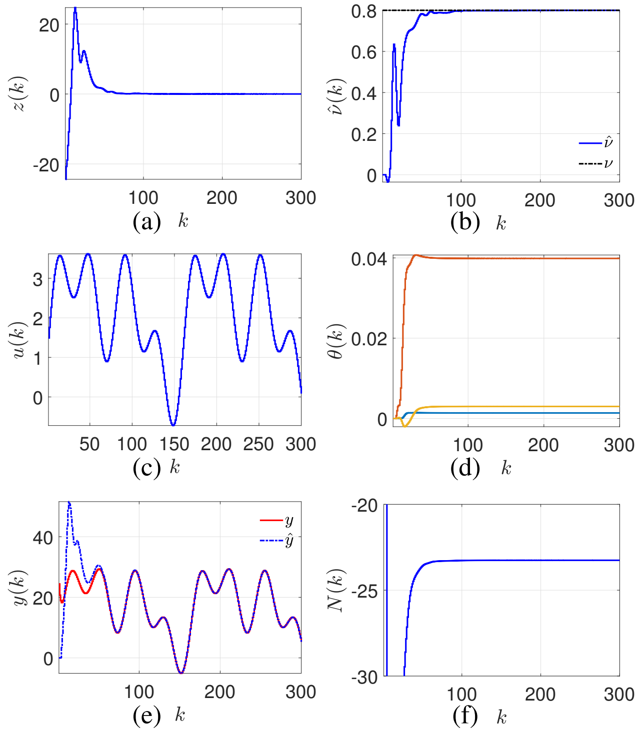

Fig. 2: RCPE estimate of the unknown parameter ν in the linear system (29), (30) with the nonlinear parameter dependence (31)-(33). (a) shows the performance z on a linear scale, (b) shows the parameter estimate $\hat{\nu}$, (c) shows the measured input u to the system, (d) shows the adapted coefficients θ of the parameter estimator, (e) shows the measured output and the output of the estimation model, and (f) shows the adapted coefficient $N(k)$ of the filter G_{f}.

To estimate the unknown EDC, we compute the TEC at every minute at the ground location 1 deg North, 45 deg East for the period starting from 00:00:00, 21-Nov-2002 to 00:00:00, 8-Dec-2002. The initial state of the upper atmosphere, comprising neutral and ion densities and temperature, is set using MSIS and IRI for the chosen start time. GITM is run for one simulation day, that is from 00:00:00, 21-Nov-2002 to 00:00:00, 22-Nov-2002 with EDC $= 1500$. At the start of the second simulation day, RCPE is switched on. Note that the delayed starting of RCPE ensures that, at the instant RCPE starts, the state of the atmosphere updated by the estimation GITM model is the not same as the GITM model used to generate the TEC measurements.

We set $n_{\mathrm{c}} = 2$, $n_{\mathrm{f}} = 1$, $R_z = 1$, and $R_\theta = 10^{-1} I_{l_\theta}$, and use RCPE to update the estimate of EDC at every minute. Figure 4 shows the estimate of EDC using RCPE. Figure 5 shows the measured and computed TEC.

Note that RCPE does not use GITM to update the parameter estimator. Instead, GITM is used as a black box model, although the EDC estimate is injected into GITM as a gray box model. In fact, the internal parameter dependence of GITM on EDC is extremely complicated; fortunately, there is no need to explicitly characterize this dependence.

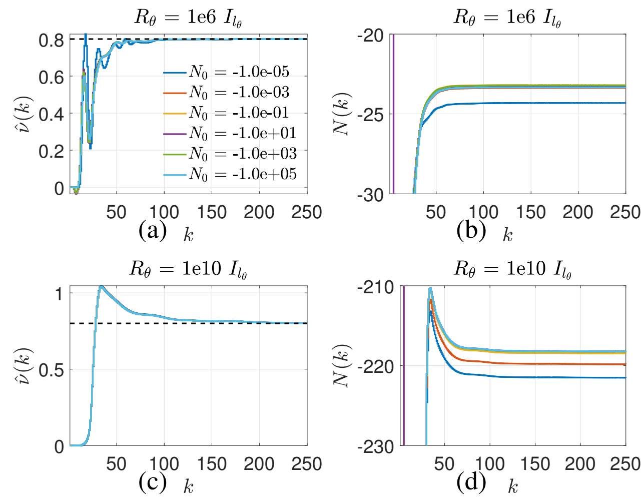

Fig. 3: Effect of the ACS filter initialization N_0 and the weight R_θ on the performance of RCPE. (a) and (c) show the parameter estimate $\hat{\nu}$ for various values of the filter initialization N_0 and the weight R_θ. (b) and (d) show the filter coefficient N for various values of the filter initialization N_0 and the weight R_θ.

