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Finite-Horizon Input Selection for System Identification*

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Abstract

The accuracy of an identified model depends on the choice of input signal. Persistency of excitation is a necessary criterion for such signals. In this paper we develop additional criteria for input signal selection, in particular, the input at each time step is chosen to minimize the predicted variance of the system estimate at the next time step. We extend the method to the finite-horizon input selection problem and demonstrate the method in simulation.

1 Introduction

Linear system identification is widely viewed as a mature subject [16, 18, 19, 22, 25]. Many system identification algorithms are designed for arbitrary persistently exciting input sequences. However, the input sequence can have a large effect on the accuracy of the identified model. There are two general guidelines for designing the input sequence: the input sequence should be persistently exciting for the system and system identification algorithm, and the input sequence should maximize the signal to noise ratio. In general, it is desirable to concentrate the input power within the frequency bands of interest, while minimizing the ratio of the peak value of the input sequence to the rms value of the input sequence. In addition, the input sequence must account for the saturation limits of the sensors and actuators. Various input sequences have been developed with these guidelines in mind, for example, pulse, stepped sine, swept sine, Schroeder multi-sine, pseudo-random binary, pseudo-random multilevel, and random input sequences [2, 3, 10–12, 19–21, 23]. These input sequences are generally applied independently of the plant dynamics and noise characteristics.

In practice, the cost of an identification experiment depends on the data length, which can also be limited by time constraints and data processing capabilities. A well-designed input sequence results in a more accurate identified model for a fixed data length. Several authors have studied the system identification and control synthesis problem of optimal feedback for identification [1, 5–7, 9, 12–14, 17, 24, 26]. In this paper we describe a different approach for input sequence selection. The proposed algorithm dynamically selects the input sequence to minimize the trace of the covariance of the estimated parameter vector. A one-step-ahead input sequence selection algorithm is developed to provide a closed form solution for the input at time $k+1$ for least squares identification. We then present a finite-horizon input selection technique and a gradient-search-based optimization approach for selecting the input sequence for time $k+1,\ldots,N$.

2 Problem Formulation

Consider the single-input, single-output, linear time-invariant system

$$y(k) = G(q^{-1})u(k) = \frac{\sum_{i=0}^{n} b_i q^{-i}}{1+\sum_{i=1}^{n} a_i q^{-i}} u(k), \quad (2.1)$$

where q^{-1} is the backward shift operator. This system can be rewritten as

$$y(k) = \sum_{i=0}^{n} b_i u(k-i) - \sum_{i=1}^{n} a_i y(k-i) = \theta^{\mathrm{T}}\phi(k), \quad (2.2)$$

where

$$\theta \stackrel{\triangle}{=} \begin{bmatrix} a_1 & \cdots & a_n & b_n & \cdots & b_0 \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{2n+1} \quad (2.3)$$

and

$$\phi(k) \stackrel{\triangle}{=} \begin{bmatrix} -y(k-1) & \cdots -y(k-n) & u(k-n) & \cdots & u(k) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{2n+1}. \quad (2.4)$$

Data for $k = 1,\ldots,\ell$ is collected as

$$y(n+1:\ell) = \theta^{\mathrm{T}}\phi(n+1:\ell). \quad (2.5)$$

The least squares cost function

$$J_\ell(u(1:\ell), y(1:\ell), \theta) \stackrel{\triangle}{=} \left\| y(n+1:\ell) - \theta^{\mathrm{T}}\phi(n+1:\ell) \right\|_{\mathrm{F}}^2 \quad (2.6)$$

is minimized by

$$\begin{aligned} \theta^\star(\ell) &= \arg\min_\theta J(u(1:\ell), y(1:\ell), \theta) \\ &= \left(\phi(n+1:\ell)\phi^{\mathrm{T}}(n+1:\ell)\right)^{-1} \phi(n+1:\ell) \times \\ &\quad y^{\mathrm{T}}(n+1:\ell), \end{aligned} \quad (2.7)$$

assuming the indicated inverse exists. The cost function at time $\ell+1$ is given by

$$\begin{aligned} &J_{\ell+1}(u(1:\ell+1), y(1:\ell+1), \theta) \\ &= J_\ell(u(1:\ell), y(1:\ell), \theta) + \left\| y(\ell+1) - \theta^{\mathrm{T}}\phi(\ell+1) \right\|_{\mathrm{F}}^2 . \end{aligned} \quad (2.8)$$

Although the measurements $y(1:\ell)$ and $u(1:\ell)$ are available at time $k = 1,\ldots,\ell$, the next input $u(\ell+1)$ is to be determined, and $y(\ell+1)$ has not yet been

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measured.

