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OPTIMAL ADAPTIVE FEEDBACK DISTURBANCE REJECTION

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ABSTRACT

Discrete-time adaptive disturbance rejection has broad engineering and scientific applications. It is most relevant in active noise and vibration control. Disturbance rejection algorithms can be of two types, feed-forward or feedback. The latter generally exhibit superior performance since they take into account the effect of the feedback path from control to measurement. In this paper we propose an optimal adaptive feedback disturbance rejection algorithm. The main result is based on a controller parameter update law derived from a retrospective cost function. The proposed algorithm requires minimal plant information and does not require a measurement of the disturbance. This work extends earlier work on ARMARKOV adaptive controllers in several ways. First, we employ an ARMA model stucture for both plant and the controller instead of a μ-MARKOV parameterization thus reducing the number of tunable parameters. Next, the step size function is replaced by an optimal gain matrix. Finally, the proposed algorithm is reformulated in a way that online computations are reduced. The effectiveness of the algorithm in rejecting tonal disturbances with unknown frequency and phase is demonstrated via simulation.

1 Introduction

A large portion of the feedback adaptive control literature is devoted to proving stability of the closed loop system and boundedness of solutions [1–6]. However in many applications the plant is open-loop stable or stability can be achieved with a controller based on a nominal model and the overriding concern is performance with respect to disturbance rejection [7–12].

In [13–15] a disturbance rejection algorithm based on ARMARKOV models was presented which has proved successful in applications and exhibits significant robustness [16]. This method uses non-minimal plant and controller parameterizations which are equivalent to successive self substitution of the model. The additional parameters introduced into the model as a result of self substitution are in fact the Markov parameters of the underlying physical model. The advantages of using μ-Markov parameterizations for system identification have been studied in the literature [17]. However the rationale for using a non-minimal controller structure is not evident and the benefits of such a formulation, if any, are unclear. Also the ARMARKOV algorithm uses the gradient method for the controller update which can lead to slow convergence.

In the present paper we utilize minimal time series models to describe both the plant and controller and thus avoid over parameterization. We also improve upon the parameter update equations developed in [13–15] by reformulating the equations for the retrospective cost function defined in [13] to obtain a linear parametric model of the retrospective cost in terms of the controller parameters. This reformulation allows the use of recursive least squares to update the parameter vector, which is an optimal estimator for a linear parametric model with constant coefficients [18].

The formulation used in this paper also divides the controller computations into an off-line component related to structural information which is assumed to be known and an online component which uses temporal information. This approach alleviates some of the burden of online computation thus making the algorithm amenable to real-time implementation.

The contents of the paper are as follows. In section 2 we develop the dynamical equations for the extended TITO model. The disturbance rejection problem based on the TITO model and retrospective performance is defined in section 3. The retrospective performance is reformulated in section 4 to bring it to the standard linear parametric form. Section 5 desribes the disturbance rejection algorithm. Advantages of the optimal algorithm compared to the ARMARKOV algorithm are demonstrated via simulation in section 6. Finally some concluding remarks are made in section 7.

2 The Extended TITO Model

Consider the linear discrete-time TITO system shown in Figure 1. Let the control vector $u(k) \in$

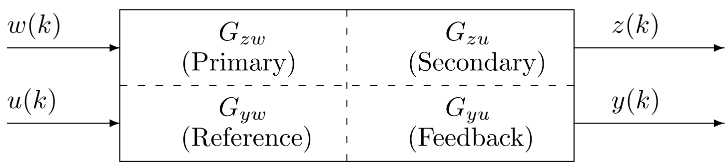

Figure 1: The Standard Problem

$\mathbb{R}^{m_u}$, the measurement vector $y(k) \in \mathbb{R}^{l_y}$, the disturbance vector $w(k) \in \mathbb{R}^{m_w}$ and the performance vector $z(k) \in \mathbb{R}^{l_z}$. The time histories of $z(k)$ and $y(k)$ can be described by the time series model

$$z(k) = \sum_{j=1}^{n} -a_j z(k-\mu-j+1) + \sum_{j=0}^{n} B_j w(k-j) + \sum_{j=0}^{n} C_j u(k-j) \tag{2.1}$$

$$y(k) = \sum_{j=1}^{n} -a_j y(k-\mu-j+1) + \sum_{j=0}^{n} D_j w(k-j) + \sum_{j=0}^{n} E_j u(k-j) \tag{2.2}$$

where $a_j \in \mathbb{R}$, $B_j \in \mathbb{R}^{l_z \times m_w}$, $C_j \in \mathbb{R}^{l_z \times m_u}$, $D_j \in \mathbb{R}^{l_y \times m_w}$, and $E_j \in \mathbb{R}^{l_y \times m_u}$.

