2018 IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, Dec. 17-19, 2018

Reconstruction of Aircraft Lateral Dynamics Using Two Transmissibility Operators

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Abstract— **Transmissibilities relate one system output to another system output without knowledge of the external excitation. The numerator and denominator of a transmissibility are thus the numerators of the respective input-output transfer functions from the external excitation to the sensor signals. In this paper, we show that a pair of transmissibilities can be used to reconstruct the underlying dynamics of the system. We apply this technique to reconstruct lateral aircraft dynamics with measurements of roll-rate and yaw-rate perturbations from steady straight-line flight and without using knowledge of the control surface deflections and thrust.**

I. Introduction

Linearized dynamics are crucial for understanding the behavior of an aircraft near steady flight conditions [1]. For each steady flight condition, which may be either straight line, circular, or helical, stability and control derivatives can be used to construct a linearized model of the aircraft dynamics. The stability and control derivatives, which appear as entries in the dynamics and control matrices, can be determined through either wind tunnel testing or computational fluid dynamics. Alternatively, it is of interest to develop techniques that can be used to estimate the required aerodynamic coefficients based on flight data [2]. These parameter estimates are based on measurements of the inputs (thrust and control-surface deflections) as well as responses measured by accelerometers, rate gyros, pitot tube, and angle-of-attack and side-slip sensors.

The goal of the present paper is to develop a model of the linearized aircraft dynamics without using knowledge of the control-surface deflections and thrust. In other words, the objective is to use only measurements from the onboard aircraft sensor to estimate the dynamics of the aircraft. Models that are based entirely on sensor data without input data are called *transmissibilities*. Transmissibilities are widely used in acoustics and structural vibration to construct models that predict sound and vibration levels at one location based on measurements at another location [3]–[8]. These models are based entirely on data from microphones and accelerometers without knowledge of the external excitation. Transmissibilities can also be used for sensor fault detection. Assuming that a transmissibility is correctly identified using healthy sensors, subsequent sensor data can be used with the estimated transmissibility to compare the output of the transmissibility with sensor measurements to determine whether one of the sensors is faulty. This is done for acoustics in [9] and aircraft sensors in [10].

From a transfer function point of view, a transmissibility can be viewed as a ratio of numerators of a pair of input-output transfer functions. To see this, let y_1 and y_2 be outputs of a system driven by an input u, so that $y_1 = G_1 u$ and $y_2 = G_2 u$. The corresponding transmissibility $T = y_2/y_1$ thus has the form $T = G_2 u/(G_1 u) = G_2/G_1 = (N_2/D)/(N_1/D) = N_2/N_1$. Consequently, the numerator and denominator of a transmissibility are the numerators of transfer functions, and thus information about the dynamics of the system (embedded in the cancelled denominator D) does not appear.

The contribution of the present paper is to show that it is possible to use sensor-only measurements to reconstruct the underlying dynamics of the system without measurements of the external inputs. This means that the poles of the input-output transfer function can be estimated based entirely on transmissibilities. To do this, we assume that a pair of transmissibilities can be constructed based on data arising from possibly unknown excitation from two separate system inputs. We then show that the resulting transmissibilities can be used to reconstruct the dynamics of the underlying system.

II. Transmissibility Operators

Consider the single-input, two-output system

$$\dot{x}(t) = Ax(t) + bu(t), \quad (1)$$

$$x(0) = x_0, \quad (2)$$

$$y_{\rm i}(t) \stackrel{\triangle}{=} c_{\rm i}x(t) + d_{\rm i}u(t), \quad (3)$$

$$y_{\rm o}(t) \stackrel{\triangle}{=} c_{\rm o}x(t) + d_{\rm o}u(t), \quad (4)$$

where $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $c_{\rm i}$, $c_{\rm o} \in \mathbb{R}^{1\times n}$, and $d_{\rm i}$, $d_{\rm o} \in \mathbb{R}$. The input signal u and the output signals $y_{\rm i}$ and $y_{\rm o}$ are scalar. Define the polynomials

$$\Gamma_{\rm i}(\mathbf{p}) \stackrel{\triangle}{=} c_{\rm i}{\rm adj}(\mathbf{p}I - A)b + d_{\rm i}\delta(\mathbf{p}), \quad (5)$$

$$\Gamma_{\rm o}(\mathbf{p}) \stackrel{\triangle}{=} c_{\rm o}{\rm adj}(\mathbf{p}I - A)b + d_{\rm o}\delta(\mathbf{p}), \quad (6)$$

$$\delta(\mathbf{p}) \stackrel{\triangle}{=} \det(\mathbf{p}I - A), \quad (7)$$

where $\mathbf{p} \stackrel{\triangle}{=} {\rm d}/{\rm d}t$, and assume that $\Gamma_{\rm i}(\mathbf{p})$ is not the zero polynomial. Combining (5), (6) yields [11]

$$\Gamma_{\rm i}(\mathbf{p})y_{\rm o} = \Gamma_{\rm o}(\mathbf{p})y_{\rm i}. \quad (8)$$