One approach to selecting $u(\ell+1)$ is to minimize (2.8) when it is evaluated at the current estimate $\theta^\star(\ell)$ of the parameter vector θ, that is,

$$J_{\ell+1}(u(1:\ell+1), y(1:\ell+1), \theta^\star(\ell)) = J_\ell(u(1:\ell), y(1:\ell), \theta^\star(\ell)) + \left\| y(\ell+1) - \theta^{\star\mathrm{T}}(\ell)\phi(\ell+1) \right\|_{\mathrm{F}}^2. \quad (2.9)$$

Since $y(\ell+1)$ has not yet been measured, we use the predicted value of the output

$$\hat{y}(\ell+1) \triangleq \theta^{\star\mathrm{T}}(\ell)\phi(\ell+1). \quad (2.10)$$

With (2.10), the cost function (2.9) becomes

$$J_{\ell+1}(u(1:\ell+1), \begin{bmatrix} y(1:\ell) & \hat{y}(\ell+1) \end{bmatrix}, \theta^\star(\ell)) = J_\ell(u(1:\ell), y(1:\ell), \theta^\star(\ell)), \quad (2.11)$$

which is independent of $u(\ell+1)$. Hence (2.10) is not helpful for choosing $u(\ell+1)$. Alternatively, we choose the future input sequence to minimize the predicted variance of the estimate of the system parameters. This is the problem we will study in the rest of this paper.

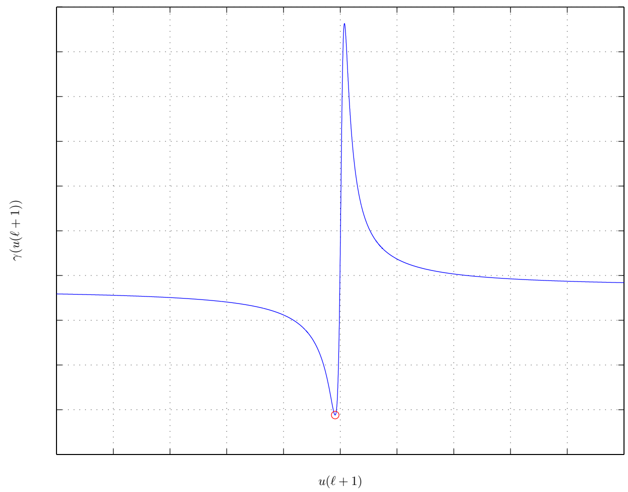

Figure 1: Plot of $\gamma(u(\ell+1))$ for Example in Section 5

3 One-Step-Ahead Input Selection

Our goal is to choose $u(\ell+1)$ to minimize the trace of the predicted value of the covariance of the estimate of the mean of the parameter vector $\mathrm{tr}\,\hat{\sigma}^2_{\theta^\star(\ell)}(\ell+1)$. To formulate this approach, we consider noisy measurements

$$y(k) = \theta^{\mathrm{T}}\phi(k) + w(k), \quad (3.1)$$

where $w(k)$ is zero mean white noise process with covariance σ_w^2. The covariance of the system estimate at time $k = \ell$ is defined by

$$\hat{\sigma}^2_{\theta^\star(\ell)}(\ell) \triangleq \sigma_w^2 \left(\phi(n+1:\ell)\phi^{\mathrm{T}}(n+1:\ell) \right)^{-1}. \quad (3.2)$$

Note that the not-yet-measured output $y(k+1)$ does not appear in (3.2). The predicted value of the covariance of the system estimate at time $k = \ell+1$ is thus

$$\hat{\sigma}^2_{\theta^\star(\ell)}(\ell+1) = \sigma_w^2 \left(Q(\ell) - \frac{Q(\ell)\phi(\ell+1)\phi^{\mathrm{T}}(\ell+1)Q(\ell)}{1 + \phi^{\mathrm{T}}(\ell+1)Q(\ell)\phi(\ell+1)} \right) \quad (3.3)$$

where

$$Q(k) = \left(\phi(n+1:k)\phi^{\mathrm{T}}(n+1:k) \right)^{-1} \quad (3.4)$$

and we have used the matrix inversion lemma ([15] pp. 18-19).