For the purpose of comparison we first develop the μ-ARMARKOV model used in [13–15]. However, later we demonstrate that the algorithm presented achieves superior performance without the use of non-minimal models i.e. with $\mu = 1$. Self substitution of (2.1) and (2.2) $\mu - 1$ times leads to the μ-ARMARKOV model

$$z(k) = \sum_{j=1}^{n} -\tilde{a}_j z(k-\mu-j+1) + \sum_{j=0}^{n+\mu-1} \tilde{B}_j w(k-j) + \sum_{j=0}^{n+\mu-1} \tilde{C}_j u(k-j) \tag{2.3}$$

$$y(k) = \sum_{j=1}^{n} -\tilde{a}_j y(k-\mu-j+1) + \sum_{j=0}^{n+\mu-1} \tilde{D}_j w(k-j) + \sum_{j=0}^{n+\mu-1} \tilde{E}_j u(k-j) \tag{2.4}$$

where $\tilde{a}_j \in \mathbb{R}$, $\tilde{B}_j \in \mathbb{R}^{l_z \times m_w}$, $\tilde{C}_j \in \mathbb{R}^{l_z \times m_u}$, $\tilde{D}_j \in \mathbb{R}^{l_y \times m_w}$, and $\tilde{E}_j \in \mathbb{R}^{l_y \times m_u}$.

Define the regressor vectors given by

$$\varphi_u(k) \triangleq \begin{bmatrix} u(k) \\ \vdots \\ u(k-n-\mu+1) \end{bmatrix} \in \mathbb{R}^{(n+\mu)m_u}, \tag{2.5}$$

$$\varphi_{zw}(k) \triangleq \begin{bmatrix} z(k-\mu) \\ \vdots \\ z(k-n-\mu+1) \\ w(k) \\ \vdots \\ w(k-n-\mu+1) \end{bmatrix} \in \mathbb{R}^{[(n+\mu-1)l_z+(n+\mu)m_w]}, \tag{2.6}$$

and

$$\varphi_{yw}(k) \triangleq \begin{bmatrix} y(k-\mu) \\ \vdots \\ y(k-n-\mu+1) \\ w(k) \\ \vdots \\ w(k-n-\mu+1) \end{bmatrix} \in \mathbb{R}^{[(n+\mu-1)l_y+(n+\mu)m_w]}. \tag{2.7}$$

Then (2.3) and (2.4) can be written as

$$z(k) = \theta_{zw}\varphi_{zw}(k) + \theta_{zu}\varphi_u(k) \tag{2.8}$$

$$y(k) = \theta_{yw}\varphi_{yw}(k) + \theta_{yu}\varphi_u(k) \tag{2.9}$$

where

$$\theta_{zw} \triangleq \begin{bmatrix} \tilde{a}_1 I_{l_z} & \cdots & \tilde{a}_n I_{l_z} & \tilde{B}_0 & \cdots & \tilde{B}_{n+\mu-1} \end{bmatrix} \in \mathbb{R}^{l_z\times[(n+\mu-1)l_z+(n+\mu)m_w]} \tag{2.10}$$

$$\theta_{yw} \triangleq \begin{bmatrix} \tilde{a}_1 I_{l_y} & \cdots & \tilde{a}_n I_{l_y} & \tilde{D}_0 & \cdots & \tilde{D}_{n+\mu-1} \end{bmatrix} \in \mathbb{R}^{l_y\times[(n+\mu-1)l_y+(n+\mu)m_w]} \tag{2.11}$$

$$\theta_{zu} \triangleq \begin{bmatrix} \tilde{C}_0 & \cdots & \tilde{C}_{n+\mu-1} \end{bmatrix} \in \mathbb{R}^{l_z\times(n+\mu)m_u} \tag{2.12}$$

$$\theta_{yu} \triangleq \begin{bmatrix} \tilde{E}_0 & \cdots & \tilde{E}_{n+\mu-1} \end{bmatrix} \in \mathbb{R}^{l_y\times(n+\mu)m_u} \tag{2.13}$$

Now define the *extended performance vector* $Z(k)$, *extended measurement vector* $Y(k)$, and *extended control vector* $U(k)$ by

$$Z(k) \triangleq \begin{bmatrix} z(k) \\ \vdots \\ z(k-p+1) \end{bmatrix} \in \mathbb{R}^{p\cdot l_z} \tag{2.14}$$

$$Y(k) \triangleq \begin{bmatrix} y(k) \\ \vdots \\ y(k-p+1) \end{bmatrix} \in \mathbb{R}^{p\cdot l_y} \tag{2.15}$$

$$U(k) \triangleq \begin{bmatrix} u(k) \\ \vdots \\ u(k-p_c+1) \end{bmatrix} \in \mathbb{R}^{p_c\cdot m_u} \tag{2.16}$$

where $p_c \triangleq \mu + n + p - 1$. Also define the *extended regressor vectors*

$$\Phi_{zw}(k) \triangleq \begin{bmatrix} z(k-\mu) \\ \vdots \\ z(k-\mu-n-p+2) \\ w(k) \\ \vdots \\ w(k-\mu-n-p+2) \end{bmatrix} \in \mathbb{R}^{(n+p-1)l_z+(n+\mu+p-1)m_w}, \tag{2.17}$$

and

$$\Phi_{yw}(k) \triangleq \begin{bmatrix} y(k-\mu) \\ \vdots \\ y(k-\mu-n-p+2) \\ w(k) \\ \vdots \\ w(k-\mu-n-p+2) \end{bmatrix} \in \mathbb{R}^{(n+p-1)l_y+(n+\mu+p-1)m_w}. \tag{2.18}$$

