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Time-Domain Errors-in-Variables Identification of Transmissibilities

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Abstract— **Transmissibility operators are time-domain operators that model the relationship between outputs of an underlying system. Since real measurements are corrupted by noise, identification of transmissibilities is an errors-in-variables identification problem. This paper applies a bias-compensated recursive least-squares algorithm to errors-in-variables identification of transmissibilities. Noncausal FIR models are used to approximate transmissibility operators. To investigate the accuracy of this approach, we consider data from an acoustic experiment. The goal is to identify the transmissibility operator that models the relationship between measurements from two microphones.**

I. Introduction

A transmissibility is a sensor-to-sensor model that describes the relationship between outputs of a system [1]–[3]. The input and output of the transmissibility are referred to as "pseudo-input" and "pseudo-output", respectively. Transmissibilities are used in applications such as structural health monitoring, fault detection, and fault localization, and are typically used in applications where the excitation signal is unknown or cannot be measured [4]–[10]. If a model of the underlying system is available, then the transmissibility of interest can be constructed mathematically. Otherwise, the transmissibility can be identified from the available measurements of the pseudo-input and pseudo-output.

Transmissibilities can be constructed in either the frequency domain or in the time domain. A frequency-domain transmissibility is called a transmissibility function [4], [5], [11], [12], whereas a time-domain transmissibility is called a transmissibility operator [3], [13], [14].

Due to leakage in frequency-domain identification, it was shown in [3], [14] that, under nonzero initial conditions of the underlying system, transmissibility functions depend on the excitation signal. In contrast, transmissibility operators are independent of the nonzero initial conditions and the excitation signal that acts on the system [3], [14].

Since real sensor measurements are noisy, measurements of the pseudo-input and pseudo-output of the transmissibility are unavoidably corrupted by noise. Moreover, if the input that excites the underlying system is a realization of a white or colored random process, then the pseudo-input and pseudo-output of the transmissibility are colored.

System identification builds mathematical models of systems from measured input and output data. Since sensor measurements are inevitably corrupted by noise, identification under noisy input and output data, which is known as errors-in-variables (EIV) identification, is a longstanding problem [15]–[21]. A challenging aspect of this problem is to obtain consistent parameter estimates, that is, parameter estimates that converge to the true parameter values with probability 1 as the amount of data increases without bound.

System identification algorithms include least squares, instrumental variables, and prediction error methods. Using least squares with an infinite impulse response (IIR) model structure for EIV identification produces biased estimates, as shown in [15]. Modifications to the least squares algorithm have been introduced to obtain consistent estimates [22], [23]. If the autocorrelation function of the noise on the input and output is known to within a scaling, consistent parameter estimation is possible by using the Koopman-Levins algorithm [22]. This approach has been revisited and refined over the years; numerous references are given in [24]. Additional bias-compensating least squares algorithms are given in [23], [25], [26]. Instrumental variables methods [27]–[30], prediction error methods [31], [32], and total least squares [28], [33] have been also used for EIV identification. Various methods used for EIV identification are discussed in [15].

Since physical systems are usually IIR, the IIR model structure is the natural choice for system identification, including EIV problems. However, using IIR models for identification requires knowledge of the system order. If the system order is unknown, an initial overestimate of the order can be used. However, for a finite data set, overestimating the system order can yield poor transfer function estimates.

An FIR model can be viewed as a truncation of the Laurent expansion of an IIR transfer function in the annulus that contains the unit circle [34]. Although physical systems are rarely FIR, an FIR model can approximate an asymptotically stable IIR system, and a noncausal FIR model can approximate an unstable system [34]. The benefit of using FIR models to identify IIR systems is that an FIR model does not require a prior estimate of the system order. EIV identification for FIR systems using FIR models is discussed in [35], where additive input and output noise are assumed to be white, and a bias-compensated least squares algorithm is used to estimate the input noise variance [35].

In the present paper, we use a modification of the bias-compensated least squares algorithm in [35] with noncausal FIR models to identify transmissibility operators whose pseudo-input and pseudo-output are corrupted by additive white noise. In particular, we show that, if the pseudo-input of the transmissibility is white or colored, and the input noise and output noise are both additive white noise, then

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consistent estimates of the noncausal FIR model parameters can be obtained. We apply this approach to an acoustic experiment consisting of an acoustic space, a speaker, and two microphones. Time-domain errors-in-variables identification of transmissibilities has not been studied before.

