Proceedings of the
2005 IEEE Conference on Control Applications
Toronto, Canada, August 28-31, 2005

Data-Based Model Refinement for Linear and Hammerstein Systems Using Subspace Identification and Adaptive Disturbance Rejection

Harish J. Palanthandalam-Madapusi, Erin L. Renk and Dennis S. Bernstein

Abstract— **First principle models and empirical models are necessarily approximate. In this paper we develop two empirical approaches that use a delta model to modify an initial model by means of cascade, parallel or feedback augmentation. A subspace based nonlinear identification algorithm and an adaptive disturbance rejection algorithm are both used to construct the delta model. Three classes of errors in the initial model, i.e. unmodeled dynamics, parametric errors and initial condition errors are considered. Some illustrative examples are presented.**

I. Introduction

Both first principle (that is, analytical) models and empirical (that is, identified) models are approximate. The required accuracy of a model is application dependent. In this paper, within the context of Hammerstein systems, we assume that an *initial model* is available and that the fidelity of the initial model is insufficient. For example, the initial model may be erroneous with regard to initial conditions, parameters, or order (due to unmodeled dynamics). Our goal is to apply identification methods and adaptive disturbance rejection methods to improve the accuracy of the initial model. To do this, we combine the initial model with a *delta model* to obtain an *augmented model*. This technique is of particular interest when the initial model is a large-scale analytical model or a computer simulation (e.g. CFD or MHD), in which case it is convenient to add a small delta model to it rather than replace the initial model.

A related approach developed in [4] corrects a model of a structure with truncated modes by appending an analytically-derived delta model in parallel. Furthermore, in [3] a method is given for modifying an existing controller based on knowledge of deviations in the plant. Several classes of plant deviations are considered including feedback, parallel and cascade. However, the aim of [3] is not to correct the model itself, but rather to correct the controller such that it handles deviations in the plant.

A delta model can be combined with the initial model in cascade, parallel, or feedback. In the present paper we consider subspace identification and adaptive disturbance rejection to construct the delta models. For the cascade and parallel augmentation case, we can use subspace identification methods to build the delta models, whereas using adaptive disturbance rejection, delta models for all three cases can be constructed.

For model refinement using subspace identification, we set up an identification problem in which we construct an empirical model of the error between the true system and the initial model, that is, the *delta system*. This approach assumes the ability to obtain input-output data by experiment. Subspace-based nonlinear identification algorithms [6] are then used to construct a delta model, that is, a model of the delta system. Subspace algorithms are desirable for this application because of their ability to provide an estimate of the delta system's initial state, and thus correct errors in the initial state. Although, in accordance with subspace algorithms, the state space basis of the identified delta model is unknown, we show that the estimated initial state can nevertheless correct errors in the initial state.

Subspace algorithms [5], [7] are used to identify state space matrices (A, B, C, D) based on the known inputs and outputs of the system. These methods are computationally tractable and naturally applicable to MIMO systems. In this paper the n4sid command in Matlab System Identification Toolbox is used for linear system identification, and the method developed in [6], [2] is used for Hammerstein system identification. With these algorithms, the system order can be manually chosen or automatically set based on numerical criteria.

For model refinement using adaptive disturbance rejection, we formulate the model refinement problem as an adaptive disturbance rejection problem. An adaptive disturbance rejection algorithm is then used to tune the delta model. In this paper, the ARMARKOV algorithm [8] is used for adaptation. Since adaptive disturbance rejection methods for Hammerstein systems are not well developed, we restrict ourselves to linear systems in this case. We also note that the adaptive disturbance rejection algorithm can be used as a system identification tool when the initial model is set to zero in the parallel augmentation framework.

In section 2 we present model refinement using subspace identification algorithms. First model equivalence concepts for Hammerstein systems are examined. These results are then used to suggest procedures for using subspace algorithms for the parallel and cascade augmentation case. In section 3 we discuss model refinement using adaptive disturbance rejection. In section 4 we present a few representative examples to illustrate the above ideas.

This research was supported by the National Science Foundation Information Technology Research initiative, through Grant ATM-0325332.

The authors are with the Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109-2140, USA

II. Model Refinement Using Subspace Identification

We first examine model equivalence for Hammerstein systems. This differs from traditional model equivalence concepts in the presence of the static nonlinearity and a non-zero initial conditions. Then we present procedures for cascade model augmentation and parallel model augmentation.