Then the extended form of (2.3) and (2.4) can be written as

$$Z(k) = W_{zw}\Phi_{zw}(k) + B_{zu}U(k) \tag{2.19}$$
$$Y(k) = W_{yw}\Phi_{yw}(k) + B_{yu}U(k) \tag{2.20}$$

where

$$\begin{aligned} W_{zw} &\in \mathbb{R}^{pl_z\times[(n+p-1)l_z+(n+\mu+p-1)m_w)]} \\ W_{yw} &\in \mathbb{R}^{pl_y\times[(n+p-1)l_y+(n+\mu+p-1)m_w)]} \\ B_{zu} &\in \mathbb{R}^{pl_z\times p_c m_u} \\ B_{yu} &\in \mathbb{R}^{pl_y\times p_c m_u} \end{aligned}$$

3 Adaptive Disturbance Rejection Problem

Now consider the TITO system with an adaptive feedback controller as shown in Figure 2. We

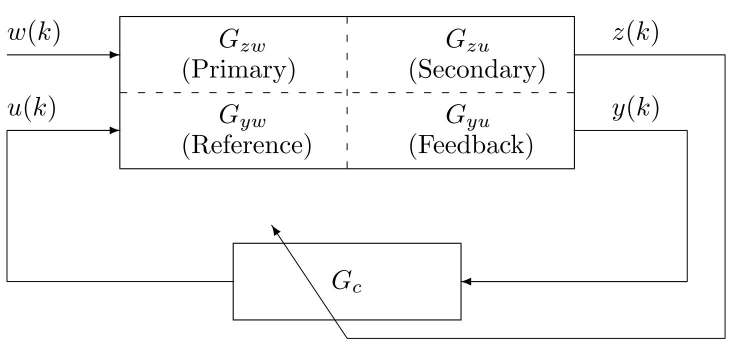

Figure 2: The Adaptive Standard Problem

make the following assumptions about the TITO plant.

Assumption 3.1. The plant is asymptotically stable.

Assumption 3.2. The order n of the plant is known.

Assumption 3.3. B_{zu} is known or can be identified.

Assumption 3.4. $y(k)$ and $z(k)$ are available for measurement.

Assumption 3.5. The disturbance $w(k)$ is not measured.

Let G_c be a strictly proper controller of order n_c with μ_c Markov parameters given by the time series model

$$u(k) = -\sum_{j=1}^{n_c} \Gamma_j u(k-\mu_c-j) + \sum_{j=1}^{n_c+\mu_c-1} \Upsilon_j y(k-j) \tag{3.1}$$

where $\Gamma_j \in \mathbb{R}^{m_u \times m_u}$ and $\Upsilon_j \in \mathbb{R}^{m_u \times l_y}$. Next define $q_1 \triangleq n_c m_u$, $q_2 \triangleq (n_c+\mu_c-1)l_y$, $q_3 \triangleq (n_c+p_c-1)m_u$, $q_4 \triangleq (n_c+\mu_c+p_c-2)l_y$, $q_5 \triangleq q_1+q_2$ and $q_6 \triangleq q_3+q_4$. Then

$$u(k) = \theta_c(k) R_1 \Phi_{uy}(k), \tag{3.2}$$

and

$$U(k) = \sum_{i=1}^{p_c} L_i \theta_c(k-i+1) R_i \Phi_{uy}(k), \tag{3.3}$$

where

$$\theta_c(k) \triangleq \begin{bmatrix} -\Gamma_1(k) & \cdots & -\Gamma_{n_c}(k) & \Upsilon_1(k) & \cdots & \Upsilon_{n_c+\mu_c-1}(k) \end{bmatrix} \in \mathbb{R}^{m_u \times q_5}, \tag{3.4}$$

$$\Phi_{uy}(k) \quad \triangleq \quad \begin{bmatrix} u(k-\mu_c) \\ \vdots \\ u(k-\mu_c-n_c-p_c+2) \\ y_{k-1} \\ \vdots \\ y(k-\mu_c-n_c-p_c+2) \end{bmatrix} \in \mathbb{R}^{q_6}, \tag{3.5}$$

$$L_i \triangleq \begin{bmatrix} 0_{(i-1)m_u \times m_u} \\ I_{m_u} \\ 0_{(p_c-i)m_u \times m_u} \end{bmatrix} \in \mathbb{R}^{p_c m_u \times m_u}, \tag{3.6}$$

and

$$R_i \triangleq \begin{bmatrix} 0_{q_1 \times (i-1)m_u} & I_{q_1 \times q_1} & 0_{q_1 \times (p_c-i)m_u} & 0_{q_1 \times (i-1)l_y} & 0_{q_1 \times q_2} & 0_{q_1 \times (p_c-i)l_y} \\ 0_{q_2 \times (i-1)m_u} & 0_{q_2 \times q_1} & 0_{q_2 \times (p_c-i)m_u} & 0_{q_2 \times (i-1)l_y} & I_{q_2 \times q_2} & 0_{q_2 \times (p_c-i)l_y} \end{bmatrix} \in \mathbb{R}^{q_5 \times q_6} \tag{3.7}$$

Now from (2.19)and (3.3)

$$Z(k) = W_{zw}\Phi_{zw}(k) + B_{zu} \sum_{i=1}^{p_c} L_i \theta_c(k-i+1) R_i \Phi_{uy}(k) \tag{3.8}$$

Also define the *retrospective performance* $\hat{Z}(k)$ function that evaluates the performance of $\theta_c(k+1)$ based on the behavior of the system during the previous p steps by

$$\hat{Z}(k) \triangleq W_{zw}\Phi_{zw}(k) + B_{zu} \sum_{i=1}^{p_c} L_i \theta_c(k+1) R_i \Phi_{uy}(k). \tag{3.9}$$

Notice that (3.9) has the same form as (3.8) but with $\theta_c(k-i+1)$ replaced by the current controller parameter block vector $\theta_c(k+1)$.