II. Transmissibility Operators

Consider the state space system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$y_{\rm i}(t) = C_{\rm i}x(t) + D_{\rm i}u(t), \tag{2}$$

$$y_{\rm o}(t) = C_{\rm o}x(t) + D_{\rm o}u(t), \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C_{\rm i} \in \mathbb{R}^{m\times n}$, $C_{\rm o} \in \mathbb{R}^{(p-m)\times n}$, $D_{\rm i} \in \mathbb{R}^{m\times m}$, and $D_{\rm o} \in \mathbb{R}^{(p-m)\times m}$.

Let $\mathbf{p} \stackrel{\triangle}{=} {\rm d}/{\rm d}t$ denote the differentiation operator. Moreover, let $y_{\rm i}$ be the pseudo-input of the transmissibility and $y_{\rm o}$ be the pseudo-output of the transmissibility. Then, for all $t \geq 0$, the transmissibility operator from $y_{\rm i}$ to $y_{\rm o}$ satisfies [3]

$$y_{\rm o}(t) = \mathcal{T}(\mathbf{p})y_{\rm i}(t), \tag{4}$$

where

$$\mathcal{T}(\mathbf{p}) \stackrel{\triangle}{=} \Gamma_{\rm o}(\mathbf{p})\Gamma_{\rm i}^{-1}(\mathbf{p}), \tag{5}$$

$$\Gamma_{\rm o}(\mathbf{p}) \stackrel{\triangle}{=} C_{\rm o}{\rm adj}\,(\mathbf{p}I - A)B + D_{\rm o}\delta(\mathbf{p}), \tag{6}$$

$$\Gamma_{\rm i}(\mathbf{p}) \stackrel{\triangle}{=} C_{\rm i}{\rm adj}\,(\mathbf{p}I - A)B + D_{\rm i}\delta(\mathbf{p}), \tag{7}$$

$$\delta(\mathbf{p}) \stackrel{\triangle}{=} \det(\mathbf{p}I - A). \tag{8}$$

Note that (4) represents the differential equation

$$\det\Gamma_{\rm i}(\mathbf{p})y_{\rm o}(t) = \Gamma_{\rm o}(\mathbf{p})\,{\rm adj}\,\Gamma_{\rm i}(\mathbf{p})y_{\rm i}(t). \tag{9}$$

A transmissibility operator has information about the zeros of the underlying system and is independent of its poles. A nonminimum-phase zero in the input channel of the transmissibility yields an unstable transmissibility operator. Moreover, if the number of zeros in the input channel of the transmissibility is more than the number of zeros in the output channel of the transmissibility, then the transmissibility is improper, that is, noncausal. Therefore, a transmissibility can be unstable and noncausal.

We assume that measurements of the output signals are obtained in discrete time. Therefore, we replace the differentiation operator $\mathbf{p}$ in (4) and (9) by the forward-shift operator $\mathbf{q}$. Moreover, we assume that $\mathcal{T}$ has no poles on the unit circle. For convenience, we assume for the rest of the paper that $p = m = 1$.

III. Using Noncausal FIR models to Approximate Transmissibilities

Noncausal FIR models have been used to approximate transmissibility operators that might be unstable, noncausal, and of unknown order [10], [34], [36]. A noncausal FIR model of $\mathcal{T}$ is a truncation of the Laurent expansion of $\mathcal{T}$ in an annulus that contains the unit circle. Let $\mathbb{A}(\rho_1,\rho_2)$ be an annulus in the complex plane centered at the origin, where $\mathcal{T}$ is analytic in $\mathbb{A}(\rho_1,\rho_2)$ and $\rho_1 < 1 < \rho_2$. Then, the Laurent expansion of $\mathcal{T}$ in $\mathbb{A}(\rho_1,\rho_2)$ is given by

$$\mathcal{T}(z) = \sum_{i=-\infty}^{\infty} H_i z^{-i}, \tag{10}$$

where, for all i, H_i is the ith coefficient of the Laurent expansion of $\mathcal{T}$ in $\mathbb{A}(\rho_1,\rho_2)$. Therefore, a noncausal FIR model of $\mathcal{T}$ is given by

$$\mathcal{T}(\mathbf{q},\theta_{r,d}) \stackrel{\triangle}{=} \sum_{i=-d}^{r} H_i\mathbf{q}^{-i}, \tag{11}$$

where

$$\theta_{r,d} \stackrel{\triangle}{=} \left[\begin{array}{ccc} H_{-d} & \cdots & H_r \end{array}\right] \in \mathbb{R}^{1\times(r+d+1)}. \tag{12}$$