The notation $\mathcal{S} \sim \left[\begin{array}{c|cc} A & B & f \\ \hline C & D & g \end{array}\right]_{x(0)}$ denotes the discrete-time Hammerstein dynamical system

$$x(k+1) = Ax(k) + Bf(u(k)), \quad \text{(II.1)}$$
$$y(k) = Cx(k) + Dg(u(k)), \quad \text{(II.2)}$$

with initial condition $x(0)$, where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^l$, $u \in \mathbb{R}^m$, $f : \mathbb{R}^m \mapsto \mathbb{R}^r$ and $g : \mathbb{R}^m \mapsto \mathbb{R}^s$. If (A, B, C) is minimal, where $A \in \mathbb{R}^{n\times n}$, then n is the *order* of $\mathcal{S}$. The notation $u_1 \equiv u_2$ means $u_1(k) = u_2(k)$ for all $k \geq 0$.

Definition II.1. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be systems with inputs u_1 and u_2 and outputs y_1 and y_2, respectively. Then $\mathcal{S}_1$ and $\mathcal{S}_2$ are *equivalent* if $y_1 \equiv y_2$ whenever $u_1 \equiv u_2$.

Proposition II.1. Consider the n^{th} order systems

$$\mathcal{S}_1 \sim \left[\begin{array}{c|cc} A_1 & B_1 & f_1 \\ \hline C_1 & D_1 & g_1 \end{array}\right]_{x_1(0)}, \mathcal{S}_2 \sim \left[\begin{array}{c|cc} A_2 & B_2 & f_2 \\ \hline C_2 & D_2 & g_2 \end{array}\right]_{x_2(0)}$$

with $f_1(0) = f_2(0) = g_1(0) = g_2(0) = 0$. Then $\mathcal{S}_1$ and $\mathcal{S}_2$ equivalent if and only if $D_1 g_1(u) = D_2 g_2(u)$ for all u, and there exists a nonsingular matrix $S \in \mathbb{R}^{n\times n}$ such that

$$A_1 = SA_2S^{-1}, \quad C_1 = C_2S^{-1}, \quad x_1(0) = Sx_2(0), \quad \text{(II.3)}$$

and

$$B_1 f_1(u) = SB_2 f_2(u) \quad \text{for all } u. \quad \text{(II.4)}$$

Proof. Sufficiency is immediate. To prove necessity, by setting $u_1 = u_2 = 0$, and using equivalence, we can show that

$$x_1(0) = Sx_2(0), \quad \text{(II.5)}$$

with $S \stackrel{\triangle}{=} (Q_1^{\mathrm{T}} Q_1)^{-1} Q_1^{\mathrm{T}} Q_2$, where Q_1 and Q_2 are the observability matrices respectively. Since (A_1, C_1) and (A_2, C_2) are both observable, Q_1 and Q_2 have full rank, and hence S and S^{-1} always exists. Further, using the above mentioned construction for S, the rest of the proof follows. □

A. Cascade Delta-Model Augmentation

Consider the system $\mathcal{S}_0 \sim \left[\begin{array}{c|cc} A_0 & B_0 & f_0 \\ \hline C_0 & D_0 & g_0 \end{array}\right]_{x_0(0)}$ and the initial model $\mathcal{S}_{\rm m} \sim \left[\begin{array}{c|cc} A_{\rm m} & B_{\rm m} & f_{\rm m} \\ \hline C_{\rm m} & D_{\rm m} & g_{\rm m} \end{array}\right]_{x_{\rm m}(0)}$. The system with input $y_{\rm m}$ and output y_Δ is the *cascade delta system* $\mathcal{S}_\Delta$, which is connected in series with the initial model to obtain the cascade-augmented model $\mathcal{S}_{\rm a} \stackrel{\triangle}{=} \mathcal{S}_\Delta \mathcal{S}_{\rm m}$ shown in Figure 1.

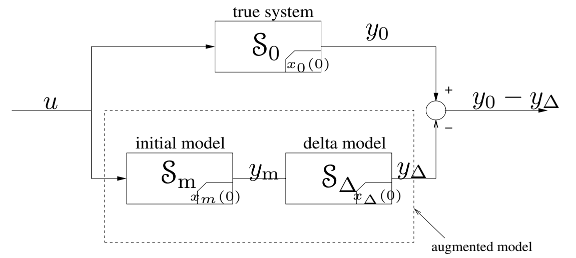

Fig. 1. Cascaded Delta-Model Augmentation

Proposition II.2. Suppose that $\mathcal{S}_\Delta \mathcal{S}_{\rm m}$ and $\mathcal{S}_0$ are equivalent. Then $y_\Delta \equiv y_0$ for all inputs u.

In view of Proposition II.2, we use subspace-based nonlinear identification algorithms [6] to construct a cascade delta model $\hat{\mathcal{S}}_\Delta$ with input $u_\Delta = y_{\rm m}$, where $y_{\rm m}$ is the initial model output, and output $y_\Delta = y_0$, where y_0 is the true system output. Then, a cascade-augmented model $\hat{\mathcal{S}}_{\rm a} \stackrel{\triangle}{=} \hat{\mathcal{S}}_\Delta \mathcal{S}_{\rm m}$ can be constructed as shown in Figure 1 to approximate the true system $\mathcal{S}_0$.