Remark 3.1. If the controller parameter vector $\theta_c(k)$ converges, then $\hat{Z}(k) - Z(k) \to 0$.

Remark 3.2. Since by assumption $w(k)$ is unavailable for measurement, $\Phi_{zw}(k)$is unknown. Therefore $\hat{Z}(k)$ cannot be computed from (3.9). However it follows from (2.19) and (3.9) that $\hat{Z}(k)$ can be computed using

$$\hat{Z}(k) = Z(k) - B_{zu}\left(U(k) - \sum_{i=1}^{p_c} L_i\theta_c(k+1)R_i\Phi_{uy}(k)\right). \tag{3.10}$$

The objective is to determine a θ_c that minimize a positive definite function of $\hat{Z}(k)$.

4 Reformulation Of Retrospective Performance

We will use the following facts from Kronecker algebra.

Fact 4.1. Let $\otimes$ denote the Kronecker product and let $W \in \mathbb{R}^{l\times m}$, $X \in \mathbb{R}^{m\times q}$, $Y \in \mathbb{R}^{q\times r}$ and $Z \in \mathbb{R}^{r\times t}$. Then

$$\mathrm{vec}\,[XYZ] = \left(Z^{\mathrm{T}} \otimes X\right)\mathrm{vec}\,[Y] \tag{4.1}$$

and

$$WX \otimes YZ = (W \otimes Y)(X \otimes Z) \tag{4.2}$$

From (3.10) it follows that

$$\begin{aligned}\hat{Z}(k) &= Z(k) - B_{zu}\left(U(k) - \sum_{i=1}^{p_c} L_i\theta_c(k+1)R_i\Phi_{uy}(k)\right) \\ &= Z(k) - B_{zu}U(k) + \sum_{i=1}^{p_c} B_{zu}L_i\theta_c(k+1)R_i\Phi_{uy}(k)\end{aligned} \tag{4.3}$$

Use (4.1) to get

$$\hat{Z}(k) = Z(k) - B_{zu}U(k) + \sum_{i=1}^{p_c}\left(\Phi_{uy}^{\mathrm{T}}(k)R_i^{\mathrm{T}}\right) \otimes (B_{zu}L_i)\mathrm{vec}\,[\theta_c(k+1)]$$

Now use (4.2) to get

$$\hat{Z}(k) = Z(k) - B_{zu}U(k) + \left(\Phi_{uy}^{\mathrm{T}}(k) \otimes B_{zu}\right)\sum_{i=1}^{p_c}\left(R_i^{\mathrm{T}} \otimes L_i\right)\mathrm{vec}\,[\theta_c(k+1)]$$

Define $q_7 \triangleq p_c m_u q_6$, $q_8 \triangleq m_u q_5$,

$$\xi(k) \triangleq Z(k) - B_{zu}U(k) \in \mathbb{R}^{pl_z}, \tag{4.4}$$

$$\Lambda_z \triangleq -\sum_{i=1}^{p_c}\left(R_i^{\mathrm{T}} \otimes L_i\right) \in \mathbb{R}^{q_7\times q_8}, \tag{4.5}$$

$$\mathcal{G}^{\mathrm{T}}(k) \triangleq \left(\Phi_{uy}^{\mathrm{T}}(k) \otimes B_{zu}\right)\Lambda_z \in \mathbb{R}^{pl_z\times q_8}, \tag{4.6}$$

and

$$\Theta(k+1) \triangleq \mathrm{vec}\,[\theta_c(k+1)] \in \mathbb{R}^{q_8}.$$

Then we obtain the linear prediction error model

$$\hat{Z}(k) = \xi(k) - \mathcal{G}^{\mathrm{T}}(k)\Theta(k+1). \tag{4.7}$$

To express the control $u(k)$ in terms of $\Theta(k+1)$ we note that

$$u(k) = I_{m_u}\theta_c(k)R_1\Phi_{uy}(k). \tag{4.8}$$

Again using (4.1) and (4.2) we have

$$u(k) = \left(\Phi_{uy}^{\mathrm{T}}(k) \otimes I_{m_u}\right)\left(R_1^{\mathrm{T}}(k) \otimes I_{m_u}\right) \operatorname{vec}\left[\theta_{\mathrm{Z}}(k+1)\right].$$

Define

$$\Lambda_u \triangleq \left(R_1^{\mathrm{T}}(k) \otimes I_{m_u}\right) \in \mathbb{R}^{m_u q_6 \times q_8}, \tag{4.9}$$

and

$$\mathcal{U}^{\mathrm{T}}(k) \triangleq \left(\Phi_{uy}^{\mathrm{T}}(k) \otimes I_{mu}\right)\Lambda_u \in \mathbb{R}^{m_u \times q_8}. \tag{4.10}$$