IV. Assumptions

Suppose that both $y_{\rm i}$ and $y_{\rm o}$ are corrupted with additive white noise, that is,

$$\hat{y}_{\rm i}(k) = y_{\rm i}(k) + v(k), \tag{13}$$

$$\hat{y}_{\rm o}(k) = y_{\rm o}(k) + w(k), \tag{14}$$

where $\hat{y}_{\rm i}$ and $\hat{y}_{\rm o}$ are measurements of $y_{\rm i}$ and $y_{\rm o}$, respectively, and v and w represent sensor noise.

We consider the following assumptions

A1) $y_{\rm i}$ is a realization of a zero-mean ergodic random process that is persistently exciting of a sufficient order.
A2) v and w are realizations of zero-mean ergodic white random processes with unknown variances.
A3) v and w are realizations of mutually uncorrelated random processes and are uncorrelated with the random process producing the realization $y_{\rm i}$.

V. Least Squares Identification

The least squares estimate $\hat{\theta}_{r,d,\ell}^{\rm LS}$ of $\theta_{r,d}$ satisfies

$$\hat{\theta}_{r,d,\ell}^{\rm LS} = \underset{\bar{\theta}_{r,d}\in\mathbb{R}^{1\times(r+d+1)}}{\arg\min} \|\Psi_{\hat{y}_{\rm o},r,d,\ell} - \bar{\theta}_{r,d}\Phi_{\hat{y}_{\rm i},r,d,\ell}\|_2, \tag{15}$$

where

$$\Psi_{\hat{y}_{\rm o},r,d,\ell} \stackrel{\triangle}{=} \left[\begin{array}{ccc} \hat{y}_{\rm o}(r) & \cdots & \hat{y}_{\rm o}(\ell-d) \end{array}\right], \tag{16}$$

$$\Phi_{\hat{y}_{\rm i},r,d,\ell} \stackrel{\triangle}{=} \left[\begin{array}{ccc} \phi_{\hat{y}_{\rm i},r,d}(r) & \cdots & \phi_{\hat{y}_{\rm i},r,d}(\ell-d) \end{array}\right], \tag{17}$$

$$\phi_{\hat{y}_{\rm i},r,d}(k) \stackrel{\triangle}{=} \left[\begin{array}{ccc} \hat{y}_{\rm i}(k+d) & \cdots & \hat{y}_{\rm i}(k-r) \end{array}\right]^{\rm T}, \tag{18}$$

and ℓ is the number of samples.

It follows from (15) that the least squares estimate $\hat{\theta}_{r,d,\ell}^{\rm LS}$ of $\theta_{r,d}$ satisfies

$$\Psi_{\hat{y}_{\rm o},r,d,\ell}\Phi_{\hat{y}_{\rm i},r,d,\ell}^{\rm T} = \hat{\theta}_{r,d,\ell}^{\rm LS}\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi_{\hat{y}_{\rm i},r,d,\ell}^{\rm T}. \tag{19}$$

Note that $\Psi_{\hat{y}_{\rm o},r,d,\ell}$ can be written as

$$\begin{aligned}\Psi_{\hat{y}_{\rm o},r,d,\ell} &= \Psi_{y_{\rm o},r,d,\ell} + \Psi_{w,r,d,\ell}\\ &= \theta_{r,d}\Phi_{y_{\rm i},r,d,\ell} + \Psi_{e,r,d,\ell} + \Psi_{w,r,d,\ell},\end{aligned} \tag{20}$$

where

$$\Phi_{y_{\rm i},r,d,\ell} \triangleq \left[\ \phi_{y_{\rm i},r,d}(r) \quad \cdots \quad \phi_{y_{\rm i},r,d}(\ell-d)\ \right], \tag{21}$$

$$\phi_{y_{\rm i},r,d}(k) \triangleq \left[\ y_{\rm i}(k+d) \quad \cdots \quad y_{\rm i}(k-r)\ \right]^{\rm T}, \tag{22}$$

$$\Psi_{w,r,d,\ell} \triangleq \left[\ w(r) \quad \cdots \quad w(\ell-d)\ \right], \tag{23}$$

$$\Psi_{e,r,d,\ell} \triangleq \left[\ e_{r,d}(r) \quad \cdots \quad e_{r,d}(\ell-d)\ \right], \tag{24}$$

$$\begin{aligned} e_{r,d}(k) &\triangleq y_{\rm o}(k) - \sum_{i=-d}^{r} H_i y_{\rm i}(k-i) \\ &= y_{\rm o}(k) - \theta_{r,d}\phi_{y_{\rm i},r,d}(k). \end{aligned} \tag{25}$$