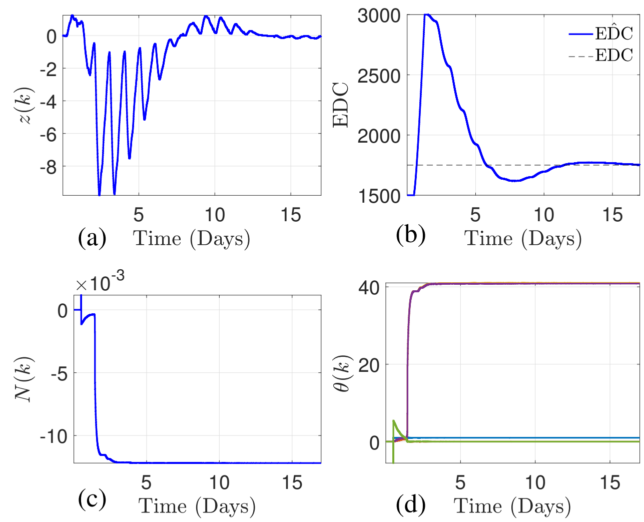

Fig. 4: RCPE estimate of the unknown EDC in GITM. (a) shows the performance z on a linear scale, (b) shows the EDC estimate, (c) shows the adapted coefficient $N(k)$ of the filter G_{f}, and (d) shows the adapted coefficients θ of the parameter estimator.

VIII. Conclusions and Future Work

This presented an extension of retrospective cost parameter estimation (RCPE) by concurrently optimizing the filter G_{f} and the parameter estimator. This technique was used to estimate an unknown parameter in a large-scale model of a physical system, namely, the eddy diffusion coefficient in the global ionosphere-thermosphere model.

Analysis of RCPE focused on the biquadratic nature of the retrospective cost function as well as the structure of the filter G_{f} for which RCPE is potentially able to estimate the unknown parameter ν. Alternating convex search algorithm was used to optimize the biquadratic retrospective cost function. In addition, it was shown that, if ν is scalar and

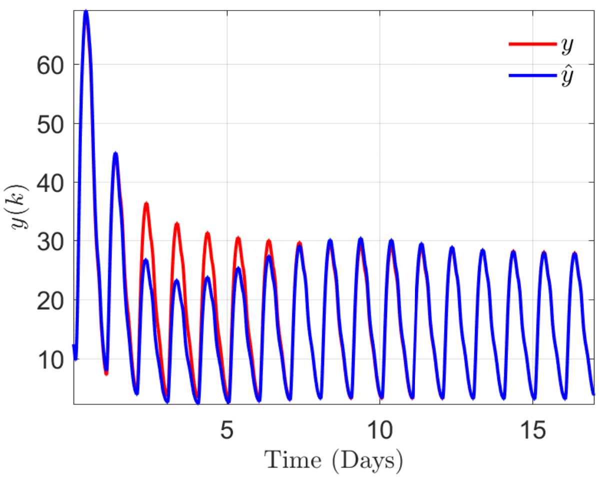

Fig. 5: Measured and computed TEC at the fictitious ground station located at 1 deg North, 45 deg East. y denotes the TEC measurements generated using GITM with a constant EDC= 1750, and $\hat{y}$ denotes the TEC computed by GITM where EDC is updated at every minute by RCPE.

$G_{\rm f}$ is a first-order FIR filter, then $\mathcal{R}(N_1^{\rm T}) = \mathbb{R}$. It thus follows from (26) that the subspace $\mathcal{R}(N_1^{\rm T})$, which contains $\hat{\nu}$, is independent of the choice of N_1. The situation is more complicated, however, in the case where ν is a vector. In this case, analysis of the retrospective cost showed that $G_{\rm f}$ must be higher order. This case is a priority for future research.

To improve the accuracy of GITM, better empirical relationships describing various physical processes, such as turbulent mixing and Joule heating, need to be embedded in GITM. One way to improve such empirical functions is to parameterize them with more than one parameter. Future work will thus focus on application of RCPE to systems with two or more uncertain parameters using higher order filters $G_{\rm f}$.

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