Then

$$u(k) = \mathcal{U}^{\mathrm{T}}(k)\Theta(k+1). \tag{4.11}$$

5 Adaptive Disturbance Rejection Algorithm

Consider the *weighted retrospective performance cost function*

$$\begin{aligned} J(k) &\triangleq \sum_{j=1}^{k} \lambda^{k-j}\left[\hat{Z}^{\mathrm{T}}(j)\hat{Z}(j)\right] \\ &= \sum_{j=1}^{k} \lambda^{k-j}\left[\xi(k) - \mathcal{G}^{\mathrm{T}}(k)\Theta(k+1)\right]^{\mathrm{T}}\left[\xi(k) - \mathcal{G}^{\mathrm{T}}(k)\Theta(k+1)\right] \end{aligned} \tag{5.1}$$

where $0 < \lambda \leq 1$ is a temporal weighting function. A recursive estimate of the $\Theta(k)$ that minimizes $J(k)$ can be easily derived. See for example [18]. The RLS estimate for $\Theta(k)$ is given by

$$\hat{\Theta}(k+1) = \hat{\Theta}(k) + \mathcal{P}(k+1)\mathcal{G}(k)\left[\xi(k) - \mathcal{G}^{\mathrm{T}}(k)\hat{\Theta}(k)\right] \tag{5.2}$$

$$\mathcal{P}(k+1) = \frac{1}{\lambda}\left[\mathcal{P}(k) - \mathcal{P}(k)\mathcal{G}(k)\left(\lambda I + \mathcal{G}^{\mathrm{T}}(k)\mathcal{P}(k)\mathcal{G}(k)\right)^{-1}\mathcal{G}^{\mathrm{T}}(k)\mathcal{P}(k)\right] \tag{5.3}$$

The adaptive disturbance rejection algorithm may be summarized as follows.

1. Compute Λ_z and Λ_u off line using (4.5) and (4.9).
2. Intialize $\Phi_{uy}(k)$, $\Theta(k)$ and $\mathcal{P}(k)$.
3. Compute $u(k)$ using (4.11).
4. Update $\Theta(k)$ and $\mathcal{P}(k)$ using (5.2) and (5.3).
5. Use $z(k)$, $y(k)$ and $u(k)$ to update $\Phi_{uy}(k)$ in accordance with (3.5).
6. Go to step 3.

6 Examples

Example 6.1. Consider the lumped parameter model of the serially connected structure shown in Figure 3. Let $m_1 = ... = m_4 = 5$ kg, $k_1 = ... = k_5 = 2$ N/m and $c_1 = ... = c_5 = 0.01$ N/m/s. Then the

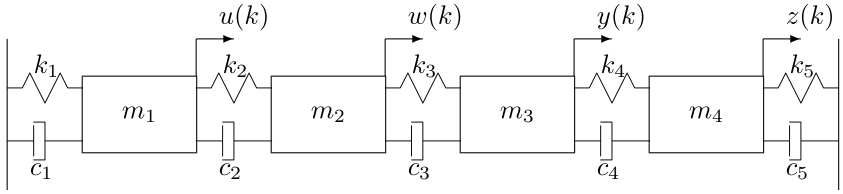

Figure 3: Serially Connected Structure

state equations for the structure are given by

$$\dot{x} = Ax + Bu + D_1 w \tag{6.1}$$

$$z = E_1 x \tag{6.2}$$

$$y = Cx \tag{6.3}$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -5.0 & 2.5 & 0 & 0 & -0.01 & 0.005 & 0 & 0 \\ 2.5 & -5.0 & 2.5 & 0 & 0.005 & -0.01 & 0.005 & 0 \\ 0 & 2.5 & -5.0 & 2.5 & 0 & 0.005 & -0.01 & 0.005 \\ 0 & 0 & 2.5 & -5.0 & 0 & 0 & 0.005 & -0.01 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The plant has modes at 0.1555 Hz, 0.2958, 0.4072 and 0.4787 Hz. The mass m_2 is excited at the modal frequency of 0.1555 Hz. The simualtion results with the ARMARKOV controller and the optimal controller described in section 5 are shown in Figure 4 and Figure 5, respectively.

Notice that the the ARMARKOV controller uses $50 + 24 = 74$ tunable tunable parameters while the RLS based controller uses 16 tunable parameters. The RLS based controller has significantly smaller transients and converges faster than the ARMARKOV algorithm.

Example 6.2. Consider the rectangular crossection acoustic duct shown in shown in Figure 6. We treat the duct as a one dimensional waveguide with spatial coordinate x, where $0 \leq x \leq L$. We use the mathematical model for the acoustic duct derived in [19], where we have assumed that the speed of acoustic waves is 343 m/s, the density of air is 1.21 kg/m^3 and the duct has five modes. Let the disturbance speaker be placed at x_d, the control speaker at x_c, the performance microphone at x_p and the measurement microphone at x_m. Then for $L = 6$ m, $x_d = 0.1$ m, $x_p = 0.15$ m, $x_m = 5.9$ m and $x_c = 0.5.95$ m the state space matrices