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We write (8) as

$$y_o = \mathcal{T}_{c_o,c_i|b}(\mathbf{p})y_i, \tag{9}$$

where the *transmissibility operator* from y_i to y_o is defined by [11]

$$\mathcal{T}_{c_o,c_i|b}(\mathbf{p}) \triangleq \frac{\Gamma_o(\mathbf{p})}{\Gamma_i(\mathbf{p})}. \tag{10}$$

Since division by $\mathbf{p}$ is not defined, (9) and (10) provide a convenient representation of the time-domain relation (8). The additional subscript b indicates that the transmissibility operator depends on the way in which the input drives the system.

Since $\mathcal{T}_{c_o,c_i|b}$ is not the forced response of a linear system, $\mathcal{T}_{c_o,c_i|b}$ is not a transfer function in the usual sense. Moreover, note that the roots of Γ_o and Γ_i are the zeros of the transfer functions from u to y_o and y_i, respectively. Finally, unlike the complex Laplace variable s, the time-domain operator $\mathbf{p}$ in (9) accounts for nonzero initial conditions [11], [12].

III. Using Transmissibilities to Reconstruct System Dynamics

Let $b_1, b_2 \in \mathbb{R}^n$. Then the transmissibilities from y_i to y_o with $b = b_1$ and $b = b_2$ are given, respectively, by

$$\mathcal{T}_{c_o,c_i|b_1}(\mathbf{p}) \triangleq \frac{\Gamma_{o,1}(\mathbf{p})}{\Gamma_{i,1}(\mathbf{p})}, \quad \mathcal{T}_{c_o,c_i|b_2}(\mathbf{p}) = \frac{\Gamma_{o,2}(\mathbf{p})}{\Gamma_{i,2}(\mathbf{p})}, \tag{11}$$

where

$$\Gamma_{i,1}(\mathbf{p}) \triangleq c_i \mathrm{adj}(\mathbf{p}I - A)b_1 + d_i\delta(\mathbf{p}), \tag{12}$$

$$\Gamma_{o,1}(\mathbf{p}) \triangleq c_o \mathrm{adj}(\mathbf{p}I - A)b_1 + d_o\delta(\mathbf{p}), \tag{13}$$

$$\Gamma_{i,2}(\mathbf{p}) \triangleq c_i \mathrm{adj}(\mathbf{p}I - A)b_2 + d_i\delta(\mathbf{p}), \tag{14}$$

$$\Gamma_{o,2}(\mathbf{p}) \triangleq c_o \mathrm{adj}(\mathbf{p}I - A)b_2 + d_o\delta(\mathbf{p}). \tag{15}$$

Using these two transmissibilities, define

$$\Delta(\mathbf{p}) \triangleq \begin{bmatrix} \Gamma_{i,1}(\mathbf{p}) & \Gamma_{i,2}(\mathbf{p}) \\ \Gamma_{o,1}(\mathbf{p}) & \Gamma_{o,2}(\mathbf{p}) \end{bmatrix}, \tag{16}$$

and note that

$$\begin{aligned} \det\Delta(\mathbf{p}) &= \Gamma_{i,1}(\mathbf{p})\Gamma_{o,2}(\mathbf{p}) - \Gamma_{o,1}(\mathbf{p})\Gamma_{i,2}(\mathbf{p}) \\ &= c_i \mathrm{adj}\,(\mathbf{p}I - A)b_1 c_o \mathrm{adj}\,(\mathbf{p}I - A)b_2 \\ &\quad - c_o \mathrm{adj}\,(\mathbf{p}I - A)b_1 c_i \mathrm{adj}\,(\mathbf{p}I - A)b_2 \\ &\quad + \delta(\mathbf{p})(d_i c_o - d_o c_i)\mathrm{adj}\,(\mathbf{p}I - A)(b_2 - b_1). \end{aligned} \tag{17}$$

The following result shows that the eigenvalues of A are elements of $\mathrm{mroots}(\det\Delta(\mathbf{p}))$. This result thus shows that the eigenvalues of A can be estimated using knowledge of two transmissibility operators.

Theorem 3.1: Let $n \geq 2$. Then exactly one of the following statements is true:

$i)$ $\det\Delta(\mathbf{p}) = 0$.

$ii)$ $\deg\det\Delta(\mathbf{p}) \geq n$.