The trace of the covariance matrix is given by

$$\gamma(u(\ell+1)) \triangleq \mathrm{tr}\,\hat{\sigma}^2_{\theta^\star(\ell)}(\ell+1) = \sigma_w^2 \Big(\alpha_0(\ell) - \frac{\alpha_1(\ell) + \alpha_2(\ell)u(\ell+1) + \alpha_3(\ell)u^2(\ell+1)}{\alpha_4(\ell) + \alpha_5(\ell)u(\ell+1) + \alpha_6(\ell)u^2(\ell+1)} \Big), \quad (3.5)$$

where

$$\psi(k) \triangleq \phi(k) - e_{2n+1}u(k) = \begin{bmatrix} -y(k-1) & \cdots & -y(k-n) & u(k-n) & \cdots & u(k-1) & 0 \end{bmatrix}^{\mathrm{T}}, \quad (3.6)$$

$$\alpha_0(\ell) \triangleq \mathrm{tr}\,Q(\ell), \quad (3.7)$$

$$\alpha_1(\ell) \triangleq \mathrm{tr}\,Q(\ell)\psi(\ell+1)\psi^{\mathrm{T}}(\ell+1)Q(\ell), \quad (3.8)$$

$$\alpha_2(\ell) \triangleq \mathrm{tr}\,Q(\ell)\left(\psi(\ell+1)e_{2n+1}^{\mathrm{T}} + e_{2n+1}\psi^{\mathrm{T}}(\ell+1) \right) Q(\ell), \quad (3.9)$$

$$\alpha_3(\ell) \triangleq \mathrm{tr}\,Q(\ell)e_{2n+1}e_{2n+1}^{\mathrm{T}}Q(\ell), \quad (3.10)$$

$$\alpha_4(\ell) \triangleq 1 + \psi^{\mathrm{T}}(\ell+1)Q(\ell)\psi(\ell+1), \quad (3.11)$$

$$\alpha_5(\ell) \triangleq \psi^{\mathrm{T}}(\ell+1)Q(\ell)e_{2n+1} + e_{2n+1}^{\mathrm{T}}Q(\ell)\psi(\ell+1), \quad (3.12)$$

$$\alpha_6(\ell) \triangleq e_{2n+1}^{\mathrm{T}}Q(\ell)e_{2n+1}, \quad (3.13)$$

and $e_i \in \mathbb{R}^{2n+1}$ is the ith column of I_{2n+1}. The terms $\alpha_i(\ell)$ are independent of $u(\ell+1)$. Note that we do not need to know σ_w^2 to minimize $\gamma(u(\ell+1))$. A plot of the trace of the covariance matrix as a function of $u(\ell+1)$ is shown in Figure 1 for the example in Section 5 with $\ell = 7$. It follows from (3.5) that the asymptotes of γ are given by

$$\lim_{u(\ell+1)\to\pm\infty} \gamma(u(\ell+1)) = \sigma_w^2 \left(\alpha_0(\ell) - \frac{\alpha_3(\ell)}{\alpha_6(\ell)} \right). \quad (3.14)$$

The derivative of the trace of the covariance matrix is given by

$$\frac{\partial\gamma(u(\ell+1))}{\partial u(\ell+1)} = \sigma_w^2 \frac{\beta_1(\ell) + 2\beta_2(\ell)u(\ell+1) - \beta_3(\ell)u^2(\ell+1)}{\left(\alpha_4(\ell) + \alpha_5(\ell)u(\ell+1) + \alpha_6(\ell)u^2(\ell+1) \right)^2} \quad (3.15)$$

where

$$\beta_1(\ell) = \alpha_2(\ell)\alpha_4(\ell) - \alpha_1(\ell)\alpha_5(\ell) \quad (3.16)$$

$$\beta_2(\ell) = \alpha_3(\ell)\alpha_4(\ell) - \alpha_1(\ell)\alpha_6(\ell) \quad (3.17)$$

$$\beta_3(\ell) = \alpha_2(\ell)\alpha_6(\ell) - \alpha_3(\ell)\alpha_5(\ell). \quad (3.18)$$

