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Experimental Comparison of Adaptive Cancellation Algorithms for Active Noise Control *

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Abstract

With the success of adaptive cancellation methods developed largely within the active noise control community, it is of interest to understand these algorithms within a more traditional feedback control framework. This paper thus has two goals, namely, to systematically describe three such algorithms (two LMS algorithms and the recently developed ARMARKOV/Toeplitz algorithm) in standard feedback control terminology, and to experimentally compare the performance of the algorithms. For experimental purposes, we use an acoustic duct testbed with both tonal and broadband disturbances.

1. Introduction

One of the main uses of feedback control is to suppress unwanted disturbances which can cause excessive vibration levels and poor system performance. Disturbances can arise from a wide variety of sources. For example, rotating machinery can cause tonal or harmonic multi-tone disturbances, while turbulence can give rise to wide-band noise. The reduction of noise and vibration levels can be an important issue in aerospace vehicles.

In recent years there has been considerable progress in developing algorithms for *adaptive disturbance cancellation.* Unlike fixed-gain control methods, these adaptive techniques generally require only limited plant models and information about the disturbance spectrum. Thus they are useful for cases in which the plant may change or may be difficult to identify. Since some of these algorithms exploit measurements of the disturbance signal and ignore the dynamics in the feedback path, they are often referred to as *feedforward algorithms.* In many cases, such algorithms have been developed outside of the traditional "feedback" control community.

The effectiveness of these adaptive cancellation algorithms is evident from the wide range of applications where they have been successfully applied [1–4]. The theoretical development of these algorithms is also quite extensive. See, for example, the recent books [5, 6] as well as the representative papers [7–9].

The present paper has two main goals. First, we briefly describe three adaptive cancellation algorithms. Two of these, which are based on the LMS algorithm, are well known and widely used [10–12], while the third, which is called the ARMARKOV/Toeplitz algorithm, was recently developed in [13]. A useful feature of this review is our description of these algorithms in a unified manner from the perspective of the *standard feedback control problem* of [14], which is commonly used in the control literature. The standard control problem has historically been used in robust fixed–gain control, but also provides a systematic framework for adaptive control schemes.

The second goal of the paper is to report experimental results that compare the performance of the various algorithms. To do this, we test each algorithm on an acoustic duct with five different disturbance spectra, namely, single tone, dual tone, moving single tone, broadband, and fan noise. In each case, we evaluate the performance of the algorithms in terms of convergence and rejection level.

2. Adaptive Control and The Standard Problem

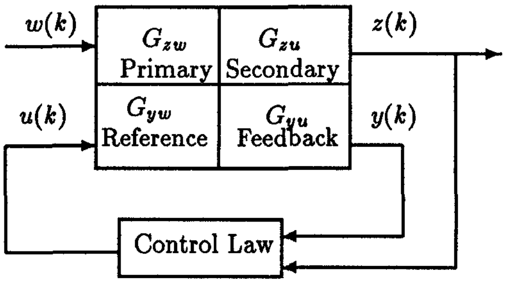

Figure 1: The Standard Problem with Performance Assumption

Consider the linear discrete–time, two–input two–output (TITO), system shown in Figure 1. An exogenous disturbance $w \in \mathcal{R}^{m_w}$ and a control signal $u \in \mathcal{R}^{m_u}$ are the inputs to the plant, while the measurement $y \in \mathcal{R}^{l_y}$, and the performance $z \in \mathcal{R}^{l_z}$ are the outputs. The transfer matrix $G \in \mathcal{R}^{(l_z+l_y)\times(m_w+m_u)}$ is defined by

$$G = \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix}, \qquad (1)$$

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and $G_c \in \mathcal{R}^{l_u \times m_y}$ is the the transfer matrix of the controller. The system is described by the input output relation

$$\begin{bmatrix} z \\ y \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix}, \qquad (2)$$

where the control signal u is given by the feedback relation

$$u = G_c y. \qquad (3)$$

All of these transfer matrices are z-domain representations. The reader should note the notational difference between the performance variable z and the complex variable z used in the z-transform.