If $ii)$ holds, then

$$\mathrm{mspec}(A) \subseteq \mathrm{mroots}(\det\Delta(\mathbf{p})). \tag{18}$$

It is important to note that $\mathrm{roots}(\det\Delta(\mathbf{p}))$ are the zeros of $\mathcal{T}_{c_o,c_i|b_1} - \mathcal{T}_{c_o,c_i|b_2}$ and $\mathcal{T}_{c_o,c_i|b_2} - \mathcal{T}_{c_o,c_i|b_1}$.

IV. Numerical Examples

We use the following example to illustrate Theorem 3.1.

Example 4.1: Consider the state space system with

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}. \tag{19}$$

$$c_i = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \quad c_o = \begin{bmatrix} -1 & 3 & -2 \end{bmatrix}, \tag{20}$$

$$b_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}, \quad b_2 = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}^{\mathrm{T}}. \tag{21}$$

Then, $\delta(\mathbf{p}) = \mathbf{p}^3 - 4\mathbf{p}^2 + 6\mathbf{p} - 3$, and thus $\mathrm{mspec}(A) = \{1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}\jmath\}$. Moreover, it follows from (12)–(15) that

$$\Gamma_{i,1} = 2\mathbf{p}^2 - 4\mathbf{p} + 3, \quad \Gamma_{o,1} = 3\mathbf{p}^2 - 6\mathbf{p} + 6, \tag{22}$$

$$\Gamma_{i,2} = 3\mathbf{p}^2 - 9\mathbf{p} + 5, \quad \Gamma_{o,2} = 5\mathbf{p}^2 - 11\mathbf{p} + 3. \tag{23}$$

Using (16), we have

$$\Delta(\mathbf{p}) = \begin{bmatrix} 2\mathbf{p}^2 - 4\mathbf{p} + 3 & 3\mathbf{p}^2 - 9\mathbf{p} + 5 \\ 3\mathbf{p}^2 - 6\mathbf{p} + 6 & 5\mathbf{p}^2 - 11\mathbf{p} + 3 \end{bmatrix},$$

and thus,

$$\det\Delta(\mathbf{p}) = \mathbf{p}^4 + 3\mathbf{p}^3 - 22\mathbf{p}^2 + 39\mathbf{p} - 21. \tag{24}$$

Noting that $\det\Delta(\mathbf{p}) = \delta(\mathbf{p})(\mathbf{p} + 7)$, it follows that

$$\mathrm{mroots}(\det\Delta(\mathbf{p})) = \{1, -7, \frac{1}{2} \pm \frac{\sqrt{3}}{2}\jmath\}, \tag{25}$$

which confirms (18). ◇

Example 4.2: Consider the state space model

$$A = \begin{bmatrix} -0.7000 & 0.2300 & 0.2390 & 0.0546 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{26}$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 4 & 1 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 8 & 1 & 1 & 0 \\ 1 & 3 & 1 & 6 \end{bmatrix}, D = 0, \tag{27}$$

where $u = [u_1\ u_2]^{\mathrm{T}}$ and $y = [y_1\ y_2\ y_3]^{\mathrm{T}}$. Suppose that $u_2 = 0$, then the transmissibility operators from y_1 to y_2 and from y_1 to y_3 are

$$\mathcal{T}_{2,1|1} = \frac{13\mathbf{q}^3 + 14.99\mathbf{q}^2 + 5.395\mathbf{q} + 0.6552}{15\mathbf{q}^3 + 15.99\mathbf{q}^2 + 4.709\mathbf{q} + 0.4914}, \tag{28}$$

$$\mathcal{T}_{3,1|1} = \frac{8\mathbf{q}^3 + 34.09\mathbf{q}^2 + 26.91\mathbf{q} + 5.39}{15\mathbf{q}^3 + 15.99\mathbf{q}^2 + 4.709\mathbf{q} + 0.4914}. \tag{29}$$

Next, assuming $u_1 = 0$, the transmissibility operators from y_1 to y_2 and from y_1 to y_3 are

$$\mathcal{T}_{2,1|2} = \frac{\mathbf{q}^3 + 3.049\mathbf{q}^2 + 0.5004\mathbf{q} + 0.1092}{3\mathbf{q}^3 + 2.394\mathbf{q}^2 - 0.0482\mathbf{q} + 0.273}, \tag{30}$$

$$\mathcal{T}_{3,1|2} = \frac{7\mathbf{q}^3 + 11.19\mathbf{q}^2 + 3.525\mathbf{q} - 2.596}{3\mathbf{q}^3 + 2.394\mathbf{q}^2 - 0.0482\mathbf{q} + 0.273}. \tag{31}$$