Using (20), (19) can be written as

$$\theta_{r,d}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} + \Psi_{e,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} + \Psi_{w,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} = \hat{\theta}^{\rm LS}_{r,d,\ell}\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell}. \tag{26}$$

Using Assumptions A1) and A2), dividing (26) by ℓ and taking the limit as ℓ tends to infinity yields

$$\theta_{r,d}\lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} + \lim_{\ell\to\infty}\frac{1}{\ell}\Psi_{e,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} + \lim_{\ell\to\infty}\frac{1}{\ell}\Psi_{w,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} = \lim_{\ell\to\infty}\hat{\theta}^{\rm LS}_{r,d,\ell}\lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell}. \tag{27}$$

Using Assumptions A2) and A3), we have

$$\lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} = \lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{y_{\rm i},r,d,\ell}, \tag{28}$$

$$\lim_{\ell\to\infty}\frac{1}{\ell}\Psi_{w,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} = 0, \tag{29}$$

$$\lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} = \lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{y_{\rm i},r,d,\ell} + \sigma_v^2 I_{r+d+1}, \tag{30}$$

where σ_v^2 is the variance of v, and I_{r+d+1} is the $(r+d+1)\times(r+d+1)$ identity matrix. Assuming that r and d are large enough such that, for all $k\geq 0$, $e_{r,d}(k)$ is negligible, it follows that

$$\lim_{\ell\to\infty}\frac{1}{\ell}\Psi_{e,r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell} \approx 0. \tag{31}$$

Defining

$$Q \triangleq \lim_{\ell\to\infty}\frac{1}{\ell}\Phi_{y_{\rm i},r,d,\ell}\Phi^{\rm T}_{y_{\rm i},r,d,\ell}, \tag{32}$$

and using (28)–(31), then (27) becomes

$$\theta_{r,d}Q = \lim_{\ell\to\infty}\hat{\theta}^{\rm LS}_{r,d,\ell}(Q+\sigma_v^2 I_{r+d+1}), \tag{33}$$

which can be written as

$$\theta_{r,d} = \lim_{\ell\to\infty}\hat{\theta}^{\rm LS}_{r,d,\ell}(I_{r+d+1}+\sigma_v^2 Q^{-1}). \tag{34}$$

It follows from (34) that $\hat{\theta}^{\rm LS}_{r,d,\ell}$ is not a consistent estimate of $\theta_{r,d}$.

VI. Errors-in-Variables Identification Algorithm

For EIV identification we use a modified version of the algorithm considered in [35]. The main difference between the algorithm we use in this paper and the algorithm in [35] is that we use a noncausal FIR model to identify an IIR transmissibility operator instead of using an FIR model to identify an FIR system.

Here we list the steps of the algorithm. The reader can refer to [35] for more details.

1) Compute $\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell}$ and $\Psi_{\hat{y}_{\rm o},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell}$.
2) Use (19) to compute the least squares estimate $\hat{\theta}^{\rm LS}_{r,d,\ell}$.
3) Set $\hat{\theta}^{\rm EIV}_{r,d,\ell,i} = \hat{\theta}^{\rm LS}_{r,d,\ell}$, where, for all $i\geq 1$,

$$\hat{\theta}^{\rm EIV}_{r,d,\ell,i} \triangleq \left[\ \hat{H}_{i,-d,\ell} \quad \cdots \quad \hat{H}_{i,r,\ell}\ \right], \tag{35}$$

and for all $j=-d,\ldots,r$, $\hat{H}_{i,j,\ell}$ is an estimate of H_j obtained from the ith iteration of the algorithm using ℓ samples of $\hat{y}_{\rm i}$ and $\hat{y}_{\rm o}$.
4) Construct the vector

$$\mathcal{H}(\hat{\theta}^{\rm EIV}_{r,d,\ell,i}) \triangleq \begin{bmatrix} \sum_{j=-d}^{r-1}\hat{H}_{i,j,\ell}\hat{H}_{i,j+1,\ell} \\ \sum_{j=-d}^{r-2}\hat{H}_{i,j,\ell}\hat{H}_{i,j+2,\ell} \\ \vdots \\ \hat{H}_{i,-d,\ell}\hat{H}_{i,r,\ell} \end{bmatrix}. \tag{36}$$