Each entry of G is a transfer matrix that represents a distinct noise or vibration path. In the adaptive noise cancellation literature, $G_{zw} \in \mathcal{R}^{l_z \times m_w}$ is the *primary path* corresponding to the transfer matrix from the disturbance w to the performance z, $G_{yw} \in \mathcal{R}^{l_y \times m_w}$ is the *reference path* corresponding to the transfer matrix from the disturbance w to the measurement y, $G_{zu} \in \mathcal{R}^{l_z \times m_u}$ is the *secondary path* corresponding to the transfer matrix from the control u to the performance z, and $G_{yu} \in \mathcal{R}^{l_y \times m_u}$ is the *feedback path* corresponding to the transfer matrix from the control u to the measurement y. Although this terminology is fairly standard in the feedforward control literature, it has not previously been defined within the standard problem framework.

In standard H_2 or H_∞ fixed-gain controller design all of the transfer matrices of G need to be known *a priori*. In general it may be difficult to identify or model all of these transfer matrices with a sufficient level of accuracy. Moreover, the plant and disturbance spectra may be time varying. In these cases, adaptive techniques may improve the system's performance, with less *a priori* modeling. This modeling and disturbance information is implicitly determined on line during the adaptation process.

One fundamental distinction between conventional fixed-gain control and adaptive control is the fact that in fixed-gain control, the performance variable z is not required as a physical measurement, but rather is used as a design variable. In adaptive control, however, little knowledge of the plant dynamics is needed at the expense of requiring real time knowledge of the performance. This requirement dictates that the performance z be measured and fed back to the controller online as shown in Figure 1. The *performance assumption* is said to be satisified when this requirement is met. Additionally, when the control has little effect on the measurement, the feedback path can be ignored, or $G_{yu} = 0$. In this case the *feedforward assumption* is satisfied. Moreover, the dynamics of the reference path are sometimes ignored, or $G_{yw} = I$. In this case the *disturbance measurement assumption* is satisfied. The secondary path is also sometimes ignored in the same manner as $G_{zu} = I$. In this case the *control feedthrough assumption* is said to be satisifed.

3. Adaptive Control Algorithms

In this section we review three adaptive control algorithms within the framework of the standard problem developed in Section 2. Each algorithm attempts to minimize on-line a performance measure based on the performance variable z.

All of the algorithms considered in this paper assume that at each instant in time the controller has a linear structure. The time-varying parameters of the controller are varied by the update algorithm through adaptation.

The adaptive control laws considered in this paper have two distinct parts. The controller $G_{c,k}(z)$ processes the reference signal $y(k)$ and outputs the actuation signal $u(k)$, while the adaptive algorithm modifies parameters within the controller. The adaptive algorithm invokes the performance assumption as a means of adjusting parameters during the adaptation process by utilizing information contained in the performance measurement $z(k)$.

3.1. Filtered–X LMS Algorithm

The filtered-x LMS algorithm (FXLMS), which is perhaps the most commonly used adaptive control algorithm in noise cancellation applications, is based on the least mean square (LMS) algorithm of [10]. This section presents the LMS algorithm and then extends it to the filtered–x LMS algorithm developed in [15] and [16].

The controller considered in this section is assumed to have a finite impulse response (FIR) structure. Because the FIR structure only incorporates zeros into the feedforward path, it is always stable. The update law adjusts the FIR filter coefficients on-line in order to minimize the performance measure.

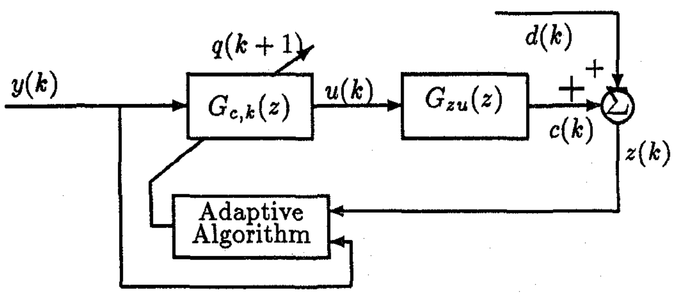

Figure 2: Block Diagram of Adaptive Control Law and Error Path

Consider the block diagram of the control law shown in Figure 2, where $y(k) \in \mathcal{R}$ is the input to the controller, $u(k) \in \mathcal{R}$ is the output of the controller, $d(k) = G_{zw}w(k) \in \mathcal{R}$ is the propagated disturbance, and $c(k) \in \mathcal{R}$ is the cancellation signal that arrives at the point of the performance measurement. The performance $z(k) \in \mathcal{R}$ is chosen as the sum of $d(k)$ and $c(k)$ and is fedback to the adaptive algorithm h. When $c(k) = -d(k)$ perfect rejection is obtained.