Note that

$$\mathcal{T}_{2,1|1} - \mathcal{T}_{2,1|2} = \tag{32}$$

$$\frac{24\mathbf{q}^6 + 14.36\mathbf{q}^5 - 9.519\mathbf{q}^4 - 6.781\mathbf{q}^3 - 0.2005\mathbf{q}^2 + 0.681\mathbf{q} + 0.125}{45\mathbf{q}^6 + 83.86\mathbf{q}^5 + 51.67\mathbf{q}^4 + 16.07\mathbf{q}^3 + 5.313\mathbf{q}^2 + 1.262\mathbf{q} + 0.1342},$$

and

$$\mathcal{T}_{3,1|1} - \mathcal{T}_{3,1|2} = \tag{33}$$
$$\frac{-81\mathbf{q}^6 - 158.4\mathbf{q}^5 - 102.9\mathbf{q}^4 + 7.533\mathbf{q}^3 + 40.3\mathbf{q}^2 + 17.58\mathbf{q} + 2.747}{45\mathbf{q}^6 + 83.86\mathbf{q}^5 + 51.67\mathbf{q}^4 + 16.07\mathbf{q}^3 + 5.313\mathbf{q}^2 + 1.262\mathbf{q} + 0.1342}.$$

Note that $\mathrm{eig}\,(A) = \{0.6, -0.7, -0.3 \pm \jmath 0.2\}$. Moreover, note that the roots of the numerator of $\mathcal{T}_{2,1|1} - \mathcal{T}_{2,1|2}$ are $\{0.6, -0.7, 0.364, -0.2625, -0.3 \pm \jmath 0.2\} \supset \mathrm{eig}\,(A)$, and the roots of the numerator of $\mathcal{T}_{3,1|1} - \mathcal{T}_{3,1|2}$ are $\{0.6, -0.7, -0.3 \pm \jmath 0.2, -0.628 \pm \jmath 0.4761\} \supset \mathrm{eig}\,(A)$. $\diamond$

V. Estimation of Transmissibility Operators

Since transmissibility operators can be unstable, noncausal, and of unknown order, noncausal FIR models can be used to approximate transmissibility operators [13]. Noncausal FIR models have been used to approximate transmissibility operators that are possibly unstable, noncausal, and of unknown order in [8], [9]. However, if the transmissibility operator has poles on the unit circle, then the Laurent expansion coefficients of the transmissibility in each annulus are bounded away from zero [13], [14]. To overcome this difficulty, composite FIR/IIR (CFI) models can be used to approximate systems with poles on the unit circle [15].

A. Transmissibility Approximation Using Composite Noncausal FIR/IIR (CFI) Models

Note that $\mathcal{T}$ can be written as

$$\mathcal{T}(\mathbf{q}) = \frac{(\mathbf{q} - z_1)\cdots(\mathbf{q} - z_m)}{(\mathbf{q} - p_1)\cdots(\mathbf{q} - p_n)}, \tag{34}$$

where n is the order of $\mathcal{T}$, $z_1, \ldots, z_m \in \mathbb{C}$ are the zeros of $\mathcal{T}$, and $p_1, \ldots, p_n \in \mathbb{C}$ are the poles of $\mathcal{T}$. Let $l \in \{0, \ldots, n\}$ be the number of poles of $\mathcal{T}$ located on the unit circle. Hence, for all $i = 1, \ldots, n - l$, $|p_i| < 1$, and, assuming $l \geq 1$, for all $i = n - l + 1, \ldots, n$, $|p_i| \geq 1$. Then, define

$$D_{\mathrm{I},l}(\mathbf{q}) \triangleq (\mathbf{q} - p_{n-l+1})\cdots(\mathbf{q} - p_n) = \mathbf{q}^l + \sum_{i=1}^{l-1} c_i \mathbf{q}^i, \tag{35}$$

where $c_1, \ldots, c_l \in \mathbb{R}$. Then $\mathcal{T}$ can be written as

$$\mathcal{T}(\mathbf{q}) = \frac{1}{D_{\mathrm{I},l}(\mathbf{q})} \mathcal{T}_l(\mathbf{q}), \tag{36}$$

where

$$T_l(\mathbf{q}) \triangleq \frac{(\mathbf{q} - z_1)\cdots(\mathbf{q} - z_m)}{(\mathbf{q} - p_1)\cdots(\mathbf{q} - p_{n-l})}. \tag{37}$$

Note that all of the poles of $\mathcal{T}_l$ are either in the open unit disk or outside the closed unit disk.