5) For all $k\geq r$, compute

$$E_{r,d,\ell}(k) \triangleq \hat{y}_{\rm o}(k) - \hat{\theta}^{\rm EIV}_{r,d,\ell,i}\phi_{\hat{y}_{\rm i},r,d}(k). \tag{37}$$

6) Construct

$$\mathcal{E}_{r,d,\ell} \triangleq \frac{1}{\ell}\left[\Psi_{E,r,d,\ell,0}\Psi^{\rm T}_{E,r,d,\ell,1} \ \cdots\ \Psi_{E,r,d,\ell,0}\Psi^{\rm T}_{E,r,d,\ell,r+d}\right]^{\rm T}, \tag{38}$$

where, for all $j=1,\ldots,r+d$,

$$\Psi_{E,r,d,\ell,j} \triangleq \left[\ E(2r+d-j) \quad \cdots \quad E(\ell-j)\ \right], \tag{39}$$

7) Compute

$$\hat{\sigma}^2_{v,r,d,\ell,i} = \frac{\mathcal{H}(\hat{\theta}^{\rm EIV}_{r,d,\ell,i})^{\rm T}\mathcal{E}_{r,d,\ell}}{\mathcal{H}(\hat{\theta}^{\rm EIV}_{r,d,\ell,i})^{\rm T}\mathcal{H}(\hat{\theta}^{\rm EIV}_{r,d,\ell,i})}. \tag{40}$$

8) Compute

$$\hat{\theta}^{\rm EIV}_{r,d,\ell,i+1} = \hat{\theta}^{\rm LS}_{r,d,\ell} + \hat{\sigma}^2_{v,r,d,\ell,i}\hat{\theta}^{\rm EIV}_{r,d,\ell,i}(\Phi_{\hat{y}_{\rm i},r,d,\ell}\Phi^{\rm T}_{\hat{y}_{\rm i},r,d,\ell})^{-1}. \tag{41}$$

9) Set $\hat{\theta}^{\rm EIV}_{r,d,\ell,i} = \hat{\theta}^{\rm EIV}_{r,d,\ell,i+1}$ and go to step 4).
10) Repeat steps 4)–9) until

$$\frac{\|\hat{\theta}^{\rm EIV}_{r,d,\ell,i+1}-\hat{\theta}^{\rm EIV}_{r,d,\ell,i}\|}{\|\hat{\theta}^{\rm EIV}_{r,d,\ell,i+1}\|} < \varepsilon, \tag{42}$$

where ε is a predetermined convergence threshold.

VII. Experimental Results

We consider the acoustic setup shown in Figures 1 and 2 consisting of an acoustic space with a speaker and two microphones. The speaker is the actuator that drives the acoustic system, and the two microphones measure the acoustic response at their locations. We add artificial noise to the measurements from the microphones to emulate input and output sensor noise.

We consider the following cases for the sensor noise: Colored output noise with noise-free input; colored input noise with noise-free output; and uncorrelated white input and output noise. For all cases, the excitation u from the speaker is a realization of a colored random process. We consider a baseline model identified using least squares with noise-free data as the truth model, and we compare the identified models to the baseline model.

A. Output noise only

We add colored, Gaussian noise to the pseudo-output with output noise variance 0.01 and signal-to-noise ratio 2.4379 dB, where the input is noise-free. Figure 3 compares the frequency response of the identified transmissibilities obtained using least squares and the Diversi algorithm [35] with the baseline model obtained using noise-free data. Figure 4 shows the distance between the frequency response of the baseline model and the frequency responses of the identified transmissbilities obtained using least squares and the Diversi algorithm [35] versus the number of samples for several values of the output noise variance, specifically, 0.006, 0.008, and 0.01. Figure 4 shows that the estimates from both algorithms are consistent, but least squares produces more accurate estimates of the transmissibility.