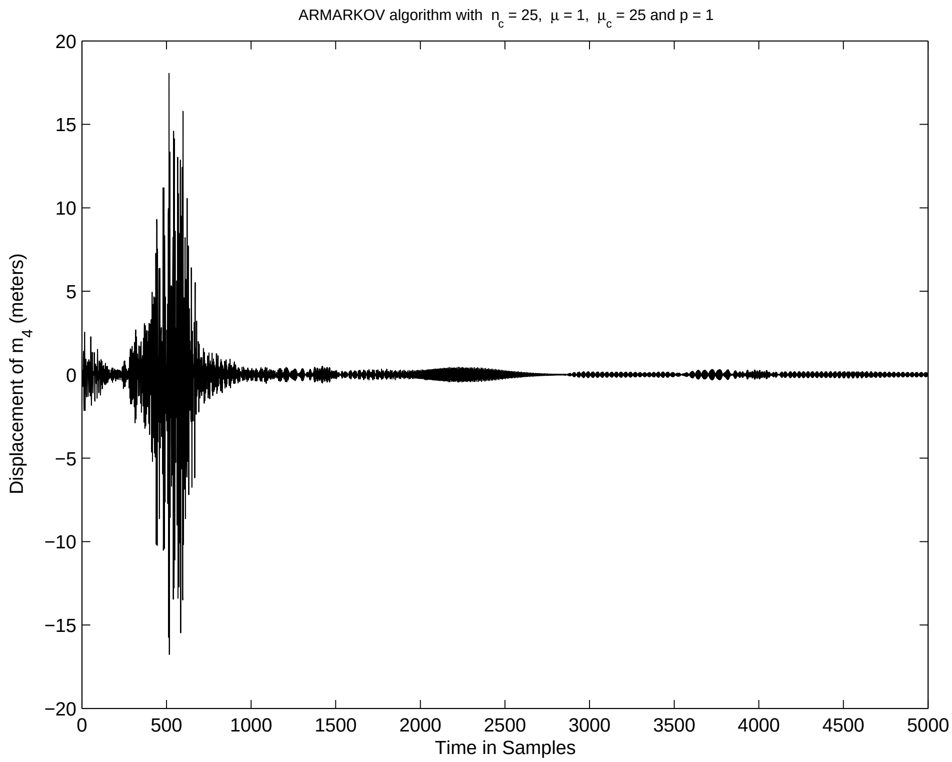

Figure 4: Closed loop response with ARMARKOV controller for Example 6.1

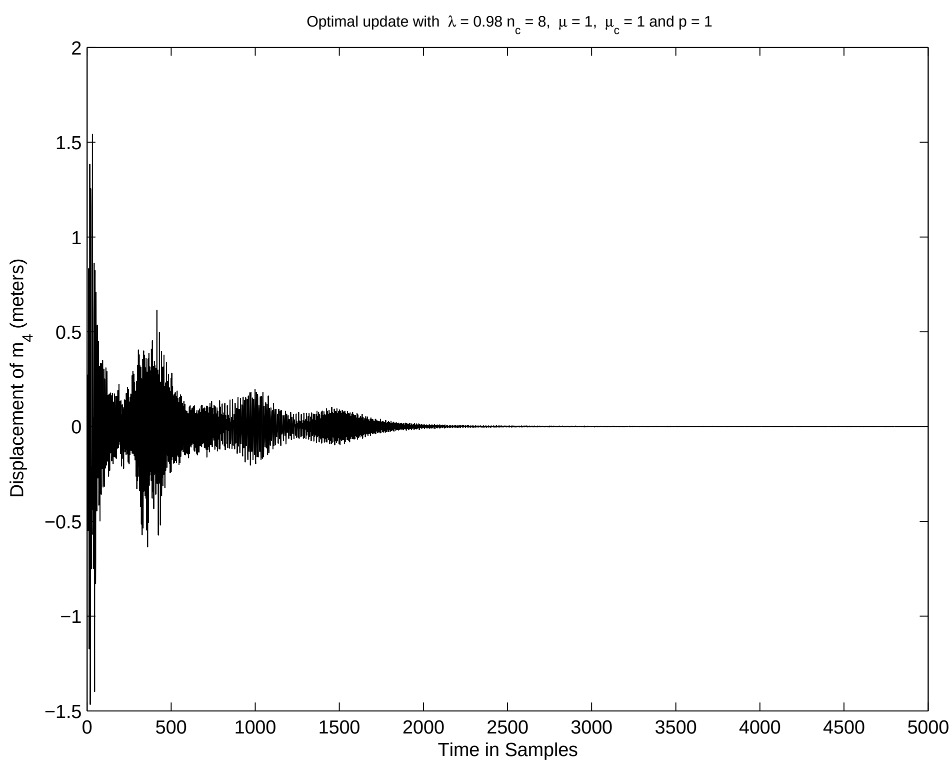

Figure 5: Closed loop response with RLS controller for Example 6.1

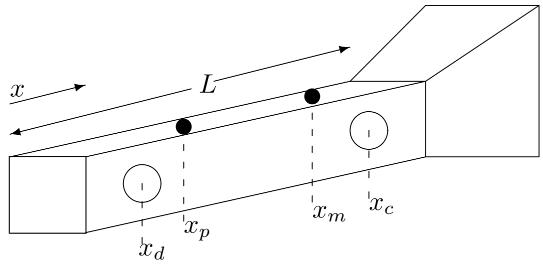

Figure 6: Acoustic Duct

for the acoustic duct model are given by

$$A = \begin{bmatrix} 0.9645 & 0.0004 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -140.4030 & 0.9383 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8618 & 0.0004 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -534.3447 & 0.8120 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7010 & 0.0004 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1115.1480 & 0.6318 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4950 & 0.0003 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1788.9550 & 0.4116 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2594 & 0.0003 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2445.8760 & 0.1682 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0.0075 \\ 0 \\ 0.0215 \\ 0 \\ 0.0332 \\ 0 \\ 0.0418 \\ 0 \\ 0.0469 \end{bmatrix}, \quad D1 = \begin{bmatrix} 0 \\ 0.0150 \\ 0 \\ 0.0429 \\ 0 \\ 0.0658 \\ 0 \\ 0.0823 \\ 0 \\ 0.0912 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 46.4308 & 0 & 138.1492 & 0 & 226.4659 & 0 & 309.2063 & 0 & 384.3330 & 0 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 30.9715 & 0 & 92.5754 & 0 & 153.1650 & 0 & 212.0764 & 0 & 268.6643 & 0 \end{bmatrix}.$$

The duct has modes at 85.4167 Hz, 170.8333 Hz, 256.25 Hz, 341.6667 Hz and 427.0833 Hz. The mass disturbance speaker is excited at the modal frequency of 427.0833 Hz. The simualtion results with the ARMARKOV controller and the optimal controller described in section 5 are shown in Figure 7 and Figure 8, respectively.

7 Conclusions

In this paper we present a method for adaptive disturbance rejection that requires minimal plant information and does not require a measurement of the disturbance. The algorithm presented uses a retrospective performance like [13–15] but does not use non-minimal plant or controller structure. Performance achieved in terms of transient response and speed of convergence is superior to the ARMARKOV algorithm. A formal proof of closed loop stability with the presented algorithm will appear in a subsequent paper.