Let $\mathbb{A}(\rho_1, \rho_2) \triangleq \{z \in \mathbb{C} : \rho_1 < |z| < \rho_2\}$ denote the open annulus centered at the origin with inner radius $0 \leq \rho_1 < 1$ and outer radius $1 \leq \rho_2$. Then, the Laurent expansion of $\mathcal{T}_l$ in $\mathbb{A}(\rho_1, \rho_2)$ is given by

$$\mathcal{T}_l(z) = \sum_{i=-\infty}^{\infty} h_i z^{-i}, \tag{38}$$

where for all $i \in \mathbb{Z}$, $h_i \in \mathbb{R}$. Using (36) and (38) implies that, for all $z \in \mathbb{A}(\rho_1, \rho_2)$,

$$\mathcal{T}(z) = \frac{1}{D_{\mathrm{I},l}(z)} \sum_{i=-\infty}^{\infty} h_i z^{-i}. \tag{39}$$

Truncating the sum in (39) yields the truncated model

$$\mathcal{T}_{l,r,d}(\mathbf{q}) \triangleq \frac{1}{D_{\mathrm{I},l}(\mathbf{q})} \mathcal{T}_{l,r,d}(\mathbf{q}), \tag{40}$$

where the noncausal FIR truncation $\mathcal{T}_{l,r,d}$ of $\mathcal{T}_l$ is defined by

$$\mathcal{T}_{l,r,d}(\mathbf{q}) \triangleq \sum_{i=-d}^{r} h_i \mathbf{q}^{-i}. \tag{41}$$

B. Least Squares Transmissibility Identification Using Noncausal CFI Models

For all $k \geq 0$, let $y_{\mathrm{i}}(k)$ and $y_{\mathrm{o}}(k)$ be the pseudo input and pseudo output of $\mathcal{T}$ at step k, respectively, and let ℓ denote the size of the data window. Furthermore, let $\theta_{l,r,d} \in \mathbb{R}^{1\times(l+r+d)}$ denote the vector of parameters of the truncated model $\mathcal{T}_{l,r,d}$ defined by (40), where

$$\theta_{l,r,d} = \begin{bmatrix} \theta_{c,l} & \theta_{h,r,d} \end{bmatrix}, \tag{42}$$
$$\theta_{c,l} = \begin{bmatrix} c_1 & \cdots & c_{l-1} \end{bmatrix}, \tag{43}$$
$$\theta_{h,r,d} = \begin{bmatrix} h_{-d} & \cdots & h_r \end{bmatrix}. \tag{44}$$

The least squares estimate $\hat{\theta}_{l,r,d,\ell}$ of $\theta_{l,r,d}$ is given by

$$\hat{\theta}_{l,r,d,\ell} = \underset{\bar{\theta}_{l,r,d}}{\arg\min} \left\| \Psi_{y_{\mathrm{o}},r,l,\ell} - \bar{\theta}_{l,r,d} \Phi_{l,r,d,\ell} \right\|_{\mathrm{F}}, \tag{45}$$

where $\bar{\theta}_{l,r,d} \in \mathbb{R}^{1\times(l+r+d-1)}$ and the components of $\hat{\theta}_{l,r,d,\ell}$ are the coefficients of the noncausal CFI model (40) given by

$$\hat{\theta}_{l,r,d,\ell} = \begin{bmatrix} \hat{\theta}_{c,r,d,\ell} & \hat{\theta}_{h,r,d,\ell} \end{bmatrix},$$
$$\hat{\theta}_{c,l,r,d,\ell} = \begin{bmatrix} \hat{c}_{1,\ell} & \cdots & \hat{c}_{l-1,\ell} \end{bmatrix},$$
$$\hat{\theta}_{h,l,r,d,\ell} = \begin{bmatrix} \hat{h}_{-d,\ell} & \cdots & \hat{h}_{r,\ell} \end{bmatrix},$$
$$\Psi_{y_{\mathrm{o}},r,l,\ell} \triangleq \begin{bmatrix} y_{\mathrm{o}}(r+l) & \cdots & y_{\mathrm{o}}(\ell - d + l) \end{bmatrix},$$
$$\Phi_{l,r,d,\ell} \triangleq \begin{bmatrix} \Phi_{y_{\mathrm{o}},l,\ell} \\ \Phi_{y_{\mathrm{i}},r,d,\ell} \end{bmatrix},$$
$$\Phi_{y_{\mathrm{o}},l,\ell} \triangleq \begin{bmatrix} \phi_{y_{\mathrm{o}},l}(r) & \cdots & \phi_{y_{\mathrm{o}},l}(\ell - d) \end{bmatrix},$$
$$\Phi_{y_{\mathrm{i}},r,d,\ell} \triangleq \begin{bmatrix} \phi_{y_{\mathrm{i}},r,d}(r+d) & \cdots & \phi_{y_{\mathrm{i}},r,d}(\ell) \end{bmatrix},$$
$$\phi_{y_{\mathrm{o}},l}(k) = \begin{bmatrix} -y_{\mathrm{o}}(k+1) & \cdots & -y_{\mathrm{o}}(k+l-1) \end{bmatrix}^{\mathrm{T}},$$
$$\phi_{y_{\mathrm{i}},r,d}(k) = \begin{bmatrix} y_{\mathrm{i}}(k) & \cdots & y_{\mathrm{i}}(k-r-d) \end{bmatrix}^{\mathrm{T}}.$$