The transfer function of the controller $G_{c,k}(z)$ is expressed instantaneously as

$$G_{c,k}(z) = q_0(k) + q_1(k)z^{-1} + \ldots + q_{n-1}(k)z^{-n+1}, \quad (4)$$

where n is the order of the controller. The transfer function of (4) can be more conveniently represented by the vector $q(k) \in \mathcal{R}^n$ which contains the filter coefficients of the FIR filter as

$$q(k) \triangleq [\ q_0(k) \quad q_1(k) \quad \ldots \quad q_{n-1}(k)\ ]^{\mathrm{T}}, \quad (5)$$

where T denotes the transpose. Furthermore, $u(k)$ can be expressed as

$$u(k) = q^{\mathrm{T}}(k)Y(k) \quad (6)$$

where $Y(k)$ is defined by

$$Y(k) \triangleq [\ y(k) \quad y(k-1) \quad \cdots \quad y(k-n+1)\ ]^{\mathrm{T}}. \quad (7)$$

Now consider the error performance measure $\widehat{J}(k) = z^2(k)$. The gradient of $\widehat{J}(k)$ with repsect to $q(k)$ is given by

$$\nabla_{q(k)}\widehat{J}(k) = 2z(k)Y(k). \quad (8)$$

and this gradient is used in the steepest-descent update law

$$q(k+1) = q(k) - \mu z(k)Y(k). \quad (9)$$

The update law (9) is the basic LMS algorithm. A detailed analysis of the LMS algorithm including choices of convergence rate μ is given in [6].

We note that the feedforward assumption is invoked, effectively ignoring propagation of the control signal to the feedback sensor. The presence of this feedback path may destabilize the closed loop system when this assumption is not valid. Additionally, the secondary path assumption is made, thus neglecting the dynamics of the secondary path G_{zu}. This assumption can adversely affect the performance of the algorithm if appreciable dynamics exist in this path.

One way to circumvent the error path assumption is through the use of the *filtered-x LMS algorithm (FXLMS)*, whose name comes from the original derivation in which the input to the controller $y(k)$ was denoted "$x(k)$" and subsequently "filtered" by G_{zu} [15].

The instantaneous gradient of the performance (with a slight abuse of notation) now involves the dynamics of G_{zu} and is given by

$$\nabla_{q(k)}\widehat{J}(k) = 2z(k)G_{zu}(z)Y(k). \quad (10)$$

Defining $Y'(k) \triangleq G_{zu}(z)Y(k)$ as the filtered version of $Y(k)$, the filtered-x LMS algorithm becomes

$$q(k+1) = q(k) - \mu z(k)Y'(k). \quad (11)$$

We note that knowledge of G_{zu} is required in order to implement the adaptive algorithm. One advantage of this method is its insensitivity to errors in the identification of G_{zu} for cases of slow adaptation, as is shown in [17].

3.2. Filtered–U Recursive LMS Algorithm

The filtered-U recursive LMS algorithm was first introduced by [11, 12] utilizing the LMS algorithm for adaptive cancellation with a controller that incorporate both non-zero poles and zeros. In Section 3.1., the FXLMS algorithm invoked the feedforward assumption, thus neglecting the presence of the feedback path. This assumption necessitates slow adaptation in order to maintain stability when the feedback path has a significant effect on the measured signal. As shown by [6], the optimal controller without the feedforward assumption may require a controller with both non-zero poles and zeros. Approximations can be obtained with FIR structures, but only at the expense of increased order. Moreover, [18] has shown that IIR structures perform better than FIR structures with an equal number of free parameters.

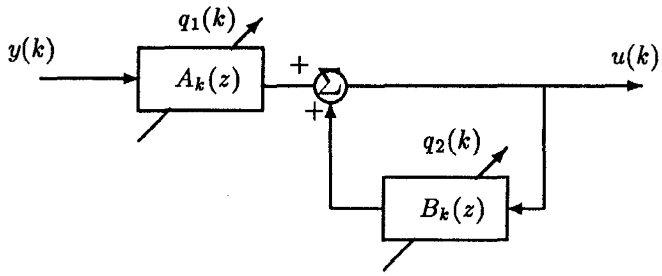

Figure 3: IIR Controller

Consider the IIR controller of Figure 3, and assume that $A_k(z)$ and $B_k(z)$ are FIR transfer functions. Define the vector containing the coefficients of $A_k(z) \in \mathcal{R}^n$ and $B_k(z) \in \mathcal{R}^m$ at time k as

$$a(k) \triangleq [\ a_0(k) \quad a_1(k) \quad \cdots \quad a_{n-1}(k)\ ]^{\mathrm{T}}, \quad (12)$$

$$b(k) \triangleq [\ b_1(k) \quad b_2(k) \quad \cdots \quad b_m(k)\ ]^{\mathrm{T}}, \quad (13)$$

and

$$U(k-1) \triangleq [\ u(k-1) \quad u(k-2) \quad \cdots \quad u(k-m)\ ]^{\mathrm{T}}. \quad (14)$$