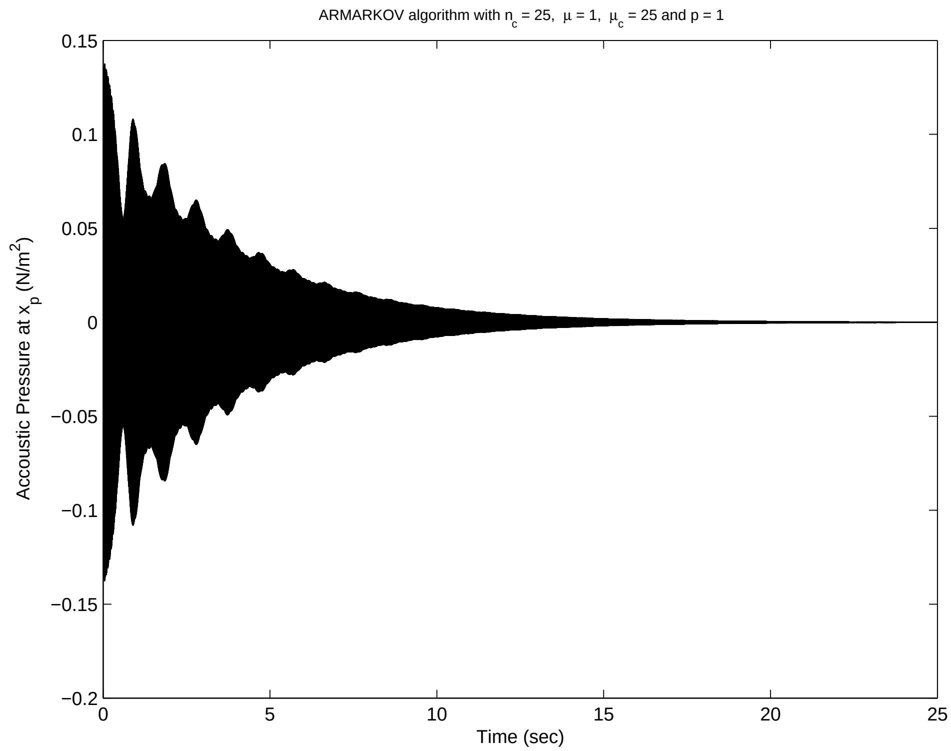

Figure 7: Closed loop response with ARMARKOV controller for Example 6.2

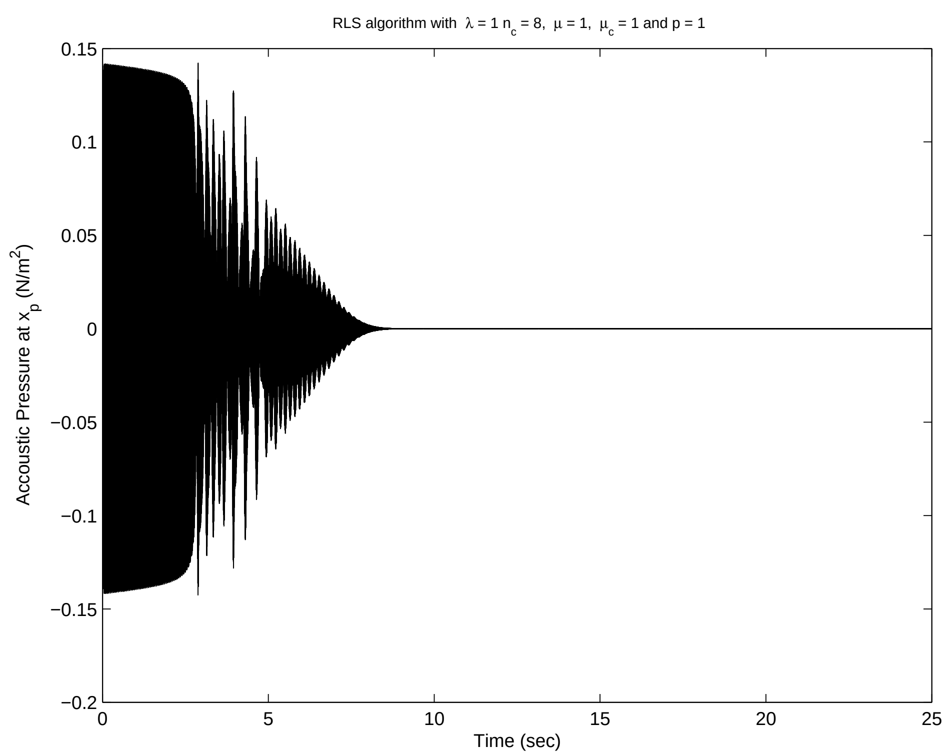

Figure 8: Closed loop response with RLS controller for Example 6.2

References

[1] G. C. Goodwin, P. J. Ramadage and P. E. Caines, "Discrete-Time Multivariable Adaptive Control," *IEEE Trans. Autom. Contr*, vol. 25, pp. 449-456, 1980.

[2] A. R. Morse, "Global Stability of Parameter Adaptive Control Systems," *IEEE Trans. Autom. Contr.*, vol. AC-25, pp. 433-440, 1980.

[3] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Prentice Hall, 1984.

[4] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice Hall, 1996.

[5] R. Johansson, "Global Lyapunov Stability and Exponential Convergence of Direct Adaptive Control," *Int. J. Cont.*, Vol. 50, No. 3, pp. 859-869, 1989.

[6] S. Akhtar and D. S. Bernstein, "Logarithmic Lyapunov Functions for Direct Adaptive Stabilization With Normalized Adaptive Laws," *Int. J. Cont.*, Vol. 77, No. 7, pp. 630-638, 2004.

[7] Y. Wei, and A. Wu, "Demonstration of Active Vibration Control of the Highes Cryocooler Testbed," *Proc. 31st IEEE Conf. Desc. Contr*, pp. 2580-2585, 1992.

[8] S. Beale, B. Shafai and E. Cusson, "Adaptive Forced Balancing for Magnetic Bearing Control Systems," *Proc. 31st IEEE Conf. Desc. Contr*, pp. 3535-3539, 1992.

[9] A. Sacks, M. Bodson and P. Kholsa, "Experimental Results of Adaptive Periodic Disturbance cancellation in High Performance Magnetic Disk Drives," *ASME. Journ. Dyn. Sys*, Vol. 118, pp. 416-424, 1996.

[10] T. J. Manayathara, T. C. Tsao, J. Bentsman and D. Ross, "Rejection of Unknown Periodic Load Disturbances in Continous Steel Casting Process Using Learning Repetitive Control Approach," *IEEE Trans. Contr. Sys. Tech.*, vol.4, pp. 259-265, 1996.