C. Constructing an IIR Transmissibility Model from a Noncausal CFI Model

In order to construct an IIR model of a transmissibility based on an approximate noncausal CFI model, we reconstruct the asymptotically stable and unstable parts of the transmissibility separately using the eigensystem realization algorithm (ERA) [13], [16]. Then, we obtain an IIR model of the transmissibility by adding the identified IIR part of the CFI model to the asymptotically stable and unstable IIR parts of the transmissibility reconstructed using the ERA algorithm. By choosing sufficiently large model orders l of the IIR part of the model, $n_{\rm s}$ of the stable component, and $n_{\rm u}$ of the unstable component of the transmissibility, we overestimate the orders of the IIR part, and the stable and unstable parts of the transmissibility. We then cancel all zeros and poles that are within 10^{-6} distance of each other.

VI. Application to Aircraft Lateral Dynamics

Let $\delta\beta$, $\dot\phi$, and $\dot\psi$ denote sideslip-angle, roll-rate, and yaw-rate perturbations, respectively. The lateral dynamics of an aircraft can be represented by

$$\dot x = Ax + Bu, \tag{46}$$

where

$$x \stackrel{\triangle}{=} \begin{bmatrix} \delta\beta \\ \dot\phi \\ \dot\psi \\ \phi \end{bmatrix}, A \stackrel{\triangle}{=} \Gamma \begin{bmatrix} \frac{Y_{\beta_0}}{U_0} & \frac{Y_{p_0}}{U_0} & \frac{Y_{r_0}-U_0}{U_0} & \frac{g}{U_0} \\ L_{\beta_0}+L_{T_{\beta_0}} & L_{p_0} & L_{r_0} & 0 \\ N_{\beta_0}+N_{T_{\beta_0}} & N_{p_0} & N_{r_0} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\Gamma \stackrel{\triangle}{=} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{I_{xx}I_{zz}}{I_{xx}I_{zz}-I_{xz}^2} & \frac{I_{xz}I_{zz}}{I_{xx}I_{zz}-I_{xz}^2} & 0 \\ 0 & \frac{I_{xz}I_{xx}}{I_{xx}I_{zz}-I_{xz}^2} & \frac{I_{xx}I_{zz}}{I_{xx}I_{zz}-I_{xz}^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

All units and descriptions of the variables above are given in [1, p. 358]. Let δa and δr be the aileron and rudder actuation signal, respectively. If the aileron is actuated, then $u = \delta a$ and $B = B_{\delta a}$, where

$$B_{\delta a} \stackrel{\triangle}{=} \Gamma \begin{bmatrix} Y_{\delta a_0} & L_{\delta a_0} & N_{\delta a_0} & 0 \end{bmatrix}^{\rm T}.$$

If the rudder is actuated, then $u = \delta r$ and $B = B_{\delta r}$, where

$$B_{\delta r} \stackrel{\triangle}{=} \Gamma \begin{bmatrix} Y_{\delta r_0} & L_{\delta r_0} & N_{\delta r_0} & 0 \end{bmatrix}^{\rm T}.$$

Consider the aircraft example given in [1, p. 358], where $U_0 = 400$, $Y_{\beta_0} = -55.4022$, $Y_{p_0} = 0$, $Y_{r_0} = 0.7689$, $g = 32.174$, $L_{\beta_0} = -4.1845$, $L_{p_0} = -0.4365$, $L_{r_0} = 0.1571$, $L_{T_{\beta_0}} = 0$, $N_{r_0} = -0.1148$, $N_{T_{\beta_0}} = 0$, $N_{\beta_0} = 2.8643$, $N_{p_0} = 0.0046$, $Y_{\delta_{a_0}} = 0$, $Y_{\delta_{r_0}} = 10.4733$, $L_{\delta_{a_0}} = 6.7714$, $N_{\delta_{a_0}} = -0.3879$, $L_{\delta_{a_0}} = 6.7714$, $L_{\delta_{r_0}} = 0.6543$, $N_{\delta_{a_0}} = -0.3879$, $N_{\delta_{r_0}} = -1.6847$, $I_{xx} = 27915$, $I_{zz} = 47085$, $I_{xz} = 450$.