B. Parallel Delta-Model Augmentation

Consider the system $\mathcal{S}_0 \sim \left[\begin{array}{c|cc} A_0 & B_0 & f_0 \\ \hline C_0 & D_0 & g_0 \end{array}\right]_{x_0(0)}$ and the initial model $\mathcal{S}_{\rm m} \sim \left[\begin{array}{c|cc} A_{\rm m} & B_{\rm m} & f_{\rm m} \\ \hline C_{\rm m} & D_{\rm m} & g_{\rm m} \end{array}\right]_{x_{\rm m}(0)}$. The system with input u and output $y_\Delta = y_0 - y_{\rm m}$ is the *parallel delta system* $\mathcal{S}_0 - \mathcal{S}_{\rm m}$, which is illustrated in Figure 2.

Proposition II.3. Consider two Hammerstein systems $\mathcal{S}_0$ and $\mathcal{S}_{\rm m}$, then the parallel delta system $\mathcal{S}_0 - \mathcal{S}_{\rm m}$ is also a Hammerstein system.

Proof. Let $u \in \mathbb{R}^m$ be the inputs to both the systems and let $y_0 \in \mathbb{R}^l$ and $y_{\rm m} \in \mathbb{R}^{l_{\rm m}}$ be the outputs of the two Hammerstein systems respectively. Now, the parallel delta system output is given by

$$\begin{aligned} y_\Delta(k) &= y_0(k) - y_{\rm m}(k) \\ &= C_0 x_0(k) + D_0 g_0(u(k)) - C_{\rm m} x_{\rm m}(k) \\ &\quad - D_{\rm m} g_{\rm m}(u(k)) \\ &= \begin{bmatrix} C_0 & -C_{\rm m} \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_{\rm m}(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} D_0 & -D_{\rm m} \end{bmatrix} \begin{bmatrix} g_0(u(k)) \\ g_{\rm m}(u(k)) \end{bmatrix}. \end{aligned} \quad \text{(II.6)}$$

So the parallel system can be written as

$$x_\Delta(k+1) = A_\Delta x_\Delta(k) + B_\Delta f_\Delta(u(k)), \quad \text{(II.7)}$$
$$y_\Delta(k) = C_\Delta x_\Delta(k) + D_\Delta g_\Delta(u(k)), \quad \text{(II.8)}$$

where, $x_\Delta(k) = \begin{bmatrix} x_0(k) \\ x_{\rm m}(k) \end{bmatrix}$, $A_\Delta = \begin{bmatrix} A_0 & 0 \\ 0 & A_{\rm m} \end{bmatrix}$, $B_\Delta = \begin{bmatrix} B_0 & 0 \\ 0 & B_{\rm m} \end{bmatrix}$, $f_\Delta(u) = \begin{bmatrix} f_0(u) \\ f_{\rm m}(u) \end{bmatrix}$, $C_\Delta = \begin{bmatrix} C_0 & -C_{\rm m} \end{bmatrix}$, $D_\Delta = \begin{bmatrix} D & -D_{\rm m} \end{bmatrix}$, $g_\Delta(u) = \begin{bmatrix} g_0(u) \\ g_{\rm m}(u) \end{bmatrix}$ and $x_\Delta(0) =$

$\begin{bmatrix} x_0(0) \\ x_{\rm m}(0) \end{bmatrix}$. Thus the resulting system is again a Hammerstein system. □