B. Input noise only

We add colored, Gaussian noise to the pseudo-input with input noise variance 0.01 and signal-to-noise ratio 3.4717 dB, where the output is noise-free. Figure 5 compares the frequency response of the identified transmissibilities obtained using least squares and the Diversi algorithm [35] with the baseline model. Figure 6 shows the distance between the frequency response of the baseline model and the frequency responses of the identified transmissbilities obtained using least squares and the Diversi algorithm [35] versus the number of samples for several input noise variances, specificlly, 0.004, 0.006, and 0.008. Figure 6 shows that the Diversi algorithm [35] estimate is consistent, and the least squares algorithm estimate is not consistent.

C. Input and output noise

We add white, Gaussian noise to the pseudo-input and pseudo-output with input noise variance 0.007 and output noise variance 0.01, where the input signal-to-noise ratio is 4.598 dB and the output signal-to-noise ratio is 2.4168. Figure 7 compares the frequency response of the identified transmissibilities obtained using least squares and the Diversi algorithm [35] with the baseline model. Figure 8 shows the distance between the frequency response of the baseline model and the frequency responses of the identified transmissbilities obtained using least squares and the Diversi algorithm [35] versus the number of samples for several input noise and output noise variances, specifically, 0.006, 0.008, and 0.01 for the input noise and 0.004, 0.006, and 0.008 for the output noise. Figure 8 shows that the Diversi algorithm [35] estimate is consistent, and the least squares algorithm is not consistent.

VIII. Conclusions

This paper applied the bias-compensated recursive least-squares algorithm introduced in [35] to errors-in-variables identification of transmissibilities. The pseudo-input of the transmissibility is colored, and the input and output noises were assumed to be white and Gaussian. Noncausal FIR models were used to model transmissibility operators. We showed that, under specified assumptions, the bias-compensated recursive least-squares algorithm is consistent. We used data from an acoustic experiment consisting of an acoustic space, a speaker, and two microphones to identify the transmissibility that models the relationship between the measurements of the two microphones. To enhance the effect of sensor noise, artificial white noise was added to the microphone data. The experimental results suggest that under output noise only, least squares gives more accurate results than the algorithm [35]. However, in the case of input noise only, or input and output noise, the algorithm [35] gives consistent and more accurate results than least squares.

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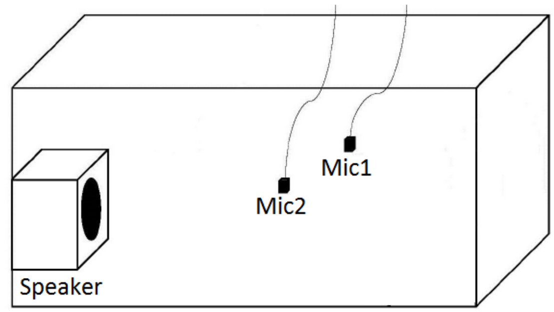

Fig. 1. A sketch of the experimental setup showing the speaker and the two microphones placement.

Fig. 2. Experimental setup.

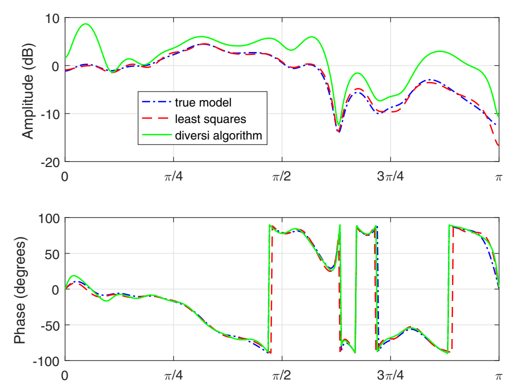

Fig. 3. Frequency response of the identified transmissibilities. The pseudo-output y_o is corrupted with colored noise, with output signal-to-noise ratio of 2.4379 dB.

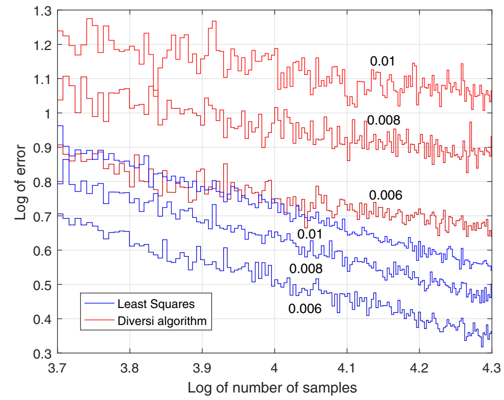

Fig. 4. Distance between the frequency response of baseline model and the frequency responses of the identified transmissbilities. The output noise is colored, where the output noise variance is 0.006, 0.008, and 0.01. Note that the estimates from both algorithms are consistent, but least squares produces more accurate estimates of the transmissibility.