The state space model (46) is in continuous time. Henceforth, we assume that measurements of the output signals are obtained in discrete time, and we consider discrete-time transmissibility operators in the forward-shift operator $\mathbf{q}$. We use a sampling time of $T_{\rm s} = 0.1$ sec for discritization.

Consider the case where the aileron is actuated, and δa is a realization of zero-mean, Gaussian, white noise with density $\mathcal{N}(0, 1)$. We use least squares with a noncausal CFI model with $l = 4$, $r = 25$, and $d = 25$ to identify the transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a}$ from $\delta\beta$ to $\dot\phi$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta a}$ from $\delta\beta$ to $\dot\psi$. Figure 1 shows the poles and Markov parameters of the identified noncausal CFI models of the transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a}$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta a}$.

Next, consider the case where the rudder is actuated, and δr is a realization of a zero-mean, Gaussian, white noise with the density $\mathcal{N}(0, 1)$. We use least squares with a noncausal CFI model with $l = 4$, $r = 25$, and $d = 25$ to identify the transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a}$ from $\delta\beta$ to $\dot\phi$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta a}$ from $\delta\beta$ to $\dot\psi$. Figure 2 shows the poles and Markov parameters of the identified noncausal CFI models of the transmissbilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta r}$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta r}$.

Next, we use ERA with $n_{\rm s} = 6$ and $n_{\rm u} = 6$ to construct IIR models of the FIR parts of the identified noncausal CFI models of the transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a}$, $\mathcal{T}_{\dot\psi,\delta\beta|\delta a}$, $\mathcal{T}_{\dot\phi,\delta\beta|\delta r}$, and $\mathcal{T}_{\dot\psi,\delta\beta|\delta r}$. Then, we multiply the identified IIR parts of the estimated noncausal CFI models of the transmissibilities by the IIR models constructed using ERA to obtain IIR models of the estimated transmissibilities. Figure 3 shows the IIR models of the estimated transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a}$, $\mathcal{T}_{\dot\psi,\delta\beta|\delta a}$, $\mathcal{T}_{\dot\phi,\delta\beta|\delta r}$, and $\mathcal{T}_{\dot\psi,\delta\beta|\delta r}$. Moreover, Figure 4 shows the pole-zero maps of the identified discrete-time IIR models of the transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a} - \mathcal{T}_{\dot\phi,\delta\beta|\delta r}$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta a} - \mathcal{T}_{\dot\psi,\delta\beta|\delta r}$, respectively. The numerical values of the zeros of the identified transmissibilities $\mathcal{T}_{\dot\phi,\delta\beta|\delta a} - \mathcal{T}_{\dot\phi,\delta\beta|\delta a}$ and $\mathcal{T}_{\dot\psi,\delta\beta|\delta a} - \mathcal{T}_{\dot\psi,\delta\beta|\delta a}$ show that both transmissibilities share the zeros $\{0.9788 \pm \jmath 0.1672, 0.9469, 0.9998\}$, which map to $\{-0.0706 \pm \jmath 1.6917, -0.0016, -0.5456\}$ in continuous time, where the eigenvalues of the A matrix in (46) are $\{-0.0706 \pm \jmath 1.6917, -0.0016, -0.5455\}$. Note that the estimated and true eigenvalues are close to each other.

VII. Conclusions

This paper provided a novel technique for reconstructing the underlying dynamics of a linear system based on estimates of two transmissibility operators. The reconstruction technique was applied to lateral aircraft dynamics with unknown aileron and rudder excitation and measurements of roll-rate and yaw-rate perturbations from steady straight-line flight. Future research will focus on the effect of sensor noise on the accuracy of the reconstructed dynamics.

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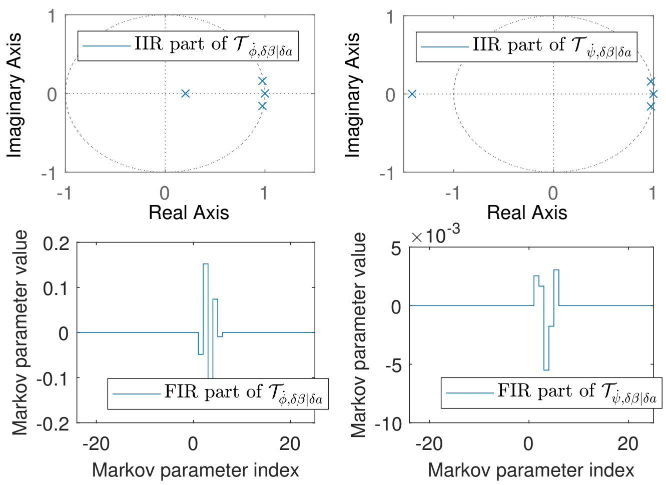

Fig. 1: Poles and Markov parameters of the estimated noncausal CFI model of the transmissibilities $\mathcal{T}_{\dot{\phi},\delta\beta|\delta a}$ and $\mathcal{T}_{\dot{\psi},\delta\beta|\delta a}$ obtained using least squares with $l = 4$, $r = 25$, and $d = 25$.