Setting $\frac{\partial\gamma(u(\ell+1))}{\partial u(\ell+1)} = 0$, it follows that the minimizer and maximizer of γ are given by

$$u_{\min}(\ell+1) = \frac{\beta_2(\ell) + \sqrt{\beta_2^2(\ell) + \beta_1(\ell)\beta_3(\ell)}}{\beta_3(\ell)}, \quad (3.19)$$

$$u_{\max}(\ell+1) = \frac{\beta_2(\ell) - \sqrt{\beta_2^2(\ell) + \beta_1(\ell)\beta_3(\ell)}}{\beta_3(\ell)}. \quad (3.20)$$

In practice, $u_{\min}(\ell+1)$ may not lie within the range $[\underline{u}, \overline{u}]$ of the actuator. In this case, the constrained minimum will be at an extreme value of $u(\ell+1)$, and we evaluate $\gamma(\overline{u})$ and $\gamma(\underline{u})$ to find the minimizing input.

4 Finite-Horizon Input Selection

In this section we extend the one-step approach of the previous section by selecting $u(\ell+1 : N)$ to minimize $\operatorname{tr}\hat{\sigma}^2_{\theta^\star(\ell)}(N)$.

To evaluate $\operatorname{tr}\hat{\sigma}^2_{\theta^\star(\ell)}(N)$, note that the expected covariance matrix at time $k = N$ is

$$\hat{\sigma}^2_{\theta^\star(\ell)}(N) = \sigma_w^2 \left(\phi(n{+}1 : N)\phi^{\mathrm{T}}(n{+}1 : N)\right)^{-1} = \sigma_w^2 \left(\phi(n{+}1{:}\ell)\phi^{\mathrm{T}}(n{+}1{:}\ell) + \phi(\ell{+}1{:}N)\phi^{\mathrm{T}}(\ell{+}1{:}N)\right)^{-1}. \quad (4.1)$$

We do not need to know σ_w^2 to minimize the trace of (4.1). On the other hand, we do need to estimate future values of the output $y(k)$ for $k \geq \ell+1$. To do this, we use the current estimate of the system parameters $\theta^\star(\ell)$.

Let

$$x(k+1) = Ax(k) + Bu(k) \quad (4.2)$$

$$y(k) = Cx(k) + Du(k) \quad (4.3)$$

represent a state space realization of the system in (2.1). Hence

$$y^{\mathrm{T}}(\ell+1 : N) = Y_0 + \Lambda u^{\mathrm{T}}(\ell+1 : N), \quad (4.4)$$

where $Y_0 \in \mathbb{R}^{N-\ell\times 1}$ and $\Lambda \in \mathbb{R}^{N-\ell\times N-\ell}$ are defined by

$$Y_0 \triangleq \begin{bmatrix} CA & CB \\ CA^2 & CAB \\ \vdots & \vdots \\ CA^{N-\ell} & CA^{N-\ell-1}B \end{bmatrix} \begin{bmatrix} x(\ell) \\ u(\ell) \end{bmatrix}, \quad (4.5)$$

$$\Lambda \triangleq \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ CA^{N-\ell-3}B & CA^{N-\ell-4}B & \cdots & 0 \\ CA^{N-\ell-2}B & CA^{N-\ell-3}B & \cdots & D \end{bmatrix}, \quad (4.6)$$

and

$$y^{\mathrm{T}}(\ell+1 : N-k) = P_k y^{\mathrm{T}}(\ell+1 : N), \quad (4.7)$$

$$u^{\mathrm{T}}(\ell+1 : N-k) = P_k u^{\mathrm{T}}(\ell+1 : N), \quad (4.8)$$

where $P_k \in \mathbb{R}^{N-\ell-k\times N-\ell}$ is defined by

$$P_k \triangleq \begin{bmatrix} I_{N-\ell-k} & 0_{N-\ell-k\times k} \end{bmatrix}. \quad (4.9)$$