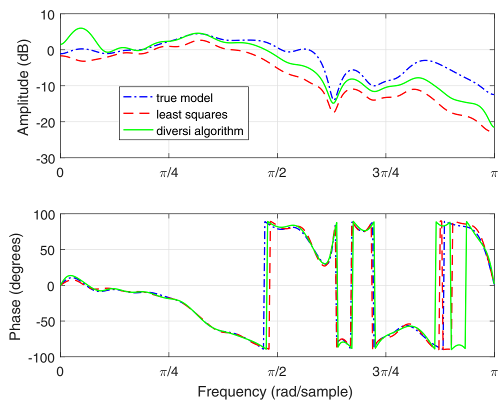

Fig. 5. Frequency response of the identified transmissbilities. The pseudo-input $y_{\rm i}$ is corrupted with colored noise, with input signal-to-noise ratio of 3.4717 dB.

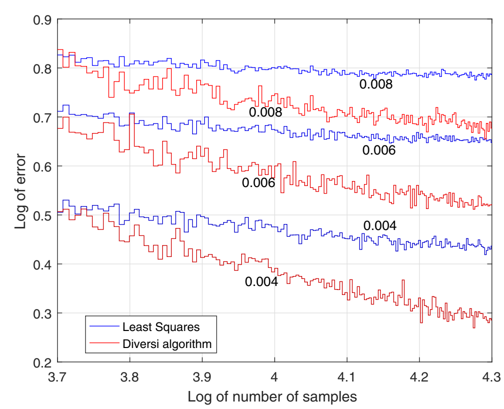

Fig. 6. Distance between the frequency response of the baseline model and the frequency responses of the identified transmissbilities. The input noise v is colored, where the input noise variance is 0.004, 0.006, and 0.008. Note that the Diversi algorithm [35] is consistent, and the least squares algorithm is not consistent.

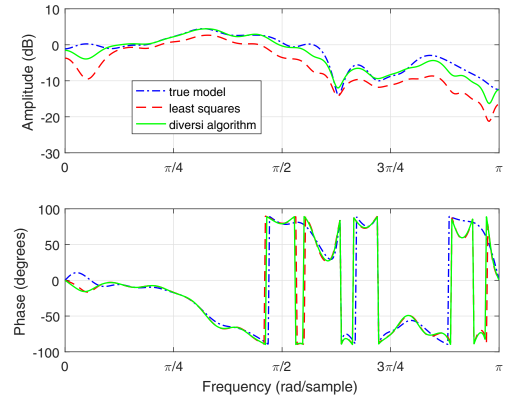

Fig. 7. Frequency response of the identified transmissibilities. The pseudo-input $y_{\rm i}$ and pseudo ouput $y_{\rm o}$ are corrupted with white, Gaussian noise, with input signal-to-noise ratio of 4.598 dB and output signal-to-noise ratio of 2.4168.

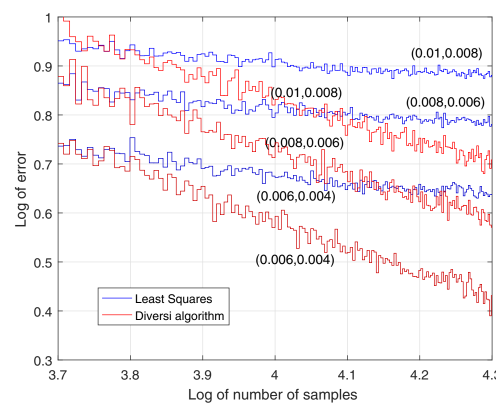

Fig. 8. Distance between the frequency response of the baseline model and the frequency responses of the identified transmissbilities. The input noise v is white, where the input noise variance is 0.006, 0.008, and 0.01. Also, the output noise w is white, where the output noise variance is 0.004, 0.006, and 0.008. Note that the Diversi algorithm [35] is consistent, and the least squares algorithm is not consistent.