The output of the IIR filter $u(k)$ is now given by given by

$$u(k) = q^{\mathrm{T}}(k)r(k). \quad (15)$$

where $q(k) \triangleq [\ a(k) \quad b(k)\ ]^{\mathrm{T}}$, $r(k) \triangleq [\ Y(k) \quad U(k-1)\ ]^{\mathrm{T}}$, and $Y(k)$ is defined in (7).

As before, the instantaneous square of the performance is to be minimized with respect to the vector $q(k)$. This minimization is performed by the method of steepest descent which involves the gradient of the performance $z(k)$. Assuming slow convergence, [6] has shown that

$$\nabla_{q(k)}\widehat{J}(k) = G_{zu}(z)r(k). \quad (16)$$

Using (16) in the steepest descent algorithm, the *filtered-U recursive LMS algorithm* becomes

$$q(k+1) = q(k) - \mu r'(k) z(k), \tag{17}$$

where $r'(k)$ is defined by

$$r'(k) \triangleq G_{zu}(z) r(k), \tag{18}$$

which is the filtered version of the reference vector $r(k)$. It should be noted that in previous derivations of this algorithm, the vector $r(k)$ was called "$u(k)$" which motivated the name of the algorithm. As in the FXLMS algorithm, the secondary path G_{zu} must be identified off–line before implementation.

A rigorous proof of convergence for this algorithm does not exist. The filtered-U recursive LMS algorithm can be ill-conditioned for large order IIR filters, so that in general the order of the filter should be chosen as small as possible.

3.3. ARMARKOV/Toeplitz Adaptive Algorithm

This algorithm uses the ARMARKOV/Toeplitz system representation described in [19], [13]. The ARMARKOV/Toeplitz system representation relates windows of input and output information and explicitly involves the Markov parameters of the system. In [20] it is shown that these models are less sensitive to measurement noise during identification.

A system described in ARMA form as

$$y(k) = -a_1 y(k-1) - \cdots - a_n y(k-n) + B_0 u(k) + \cdots + B_n u(k-n), \tag{19}$$

can be written in ARMARKOV form with η Markov parameters as

$$y(k) = \sum_{j=1}^{n} -\alpha_j y(k-\eta-j+1) + \sum_{j=1}^{\eta} H_{j-1} u(k-j+1) + \sum_{j=1}^{n} \mathcal{B}_j u(k-\eta-j+1), \tag{20}$$

where the coefficients $\alpha_j \in \mathcal{R}$, $\mathcal{B}_j \in \mathcal{R}^{l_y \times m_u}$, $j = 1, \ldots, n$ and the Markov parameters $H_j \in \mathcal{R}^{l_y \times m_u}$. The Markov parameters of a system are the impulse response coefficients of a system. As shown in [13], the ARMARKOV representation of the standard problem described in Section 2 is

$$Z(k) = W_{zw}\Phi_{zw}(k) + B_{zu}U(k), \tag{21}$$

$$Y(k) = W_{yw}\Phi_{yw}(k) + B_{yu}U(k), \tag{22}$$

where the *extended performance, measurement and control* vectors are defined by

$$Z(k) \triangleq [z(k) \ \cdots \ z(k-p+1)]^{\mathrm{T}}, \tag{23}$$

$$Y(k) \triangleq [y(k) \ \cdots \ y(k-p+1)]^{\mathrm{T}}, \tag{24}$$

$$U(k) \triangleq [u(k) \ \cdots \ u(k-p_c+1)]^{\mathrm{T}}, \tag{25}$$

and the *ARMARKOV regressor vectors* by

$$\Phi_{zw} \triangleq [z(k-\eta) \ \cdots \ z(k-\eta-n-p+2) \quad w(k) \ \cdots \ w(k-\eta-n-p+2)]^{\mathrm{T}}, \tag{26}$$

$$\Phi_{yw} \triangleq [y(k-\eta) \ \cdots \ y(k-\eta-n-p+2) \quad w(k) \ \cdots \ w(k-\eta-n-p+2)]^{\mathrm{T}}, \tag{27}$$

with the positive integer p representing the length of the data window, and $p_c = \eta + n + p - 1$. The matrices W_{zw}, B_{zu}, W_{yw} and B_{yu} are block-Toeplitz matrices which contain the ARMARKOV coefficients and Markov parameters of the system.