The state $x(\ell)$ in (4.5) can be estimated using a Kalman filter [8] designed with the current parameter estimate $\theta^\star(\ell)$ and corresponding state space realization. Alternatively, the state

$$x(k) \triangleq \begin{bmatrix} y(k) \cdots y(k-n+1) & u(k) \cdots u(k-n+1) \end{bmatrix}^{\mathrm{T}}, \quad (4.10)$$

of the non-minimal (order $2n$), non-causal, realization [4]

$$x(k+1) = \begin{bmatrix} -a_1 & \cdots & -a_{n-1} & -a_n & b_1 & \cdots & b_{n-1} & b_n \\ 1 & & 0 & 0 & 0 & \cdots & 0 & 0 \\ & \ddots & & \vdots & \vdots & & \vdots & \vdots \\ 0 & & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 & & 0 & 0 \\ \vdots & & \vdots & \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} b_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(k+1), \quad (4.11)$$

$$y(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x(k) \quad (4.12)$$

can be determined directly in terms of input and output measurements. Now we expand $\phi(\ell+1 : N)$ in terms of $u(\ell+1 : N)$ as

$$\begin{aligned} \phi(\ell+1 : N) &= \phi_0(\ell) + \sum_{i=0}^{n} e_{2n+1-i} u(\ell+1 : N-i) M_i \\ &\quad - \sum_{j=1}^{n} e_j y(\ell+1 : N-j) M_j \\ &= \phi_0(\ell) + \sum_{i=0}^{n} e_{2n+1-i} u(\ell+1 : N) P_i^{\mathrm{T}} M_i \\ &\quad - \sum_{j=1}^{n} e_j y(\ell+1 : N) P_j^{\mathrm{T}} M_j \\ &= \phi_0(\ell) + \sum_{i=0}^{n} e_{2n+1-i} u(\ell+1 : N) P_i^{\mathrm{T}} M_i \\ &\quad - \sum_{j=1}^{n} e_j \left(Y_0^{\mathrm{T}} + u(\ell+1 : N)\Lambda^{\mathrm{T}}\right) P_j^{\mathrm{T}} M_j \\ &= \phi_0(\ell) + \sum_{i=0}^{n} e_{2n+1-i} u(\ell+1 : N) P_i^{\mathrm{T}} M_i \\ &\quad - \sum_{j=1}^{n} e_j Y_0^{\mathrm{T}} P_j^{\mathrm{T}} R_j + e_j u(\ell{+}1{:}N)\Lambda^{\mathrm{T}} P_j^{\mathrm{T}} M_j \\ &= \Phi_0 + \sum_{i=0}^{2n} L_i u(\ell+1 : N) R_i, \end{aligned} \quad (4.13)$$

where $\phi_0(\ell) \in \mathbb{R}^{2n+1\times N-\ell}$, $M_i \in \mathbb{R}^{N-\ell-i\times N-\ell}$, $\Phi_0 \in \mathbb{R}^{2n+1\times N-\ell}$, $L_i \in \mathbb{R}^{2n+1\times 1}$, and $R_i \in \mathbb{R}^{N-\ell\times N-\ell}$ are given by

$$M_i = \begin{bmatrix} 0_{N-\ell-i\times i} & I_{N-\ell-i} \end{bmatrix}, \tag{4.14}$$

$$\Phi_0 = \phi_0(\ell) - \sum_{j=1}^{n} e_j Y_0^{\mathrm{T}} P_j^{\mathrm{T}} M_j, \tag{4.15}$$

$$L_i = \begin{cases} e_{2n+1-i} & \text{if} \quad 0 \le i \le n, \\ -e_{i-n} & \text{if} \quad n+1 \le i \le 2n, \end{cases} \tag{4.16}$$

$$R_i = \begin{cases} P_i^{\mathrm{T}} M_i & \text{if} \quad 0 \le i \le n, \\ \Lambda^{\mathrm{T}} P_{i-n}^{\mathrm{T}} M_{i-n} & \text{if} \quad n+1 \le i \le 2n, \end{cases} \tag{4.17}$$

$$\phi_0(\ell) = \left[\begin{array}{cccc|c} -y(\ell) & 0 & \cdots & 0 & \\ -y(\ell-1) & \ddots & \ddots & \vdots & \\ \vdots & & \ddots & 0 & \\ -y(\ell-n+1) & \cdots & \cdots & -y(\ell) & \\ u(\ell-n+1) & \cdots & \cdots & u(\ell) & 0_{2n+1\times N-\ell-n} \\ \vdots & & \cdot^{\cdot^{\cdot}} & 0 & \\ u(\ell-1) & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & \vdots & \\ u(\ell) & 0 & \cdots & 0 & \\ 0 & 0 & \cdots & 0 & \end{array}\right]. \tag{4.18}$$

