Rhymes with Mouth

As I head toward class on a sunny Friday morning, I keep repeating to myself the three most important elements of teaching: motivate, motivate, and motivate. But how could I motivate the interest of students brought up on PCs in a stability test devised in 1877? I needed a good idea fast.

"Suppose you're shipwrecked on a remote island in the South Pacific with no hope of rescue," I begin. "Yet, amazingly, you find everything you need to build an airplane except for one thing: You have no computer to analyze the aircraft's stability. What do you do?"

Almost immediately, I hear, "Get a 'palm pilot'!"

"All right, good try, but suppose that's not an option. Come to think of it, this scenario is pretty ridiculous, isn't it?"

"'Flight of the Phoenix!'"

"What do you mean?" I ask.

"That's what happens in the movie."

"All right," I say. "You're on the island, you've built an airplane, you've derived the longitudinal and lateral dynamics, and you've worked out the two fourth-order characteristic equations. How are you going to check stability?"

No response.

"OK, consider this," I say as I write on the board:

Suppose that all of the roots of the polynomial

$$p(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

are in the open left-half plane. Then $a_0, \ldots, a_{n-1}$ are positive.

When everyone finishes writing I ask, "So does this mean that the polynomial

$$p(s) = s^6 + 2s^5 + 3s^4 + .1s^3 + 100s^2 + 19s + .42$$

is stable?"

"No," I hear.

"Why not?"

"Because you gave a necessary condition for stability, not a sufficient condition."

Excellent, I say to myself, the math department is hot today.

"Precisely. In fact, if you had Matlab you would find that this polynomial isn't stable. But even without Matlab, there's something you can do. Suppose that $n = 2$. Then we have this:"

Both roots of

$$p(s) = s^2 + a_1 s + a_0$$

are in the open left-half plane if and only if a_0 and a_1 are positive.

"How would you prove this?" I ask.

"Use the quadratic formula?"

"Correct. That will be on the homework."

Groan.

"Now, here's the result for $n = 3$:"

All three roots of

$$p(s) = s^3 + a_2 s^2 + a_1 s + a_0$$

are in the open left-half plane if and only if a_0, a_1, a_2 are positive and

$$a_0 < a_1 a_2.$$

"So, is $p(s) = s^3 + 3s^2 + 2s + 5$ stable?"

"Yes!"

"Did you use Matlab?"

"No, I just used what you wrote.""

"Great. We're almost off the island. But what about $p(s) = 2s^3 + 3s^2 + 2s + 5$?"

"Does the 2 in front of the s^3 matter?"

"You bet."

"Then that one isn't stable."

"Good. Be sure to make a note that the condition I gave is for monic polynomials. When the leading coefficient isn't 1 you need to divide the polynomial by the leading coefficient to make it monic. That's a trick question I save for tests." Everyone immediately writes down what I just said.

"Now we do the fourth-order case:"

All four roots of

$$p(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

are in the open left-half plane if and only if a_0, a_1, a_2, a_3 are positive and

$$a_0 a_3^2 + a_1^2 < a_1 a_2 a_3.$$

"So, is $p(s) = s^4 + 2s^3 + 3s^2 + 2s + 1$ stable?"

"Yes, definitely."

"Did you use Matlab?"

"No, just what you wrote."

"Great. You now know the Routh test for polynomials of order 2, 3, and 4. There are Routh tests for polynomials of all orders, but in this course you only need the results I gave today. And now that you can tell whether a fourth-order polynomial is stable without computing any roots, you're ready to take off.

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"But before you leave, let me tell you a story. You've probably seen a governor on an old steam engine. It's a pair of metal spheres that swing outward as they spin. The motion of the spheres opens and shuts a valve that controls the speed of the engine. Maxwell—the physicist who discovered Maxwell's equations you learn about in EE classes—got interested in governors and derived their equations. When Maxwell linearized the equations, he got a polynomial of order 5. Since he didn't have Matlab and Routh hadn't invented his test yet, Maxwell couldn't figure out whether the polynomial was stable. Since he was stuck, he decided to work on another problem, which was the stability of the rings of Saturn. This time he got a fourth-order polynomial. He still didn't have Routh or Matlab, but he was lucky since the 4th-order polynomial factored as the product of two quadratics, and he could solve those. So what's the moral of the story?"

"It's easier to analyze the rings of Saturn than it is to figure out how a governor works?"

"Something like that. Have a good weekend."

JASON BERNSTEIN

Mohammed Zohdy of Oakland University and Dennis Bernstein meeting by chance at Michigan State University in August 2006, where Mo's youngest daughter and Dennis's younger son enrolled as freshmen.

Early Experiments

There is an incident from our childhood on 67th Street that I still remember vividly. We were both about 6 years old at the time. It was a winter evening and our parents were in the kitchen chatting over coffee. Peter and I were in the living room where Peter had unplugged 2 lamps and was examining the electrical outlets. Now, 66 years later, I can still remember his exact words – "I wonder what would happen if I pour some lemon juice into the wall plug?" Then he proceeded to do just that! The building, all 36 apartments and corridors, went black. He had just "shorted" the entire building. Wow!! Peter was really surprised. I was frightened, expecting him to be severely punished. But, his mother got a flashlight and just said "che biricchin!"-Piedmontese for "what a mischief-maker." No scolding, just a "don't do that again." Rosina was truly an exceptional mom.

So, I have to conclude that on that evening Peter, fearless and guilt-free, was launched on his future avocation—electrical engineering.

—J.C. Weiss, "67th Street Memories," in Systems and Control: A Tribute to Professor Peter Dorato, *M. Jamshidi, Ed. San Antonio, TX: TSI Press, 2006, p. 185*