Let $\theta(k) \in \mathcal{R}^{l_u, \times n_c * l_u + (n_c + \eta_c - 1) * l_y)}$ denote the matrix of ARMARKOV parameters of an n_cth order controller with η_c Markov parameters such that

$$U(k) = \Sigma_{j=1}^{p_c} L_j \theta(k-j+1) R_j \Phi_{uy}(k). \tag{28}$$

L_j and R_j are constraint matrices that maintain the block-Toeplitz structure of the matrix $\Sigma_{j=1}^{p_c} L_j \theta(k-j+1) R_j$ and

$$\Phi_{uy}(k) \triangleq [u(k-\eta_c) \ \cdots \ u(k-\eta c - n_c - p_c + 2) \quad y(k-1) \ \cdots \ y(k-\eta_c-n_c-p_c+2)]^{\mathrm{T}} \tag{29}$$

We now define the *estimated performance* $\hat{Z}(k)$ and *performance measure* $\hat{J}(k)$ by

$$\hat{Z}(k) \triangleq W_{zw}\Phi_{zw}(k) + B_{zu}\Sigma_{j=1}^{p_c} L_j \theta(k) R_j \Phi_{uy}(k) \tag{30}$$

$$\hat{J}(k) \triangleq \frac{1}{2}\hat{Z}^{\mathrm{T}}(k)\hat{Z}(k). \tag{31}$$

The estimated performance is based on past data with the present controller parameters. We note that $\hat{Z}(k)$ can be calculated without access to $w(k)$ as

$$\hat{Z}(k) = Z(k) + B_{zu}\left(\Sigma_{j=1}^{p_c} L_j \theta(k) R_j \Phi_{uy}(k) - U(k)\right). \tag{32}$$

The gradient based update law for $\theta(k)$ to minimize $\hat{J}(k)$ is given by

$$\theta(k+1) = \theta(k) - \mu(k)\Sigma_{j=1}^{p_c} L_j^{\mathrm{T}} B_{zu}^{\mathrm{T}} \hat{Z}(k) \Phi_{uy}(k)^{\mathrm{T}} R_j^{\mathrm{T}}, \tag{33}$$

where the adaptive step-size $\mu(k) = \frac{1}{\|B_{zu}\|_F^2 \|\Phi_{uy}(k)\|_2^2}$. This adaptive step-size ensures the $\|\theta^* - \theta(k)\|_F$, where θ^* minimizes $\hat{J}(k)$, is always decreasing.

The use of the adaptive step-size yields fast convergence and stable adaptation. Like LMS algorithms, the ARMARKOV/Toeplitz alogrithm requires knowledge of the matrix B_{zu} which represents the path G_{zu}.

4. Experimental Results

All of the algorithms presented in Section 3. were tested on an acoustic duct with a circular cross section. The duct is 80 inches in length and has a diameter of 4 inches. Speakers are used as actuators and their interface to the duct is through a $\frac{1}{2}$ inch diameter opening, while

Algorithm	Free Parameters	Convergence Rate	Type
FXLMS	30	$\mu = 10^{-5}$ (fixed)	FIR
FURLMS	58	$\mu = 10^{-5}$ (fixed)	IIR
ARM/Toep	14	automatic	IIR

Table 1: Algorithm parameters

Algorithm	Single Tone	Dual Tones	White Noise	Fan Noise
FXLMS	29.3dB	26.5 dB	unstable	unstable
FURLMS	56.3 dB	24.3 dB	unstable	unstable
ARM/Toep	89.5 dB	24.9 dB	6.5 dB	60.2 dB

Table 2: Disturbance Attenuation Comparison

microphones are used as sensors. The disturbance speaker is positioned at the closed end of the duct and the measurement microphone is located 4 inches away. The performance microphone is located 6 inches from the open end of the duct and the control speaker is located 16 inches from the open end.