Using (4.13), the expected covariance matrix (4.1) can be written as

$$\hat{\sigma}^2_{\theta^\star(\ell)}(N) = \sigma_w^2 \Bigg(\phi(n+1:\ell)\phi^{\mathrm{T}}(n+1:\ell) + \Bigg\| \Phi_0 + \sum_{i=0}^{2n} L_i u(\ell+1:N) R_i \Bigg\|_{\mathrm{F}}^2 \Bigg)^{-1}. \tag{4.19}$$

Differentiating (4.19) with respect to the future input sequence yields

$$\frac{\partial \mathrm{tr}\, \hat{\sigma}^2_{\theta^\star(\ell)}(N)}{\partial u(\ell+1:N)} = -\frac{2}{\sigma_w^2} \sum_{j=0}^{2n} L_j^{\mathrm{T}} \hat{\sigma}^4_{\theta^\star(\ell)}(N) \phi(\ell+1:N) R_j^{\mathrm{T}}. \tag{4.20}$$

Unlike the one step ahead input selection problem, a closed-form solution for the finite-horizon input selection problem is not available. However, (4.19) and (4.20) can be used in a gradient-search based optimization algorithm to minimize the trace of (4.19) and select $u(\ell+1:N)$, with the added constraint that the input remain within the actuator constraints.

While we have no guarantee of convergence or bound on computation time, there are several options for real-time implementation. We can use the solution obtained after a fixed number of iterations of the optimization algorithm to select $u(\ell+1)$. Then, at the next time step repeat the process to select $u(\ell+2)$. Another approach would be to allow the optimization to run for q time steps, then implement $u(\ell+1:\ell+q)$. A third strategy would be to implement the finite-horizon input selection approach for a simple identification scheme and assuming a small system order to obtain a good set of data that can then be used off-line in a more complex identification scheme.

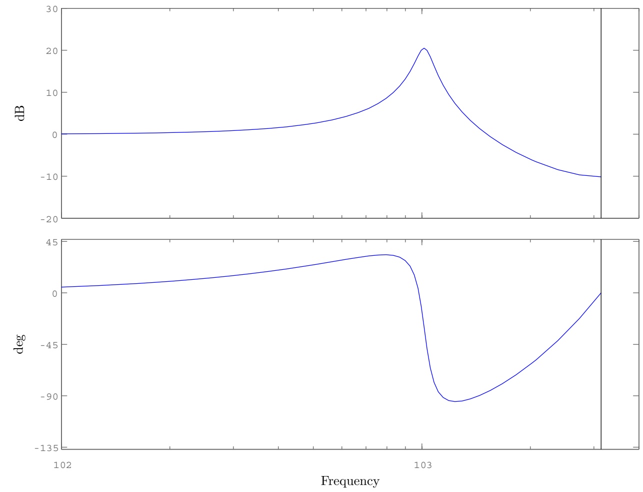

Figure 2: Bode plot of the test system

5 Example

Consider the system

$$y(k) = \frac{0.9q^{-2}}{1-q^{-1}+0.9q^{-2}} u(k) + \frac{1}{1-q^{-1}+0.9q^{-2}} w(k), \tag{5.1}$$

see Figure 2. To initialize the algorithm, the system is excited with a realization of a zero mean unit variance gaussian random variable for $k = 1, \ldots, 7$. This is exactly enough data to allow the computation of the covariance of the estimate of the parameter vector. The disturbance $w(k)$ is a realization of a zero mean gaussian random variable with variance $\sigma_w^2 = 0.1$. For $k \ge 8$, $u(k)$ is selected according to the finite-horizon input selection algorithm. The actuator limits are set at ± 10, and the experiment length is set to $N = 32$. We compare the performance of the finite-horizon algorithm with the results obtained using a random input sequence for the entire experiment, see Figure 3. We compute the matrix Y_0 at each time step using a Kalman filter. For comparison, we normalize the total input energy to be the same as that of the input sequence generated by the finite-horizon input selection algorithm.