[11] R. J. Fuentes, K. N. Schrader, M. J. Balas and R. S. Erwin, "Direct Adapative Disturbance Rejection and Control for Deployable Space Telescope, Theory and Application," *Proc. Amerc. Contr. Conf*, pp. 3980-3985, 2001.

[12] D. patt, L. Liu and P. P. Friedmann, "Achieving Simulataneous Reduction of Rotorcraft Vibration and Noise Using Simulation," *Proc. 29th Euro. Rotorcraft. Conf*, France, 2004.

[13] R. Venugopal, and D. S. Bernstein, "Adaptive Disturbance Rejection Using ARMARKOV/Toeplitz Models," *IEEE Trans. Contr. Sys. Tech.*, vol.8, pp. 257-269, 2000.

[14] R. Venugopal, and D. S. Bernstein, "United states Patent 6,208,739 Noise and vibdration suppression method and system," *IEEE Trans. Autom. Contr. Tech.*, March 27, 2001.

[15] H. R. Sane, R. Venugopal, and D. S. Bernstein, "Disturbance Rejection Using ARMARKOV Adaptive Control with Simultaneous Identification," *IEEE Trans. Contr. Sys. Tech.*, vol.9, pp. 101-106, 2001.

[16] H. R. Sane and D. S. Bernstein, "Robustness of ARMARKOV Adaptive Control Disturbance Rejection Algorithm" *Proc. Amerc. Contr. Conf*, pp. 2035-2039, 1999.

[17] T. H. van Pelt, and D. S. Bernstein, "Least Squares Identification Using μ-Markov Parameterizations," *Proc. 37st IEEE Conf. Desc. Contr*, pp. 618-619, 1998.

[18] K. J. Astrom and Bjorn Wittenmark, *Adaptive Control*. Addison Wesley, 1995.

[19] J. Hong, J. C. Ackers, R. Venugopal, M. Lee, A.G. Sparks, P. D. Washabaugh and D. S. Bernstein, "Modelling, Identification, and Feedback Control of Noise in an Acoustic Duct," *IEEE Trans. Contr. Sys. Tech.*, vol.4, pp. 283-291, 1996.