The performance and measurement signals are passed through a low pass anti-aliasing filter, which rolls off at 315 Hz, while a dSPACE ds1102 real time controller running a C30 DSP processor samples at 800 Hz.

Each algorithm was tested with the following disturbance signals: a single tone at 115 Hz, dual tones at 115 Hz and 125 Hz, a moving tone swept from 115 Hz to 125 Hz, band-limited white noise and fan noise.

The algorithm parameters are described in Table 1. Some of the representative open-loop and closed-loop frequency response plots are shown in Figures 4 - 7 and the experimental results are summarized in Table 2.

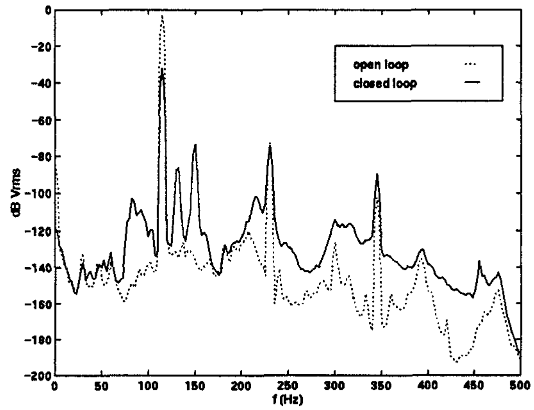

Figure 4: Performance Spectrum z - FXLMS with a single tone

5. Comments

Three adaptive algorithms have been presented in the standard feedback control framework and implemented on

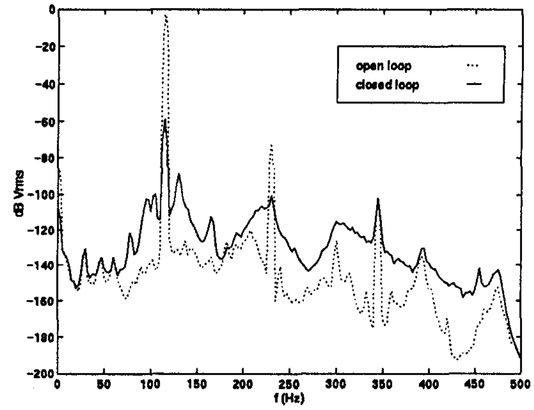

Figure 5: Performance Spectrum z - FUREC with a single tone

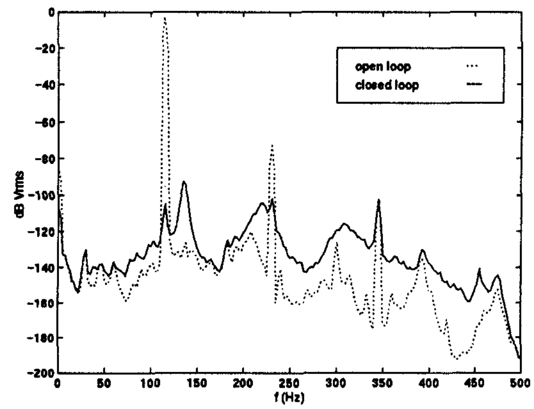

Figure 6: Performance Spectrum z - ARMARKOV with a single tone

an acoustic duct experiment. All of the algorithms were successful in rejecting a computer-generated single tone, moving tones and dual tones to varying degrees. The convergence rate for the LMS algorithms was fixed at the maximum value for which the algorithm was stable. Only the ARMARKOV/Toeplitz algorithm was capable of rejecting the computer generated band-limited white disturbance and the fan disturbance. Also, the ARMARKOV/Toeplitz algorithm converged faster in general as a result of the use of an adaptive step-size.

All of the algorithms introduced high gain at the disturbance frequency, in turn creating a notch in the closed-loop transfer function at the this frequency. Some of the controllers exhibited spillover, making parts of the spectrum louder, but this only occurred near the floor of the background noise. Because this spillover occurred in frequency bands that did not overlap with the disturbance spectrum it had only minor effects on the performance.