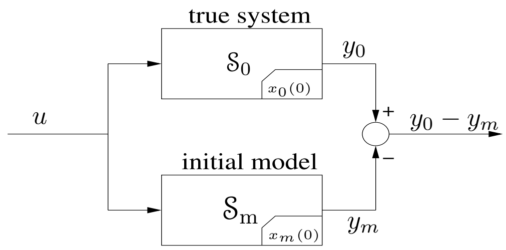

Fig. 2. Parallel Delta System

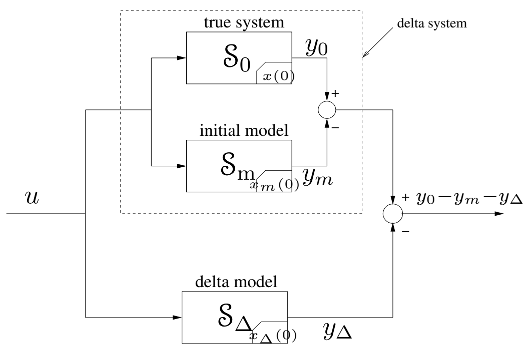

Fig. 3. Delta-Model Identification for Parallel Augmentation

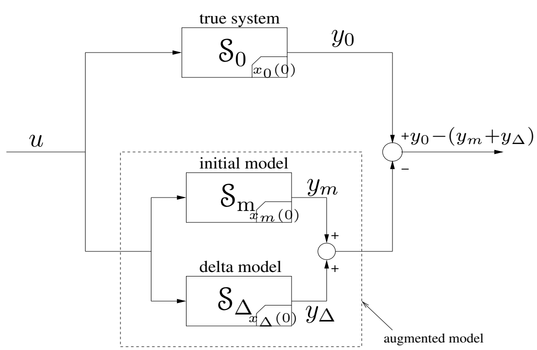

Fig. 4. Parallel Model Augmentation

Proposition II.4. Suppose $\mathcal{S}_0 - \mathcal{S}_{\rm m}$ and $\mathcal{S}_\Delta$ are equivalent with outputs $y_0 - y_{\rm m}$ and y_Δ, respectively. Then the output $y_0 - y_{\rm m} - y_\Delta$ of the system $\mathcal{S}_0 - \mathcal{S}_{\rm m} - \mathcal{S}_\Delta$ is identically zero for all inputs u.

Proposition II.5. Suppose that $\mathcal{S}_0 - \mathcal{S}_{\rm m}$ and $\mathcal{S}_\Delta$ are equivalent. Then $\mathcal{S}_0$ and $\mathcal{S}_{\rm m} + \mathcal{S}_\Delta$ are equivalent.

Proof. Since $\mathcal{S}_0 - \mathcal{S}_{\rm m}$ and $\mathcal{S}_\Delta$ are equivalent it follows that $y_\Delta \equiv y_0 - y_{\rm m}$. Therefore $y_0 \equiv y_{\rm m} + y_\Delta$ and hence $\mathcal{S}_0$ and $\mathcal{S}_{\rm m} + \mathcal{S}_\Delta$ are equivalent. □

To construct a parallel delta model, consider the initial model $\mathcal{S}_{\rm m}$ of the true system $\mathcal{S}_0$. From Prop. II.3, the delta system $\mathcal{S}_\Delta = \mathcal{S}_0 - \mathcal{S}_{\rm m}$ is a Hammerstein system. The subspace based nonlinear identification algorithm [6] can thus be used to construct a model of the delta system based on the input u and the output $y_0 - y_{\rm m}$. This is illustrated in Figure 3. The identified model $\hat{\mathcal{S}}_\Delta$ is an approximation of $\mathcal{S}_0 - \mathcal{S}_{\rm m}$. Next, a Hammerstein parallel-augmented model $\hat{\mathcal{S}}_{\rm a} \stackrel{\triangle}{=} \mathcal{S}_{\rm m} + \hat{\mathcal{S}}_\Delta$ can be constructed as shown in Figure 4 to approximate the true system $\mathcal{S}_0$.

Since linear systems are special cases of Hammerstein systems, with $f(u) = u$, all the above arguments apply even if we have a linear initial model $\mathcal{L}_{\rm m}$ for a Hammerstein system $\mathcal{S}_0$.

III. Model Refinement Using Adaptive Disturbance Rejection

Adaptive disturbance theory for Hammerstein systems are not well developed, so in this section we restrict our discussion to linear systems only. We discuss model refinement using adaptive disturbance rejection with cascade, parallel, feedback and combined cascade-feedback architectures.

Consider a linear system $\mathcal{L}_0$ described by the discrete-time state-space equations

$$x_0(k+1) = A_0 x_0(k) + D_{10} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}, \quad \text{(III.1)}$$

$$y_0(k) = C_0 x_0(k) + D_{20} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}, \quad \text{(III.2)}$$

and an initial model $\mathcal{L}_{\rm m}$ with the equations

$$x_{\rm m}(k+1) = A_{\rm m} x_{\rm m}(k) + D_{1m} w_1(k) + B_{\rm m} u(k), \quad \text{(III.3)}$$

$$y_{\rm m}(k) = C_{\rm m} x_{\rm m}(k) + D_{2m} w_1(k) + D_{\rm m} u(k), \quad \text{(III.4)}$$

where $w_1(k)$ is a known input signal, $u(k)$ is the output from the delta model, and $w_2(k)$ is an unmeasured disturbance in the true system $\mathcal{L}_0$.

The objective of delta modeling is to construct a *delta model* in cascade, parallel, or feedback with the initial model, so that the resulting augmented model matches the true system. In the case of a parallel delta model, B_m and D_m are zero since the output from the delta model is directly added to the output of the initial model. In Section 2 subspace identification was used to construct the delta model. In the case of feedback interconnection, however, subspace identification cannot be used since the required $u(k)$ to achieve model matching is unknown. Instead, the problem is recast as an adaptive disturbance rejection problem.