The experiment is run 1000 times, each experiment having a different realization of the initial state, input, disturbance, and normalized comparison input sequences. For each run we calculate

$$e_\sigma \triangleq \frac{\mathrm{tr}\, \sigma^2_{\mathrm{comp}} - \mathrm{tr}\, \sigma^2_{\mathrm{FH}}}{\mathrm{tr}\, \sigma^2_{\mathrm{FH}}}, \tag{5.2}$$

$$e_J \triangleq \frac{J_{\mathrm{comp}} - J_{\mathrm{FH}}}{J_{\mathrm{FH}}}, \tag{5.3}$$

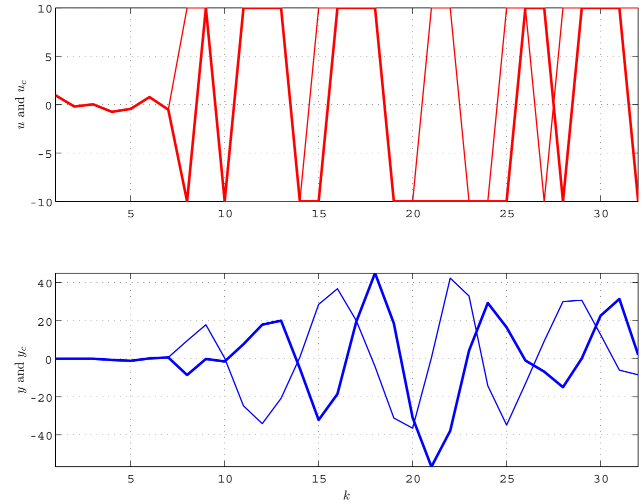

Figure 3: The input and output sequences obtained using the finite-horizon input selection algorithm (u and y, thick lines), and a random input sequence (u_c and y_c, thin lines)

$$e_\theta \triangleq \frac{\frac{\|\theta_{\rm comp}-\theta\|}{\|\theta\|}-\frac{\|\theta_{\rm FH}-\theta\|}{\|\theta\|}}{\frac{\|\theta_{\rm FH}-\theta\|}{\|\theta\|}} = \frac{\|\theta_{\rm comp}-\theta\|-\|\theta_{\rm FH}-\theta\|}{\|\theta_{\rm FH}-\theta\|}, \tag{5.4}$$

where the subscript "FH" refers to the results obtained using the finite-horizon input selection algorithm, and the subscript "comp" refers to the results obtained using the normalized comparison input sequence. We plot e_σ in Figure 4. The averages and standard deviations are presented in Table 1. For the data shown, the finite-horizon input selection algorithm reduces the variance of the estimated parameter vector by an average of 45%. It is interesting to note that the optimal input sequence is bang-bang for this example. However, it is not clear that this behavior is generic.

e_σ	0.4529 ± 0.4318
e_J	0.0110 ± 0.1530
e_θ	0.3762 ± 0.8556

Table 1: Performance Comparison of the Finite-Horizon Input Selection Algorithm.

6 Conclusion

We proposed a method for selecting the input command at time $k+1$ for system identification, and extended the approach to the finite-horizon input selection problem. Specifically, we select the input sequence to minimize the predicted covariance of the estimate of the model parameters. The method is initialized with a small amount of data such that the variance of the estimate can be computed. After this point the input sequence is selected to minimize the predicted covariance of the estimated parameter vector. For the least squares identification algorithm, we were able to find analytical expressions for the global minimizer, minimum, maximizer, maximum, and asymptotes of the predicted covariance of the estimate of the model parameters in the one-step-ahead case. We accounted for actuator saturation to select the implementable command that minimizes the predicted covariance of the estimate of the model parameters. We demonstrated the finite-horizon input selection algorithm on a least squares identification problem and compared the results to the standard results obtained using a random input sequence.

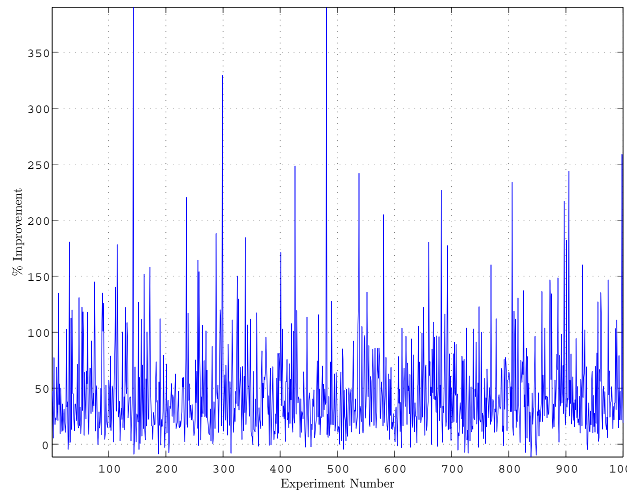

Figure 4: e_σ for each experiment

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