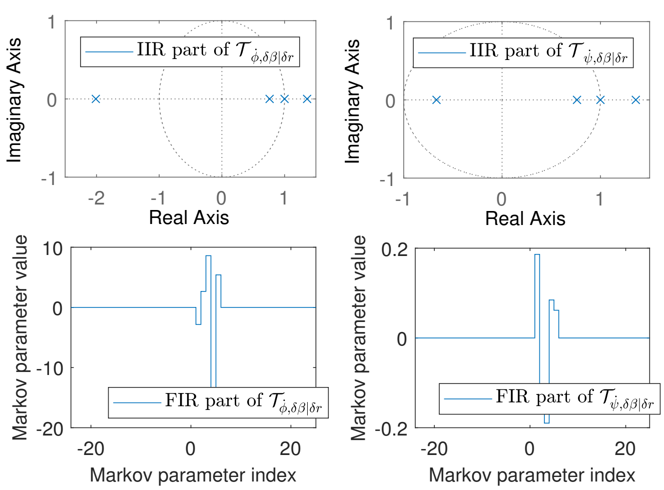

Fig. 2: Poles and Markov parameters of the estimated noncausal CFI model of the transmissibilities $\mathcal{T}_{\dot{\phi},\delta\beta|\delta r}$ and $\mathcal{T}_{\dot{\psi},\delta\beta|\delta r}$ obtained using least squares with $l = 4$, $r = 25$, and $d = 25$.

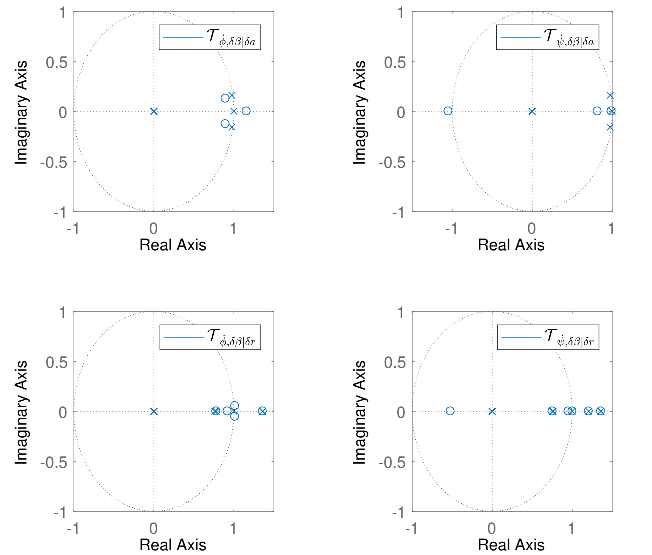

Fig. 3: Pole-zero maps of the IIR models of the estimated transmissibilities $\mathcal{T}_{\dot{\phi},\delta\beta|\delta a}$, $\mathcal{T}_{\dot{\psi},\delta\beta|\delta a}$, $\mathcal{T}_{\dot{\phi},\delta\beta|\delta r}$, and $\mathcal{T}_{\dot{\psi},\delta\beta|\delta r}$.

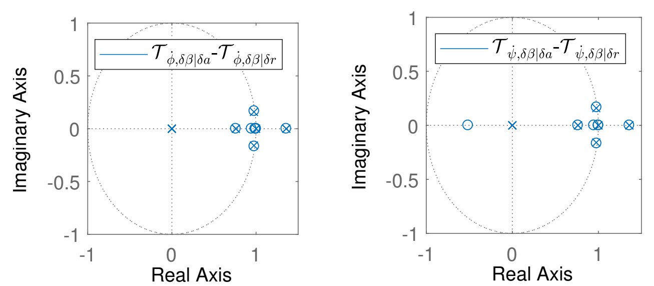

Fig. 4: Pole-zero maps of the IIR models of the estimated transmissibilities $\mathcal{T}_{\dot{\phi},\delta\beta|\delta a} - \mathcal{T}_{\dot{\phi},\delta\beta|\delta r}$ and $\mathcal{T}_{\dot{\psi},\delta\beta|\delta a} - \mathcal{T}_{\dot{\psi},\delta\beta|\delta r}$.