Consider again the true system (III.1) and (III.2), and the initial model (III.3) and (III.4). To achieve model matching we require that the error $y_0(k) - y_{\rm m}(k)$ be small. Hence the control inputs $u(k)$ have to be modified in a way that makes $y_{\rm m}(k)$ equal to $y_0(k)$. This problem can be viewed as a disturbance rejection problem where the performance variable $z \stackrel{\triangle}{=} y_0 - y_{\rm m}$ is minimized in the presence of external disturbances $w_1(k)$ and $w_2(k)$.

Four different interconnection structures are considered for the delta model, represented by $\mathcal{L}_{\rm c} \sim \left[\begin{array}{c|c} A_{\rm c} & B_{\rm c} \\ \hline C_{\rm c} & D_{\rm c} \end{array}\right]$. In all four cases the signal used to tune the controller parameters is the error between the outputs of the true system and the model,

$z \triangleq y_0 - y_{\mathrm{m}}$. Defining $\tilde{x}(k) \triangleq \begin{bmatrix} x_0(k) \\ x_{\mathrm{m}}(k) \end{bmatrix}$ and $w(k) \triangleq \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$, it will be shown that the delta modeling problem can be written in the standard control architecture form shown in Figure 5. In the context of adaptive disturbance rejection, the delta model $\mathfrak{L}_{\mathrm{c}}$ is the adaptive controller, the initial model is the plant $\mathfrak{L}_{\mathrm{m}}$, and knowledge of $w_2(k)$ or the true system $\mathfrak{L}_0$ is not required. Thus, adaptively controlling the initial model $\mathfrak{L}_{\mathrm{m}}$ to minimize the performance variable z, achieves the objective of constructing an augmented model to match the true system $\mathfrak{L}_0$. So, once the model refinement problem is recast in the form of Figure 5, a standard adaptive disturbance rejection algorithm like ARMARKOV [8] can be used to tune the delta model.

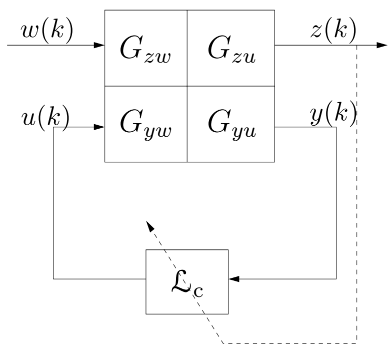

Fig. 5. Control architecture for the standard problem

A. Cascade Interconnection

In the cascade case, the input to the delta model $\mathfrak{L}_{\mathrm{c}}$ is $w_1(k)$, and thus we define the controller inputs $y(k) \triangleq w_1(k)$. This scheme shown in Figure 6 can be represented in the standard form as

$$\begin{bmatrix} z(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} \mathfrak{L}_0 - \begin{bmatrix} \mathfrak{L}_{\mathrm{m}} & 0 \end{bmatrix} & -\mathfrak{L}_{\mathrm{m}} \\ \begin{bmatrix} I & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} w(k) \\ u(k) \end{bmatrix}, \tag{III.5}$$

with the state space realization

$$\begin{bmatrix} \tilde{x}(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \left[\begin{array}{cc|cc} A_0 & 0 & D_{10} & 0 \\ 0 & A_{\mathrm{m}} & \begin{bmatrix} D_{1\mathrm{m}} & 0 \end{bmatrix} & B_{\mathrm{m}} \\ \hline C_0 & -C_{\mathrm{m}} & D_{20} - \begin{bmatrix} D_{2\mathrm{m}} & 0 \end{bmatrix} & -D_{\mathrm{m}} \\ 0 & 0 & \begin{bmatrix} I & 0 \end{bmatrix} & 0 \end{array}\right] \begin{bmatrix} \tilde{x}(k) \\ w(k) \\ u(k) \end{bmatrix}. \tag{III.6}$$

B. Parallel Interconnection

In parallel interconnection, the delta model $\mathfrak{L}_{\mathrm{c}}$ is connected as shown in Figure 7. The parallel architecture represented in the standard form has the stet space realization

$$\begin{bmatrix} \tilde{x}(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \left[\begin{array}{cc|cc} A_0 & 0 & D_{10} & 0 \\ 0 & A_{\mathrm{m}} & \begin{bmatrix} D_{1\mathrm{m}} & 0 \end{bmatrix} & 0 \\ \hline C_0 & -C_{\mathrm{m}} & 0 & -I \\ 0 & 0 & \begin{bmatrix} I & 0 \end{bmatrix} & 0 \end{array}\right] \begin{bmatrix} \tilde{x}(k) \\ w(k) \\ u(k) \end{bmatrix}. \tag{III.7}$$

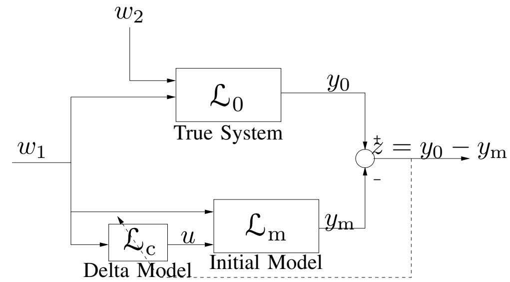

Fig. 6. Cascade interconnection for model refinement.