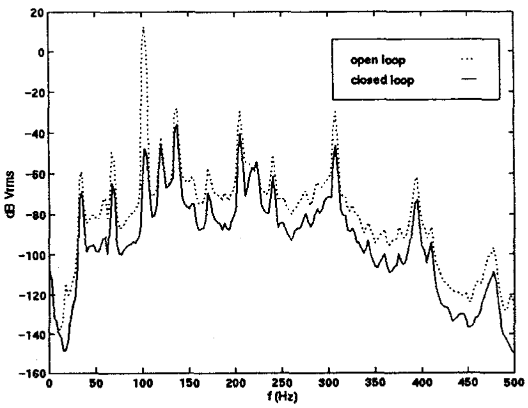

Figure 7: Performance Spectrum z - ARMARKOV with fan

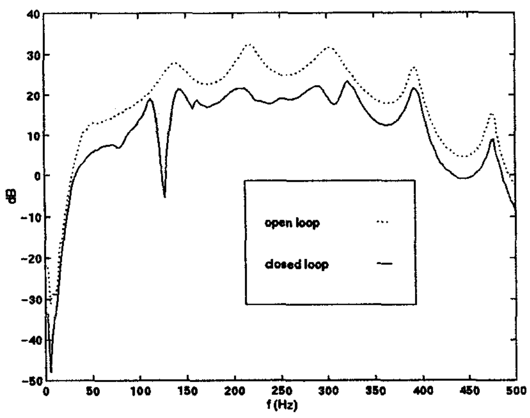

Figure 8: G_{zw} - ARMARKOV with white noise

References

[1] P. A. Nelson and S. J. Elliot. *Active Control of Sound and Vibration.* Academic Press, 1992.

[2] I. M. Stothers S. J. Elliot and P. A. Nelson. A multiple error lms algorithm and its applications to the active control of noise and vibration. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-35:1423–1434, 1987.

[3] U. Emborg. Application of active noise control in saab 340 and saab 2000. In *Proceedings of the Nordic Conference on Vehicle and Machinery Vibrations*, 1994.

[4] C. A. Rogers C. R. Fuller and H. H. Robertshaw. Control of sound radiation with active/adaptive structures. *Journal of Sound and Vibration*, 157:19–39, 1992.

[5] B. Widrow and E. Walach. *Adaptive Inverse Control.* Prentice Hall P T R, 1996.

[6] S. M. Kuo and D. R. Morgan. *Active Noise Control Systems.* Wiley, 1996.

[7] N. J. Bershad P. L. Feintuch and A. K. Lo. A frequency domain model for 'filtered' lms algorithms - stability analysis, design, and elimination of the training mode. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, ASSP-41:1518–1531, 1993.

[8] D. R. Morgan and C. Sanford. A control theory approach to the stability and transient analysis of the filtered-x lms adaptive notch filter. *IEEE Transactions on Signal Processing*, 40:2341–2346, 1992.

[9] D. G. MacMartin. A feedback perspective on the lms disturbance feedforward algorithm. In *Proceedings of the American Control Conference*, pages 1632–1636, 1994.

[10] B. Widrow. Adaptive filters. *Aspects of Network and Systems Theory*, 1970.

[11] L. J. Eriksson. *Active Sound Attenuation Using Adaptive Digital Signal Processing Techniques.* PhD thesis, University of Wisconsin, 1985.

[12] L. J. Eriksson. The selection and application of an iir adaptive filter for use in active sound attenuation. *IEEE Trans. Acoust., Speech, Signal Processing*, 19987.

[13] R. Venugopal and D. S. Bernstein. Adaptive disturbance rejection using armarkov/toeplitz models. In *Proceedings of the American Control Conference*, pages 1657–1661, 1997.

[14] B. Francis. *A Course in H_∞ Control Theory.* Springer-Verlag, 1987.

[15] D. Schur B. Widrow and S. Shaffer. On adaptive inverse control. In *Proc. 15th Asilomar Conf.*, 1981.

[16] J. C. Burgess. Active adaptive sound control in a duct: A computer simulation. *J. Acoust. Soc. Am.*, 1981.

[17] D. R. Morgan. An analysis of multiple correlation cancellation loops with a filter in the auxiliary path. *IEEE Trans. Acoust., Speech, Signal Processing*, 1980.

[18] S. Laugesen and S. J. Elliot. Multichannel active control of random noise in a small reberverant room. *IEEE Trans. Signal Processing*, 1993.

[19] J. C. Akers and D. S. Bernstein. Time-domain identification using armarkov/toeplitz models. In *Proceedings of the American Control Conference*, pages 191–195, 1997.

[20] W. M. Haddad D. C. Hyland, E. G. Collins Jr and D. L. Hunter. Neural network system identification for improved noise rejection. In *Proceedings of the American Control Conference*, pages 345–349, 1995.