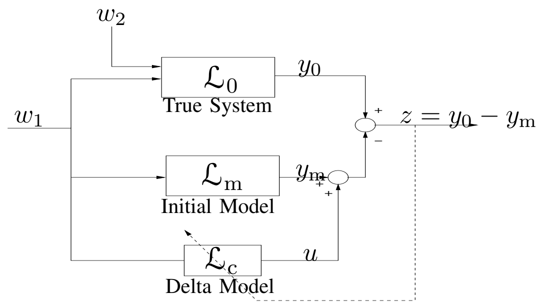

Fig. 7. Parallel interconnection for model refinement.

C. Feedback Interconnection

The feedback interconnection problem is shown in Figure 8. In this case the controller inputs $y(k) \triangleq y_{\mathrm{m}}(k)$. Thus the standard form state space representation is

$$\begin{bmatrix} \tilde{x}(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \left[\begin{array}{cc|cc} A_0 & 0 & D_{10} & 0 \\ 0 & A_{\mathrm{m}} & \begin{bmatrix} D_{1\mathrm{m}} & 0 \end{bmatrix} & B_{\mathrm{m}} \\ \hline C_0 & -C_{\mathrm{m}} & D_{20} - \begin{bmatrix} D_{2\mathrm{m}} & 0 \end{bmatrix} & -D_{\mathrm{m}} \\ 0 & C_{\mathrm{m}} & \begin{bmatrix} D_{2\mathrm{m}} & 0 \end{bmatrix} & D_{\mathrm{m}} \end{array}\right] \begin{bmatrix} \tilde{x}(k) \\ w(k) \\ u(k) \end{bmatrix}. \tag{III.8}$$

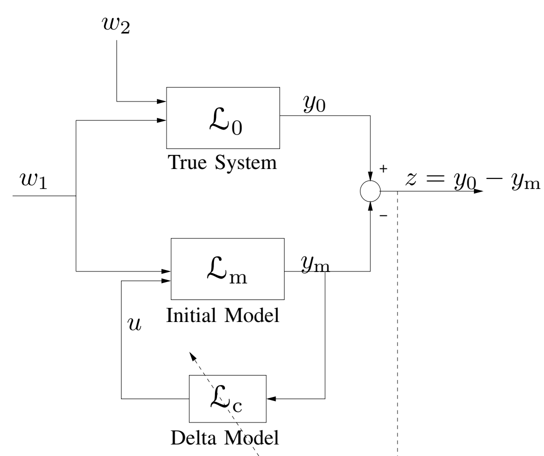

Fig. 8. Feedback interconnection for model refinement.

D. Cascade and Feedback Interconnection

In the feedback interconnection an additional feedforward path can be included to obtain a combined feedback and cascade interconnection, as illustrated in Figure 9. Noting that $y(k) \triangleq \begin{bmatrix} y_{\mathrm{m}}(k) \\ w_1(k) \end{bmatrix}$, the standard problem can be written in the state space form as

$$\begin{bmatrix} \tilde{x}(k+1) \\ z(k) \\ y(k) \end{bmatrix} = \left[\begin{array}{cc|cc} A_0 & 0 & D_{10} & 0 \\ 0 & A_{\mathrm{m}} & \begin{bmatrix} D_{1\mathrm{m}} & 0 \end{bmatrix} & B_{\mathrm{m}} \\ \hline C_0 & -C_{\mathrm{m}} & D_{20} - \begin{bmatrix} D_{2\mathrm{m}} & 0 \end{bmatrix} & -D_{\mathrm{m}} \\ 0 & C_{\mathrm{m}} & D_{2\mathrm{m}} \quad 0 & D_{\mathrm{m}} \\ 0 & 0 & I \quad 0 & 0 \end{array}\right] \begin{bmatrix} \tilde{x}(k) \\ w(k) \\ u(k) \end{bmatrix}. \tag{III.9}$$

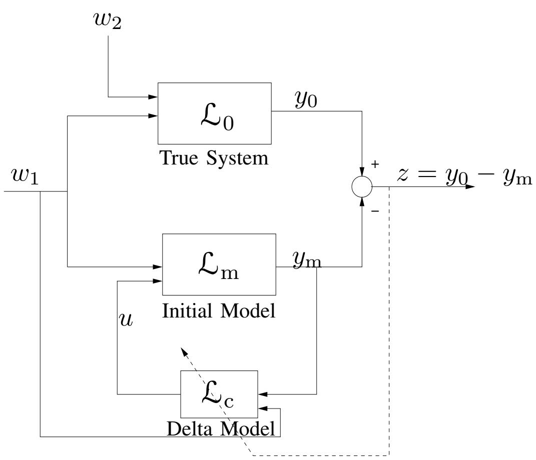

Fig. 9. Cascade and feedback interconnection for model refinement.

E. Identification using Adaptive Disturbance Rejection

Identification of linear systems can be considered a special case of model refinement, with the initial model being the zero model. Thus setting $\mathcal{L}_{\mathrm{m}} = 0$ in Figure 7, recovers the classical identification framework, with the adaptive controller $\mathcal{L}_c$ acting as the identified model. Casting the problem in this form, an adaptive disturbance rejection algorithm can be used to perform system identification.

IV. Examples

A. Model Refinement Using Subspace Identification

1) Unmodeled Dynamics Example: Here we consider the equations of an acoustic duct. The equations and state space realizations are given in [1], where the speed of acoustic waves is 343 m/s, the density of air is $1.21 kg/m^3$, length of the duct is $L = 2m$, and the duct model includes four modes.

To emulate unmodeled dynamics the initial model $\mathcal{S}_{\mathrm{m}}$ of the acoustic duct is obtained by deleting the states associated with the second mode. Hence $\mathcal{S}_0$ is 8^{th} order and $\mathcal{S}_{\mathrm{m}}$ is 6^{th} order. The system parameters and initial conditions of the retained states are assumed to be known. The frequency responses of the true system and the initial model are shown in Figure 10.

The fit errors using cascade augmentation for the various orders of the cascade delta model are shown in Table I, from which it is clear that a 6^{th}-order cascade delta model is needed for an accurate fit of the forced and free responses.

Cascade Delta-Model Order	Free Response Error	Forced Response Error
6	$6.0578e-10$	$3.5481e-09$
4	2.3649	4.5346
2	12.9163	27.5859
0	7.0867	32.5159

TABLE I

RESPONSE ERRORS OF AUGMENTED MODELS WITH REDUCED-ORDER CASCADE DELTA MODELS, WHEN THE INITIAL STATES OF $\mathcal{S}$ ARE KNOWN, FOR UNMODELED DYNAMICS

For parallel augmentation, an accurate fit for the forced and free responses is obtained with a 2^{nd}-order parallel delta model as shown in Figure 11 and Table II.

Parallel Delta-Model Order	Free Response Error	Forced Response Error
2	$3.6216e-11$	$1.2049e-10$
0	7.0867	32.5159

TABLE II

RESPONSE ERRORS OF THE PARALLEL AUGMENTED MODELS WITH REDUCED-ORDER PARALLEL DELTA MODELS, FOR UNMODELED DYNAMICS

2) Parametric Error and Initial Conditions Error Example: We consider a spring mass damper system with an input nonlinearity. Defining the states as $x \triangleq \begin{bmatrix} q & \dot{q} \end{bmatrix}^{\mathrm{T}}$, where q is the position of the mass and $\dot{q}$ is the velocity, the state space matrices are

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

where m is the mass, k is the spring stiffness and c is the damping constant. The input nonlinearity is chosen to be $f(u) = u^2 + u^3$. We use $k = 2$, $m = 5$, and $c = 3$ and a square wave as the input sequence u.

The N4SID command in Matlab is used to identify a linear model of the Hammerstein system described above. This identified model is then used as the initial model, and Hammerstein delta-model augmentation is performed on this initial model. The initial model is erroneous in the nonlinearity and the linear dynamics. Table III shows the error in the forced response for both the initial and the parallel augmented model.

B. Model Refinement Using Adaptive Disturbance Rejection

Consider the same spring mass system used in the previous example. The initial model has a parametric error in the

Parallel Delta-Model Order	Forced Response Error
0	21.8959
2	2.0939×10^{-7}

TABLE III

FORCED RESPONSE ERRORS FOR PARALLEL-AUGMENTED MODELS FOR THE LOW ORDER SYSTEM EXAMPLE.

spring stiffness and the damping coefficient. The true values are chosen to be $m = 8$, $k = 20$, $b = 1$ and the erroneous values in the initial model are $m = 8$, $k = 44$, $b = 4$. A white noise input drives the true system and the initial model. The frequency response curves of the true system, initial model and the augmented model is shown in Figure 12. The combined feedback and cascade architecture is employed in this example.

For identification of the spring mass damper system, the initial model is set to zero in the parallel interconnection framework. The frequency response curves of the true system and the identified model are shown in Figure 13.

V. CONCLUSION

In this paper we developed and illustrated two approaches to improve model accuracy by using a delta model combined in cascade, parallel or feedback with an initial model. In the case of model refinement using subspace algorithms, by identifying the initial fit error system, the identified model was combined with the initial model to construct an augmented model. This technique was shown to be only partially effective for correcting initial conditions and parametric errors. However, the method is more effective for correcting unmodeled dynamics. In the case of model refinement using adaptive disturbance rejection, the problem was recast as a disturbance rejection problem and the ARMARKOV adaptive disturbance rejection algorithm was used to tune the delta model. This technique was found to be more effective for correcting parametric errors.

REFERENCES

[1] J. Hong, J. C. Akers, R. Venugopal, M.-N. Lee, A. G. Sparks, P. D. Washabaugh, and D. S. Bernstein. Modeling, Identification, and Feedback Control of Noise in an Acoustic Duct. *IEEE Trans. Contr. Sys. Tech.*, 4:283–291, 1996.

[2] S. L. Lacy and D. S. Bernstein. Subspace Identification for Nonlinear Systems That Are Linear in Unmeasured States. In *Proc. Conf. Dec. Contr.*, pages 3518–3523, Orlando, Florida, December 2001.

[3] S. Mijanovic, G. E. Stewart, G. A. Dumont, and M. S. Davies. A controller Perturbation Technique for Transferring Closed-Loop Stability Between Systems. *Automatica*, 39:1783–1791, 2003.

[4] S. O. Reza Moheimani. Model correction for sampled-data models of structures. 24(3):634–637, 2001.

[5] M. Moonen, B. De Moor, L. Vandenberghe, and J. Vandewalle. On- and Off-Line Identification of Linear State-Space Models. *Int. J. Contr.*, 49(1):219–232, 1989.

[6] H. Palanthandalam-Madapusi, J.B. Hoagg, and D. S. Bernstein. Basis-function optimization for subspace-based nonlinear identification of systems with measured-input nonlinearities. In *Proc. Amer. Contr. Conf.*, pages 4788–4793, Boston, MA, July 2004.

[7] P. Van Overschee and B. De Moor. *Subspace Identification for Linear Systems: Theory, Implementation, Applications*. Kluwer, 1996.

[8] R. Venugopal and D. S. Bernstein. Adaptive Disturbance Rejection Using ARMARKOV/Toeplitz Models. *IEEE Trans. Contr. Sys. Tech.*, 8:257–269, 2000.

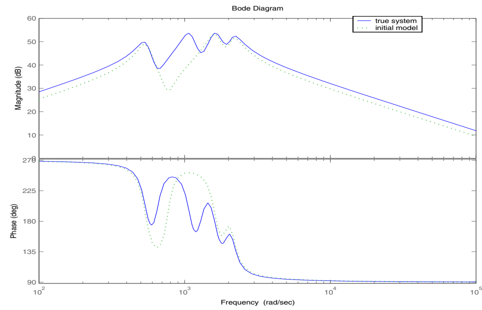

Fig. 10. Unmodeled Dynamics Example. Frequency responses of the true system and initial model.

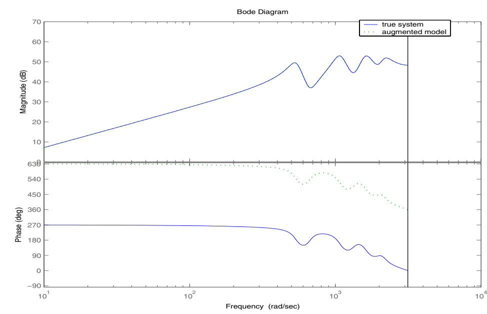

Fig. 11. Unmodeled Dynamics Example. Discrete-time frequency responses of the true system and the parallel augmented model.

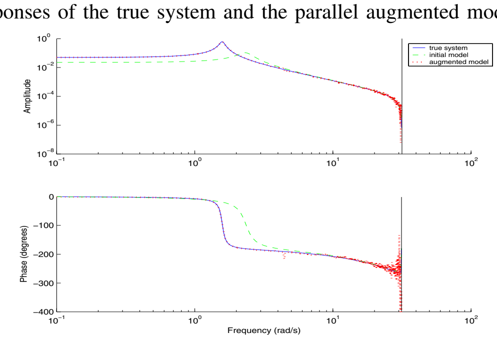

Fig. 12. Adaptive Disturbance Rejection Example. Frequency responses of the true system, initial model and the augmented model for a combined cascade and feedback interconnection.

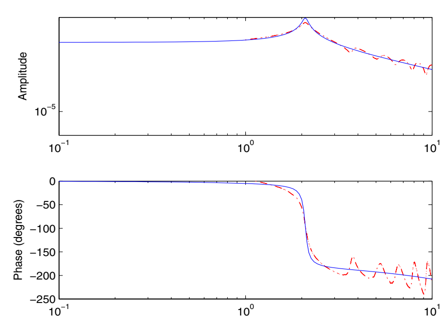

Fig. 13. Identification Example. Frequency responses of the true system, the identified model for the spring-mass system.