# Pointillism

Mathematics and life love continuity. This qualitative property allows us to infer that, nearby, things don't change drastically. We expect smooth transitions from one place to another, and we become accustomed to the slow evolution of the world from day to day and year to year.

Of course, since continuity is only qualitative, we accept that large changes can occur. The extreme sensitivity of chaotic systems is a reminder that even continuity has its practical limits, but at least continuity remains the paradigm.

Abrupt change, on the other hand, is difficult to accept. When we witness what looks like discontinuity, we dissect it to uncover the underlying continuity. The caterpillar can only become a butterfly through a sequence of steps, each connected to the other through yet more intermediate steps. Ultimately continuity must be at work.

## Contributors

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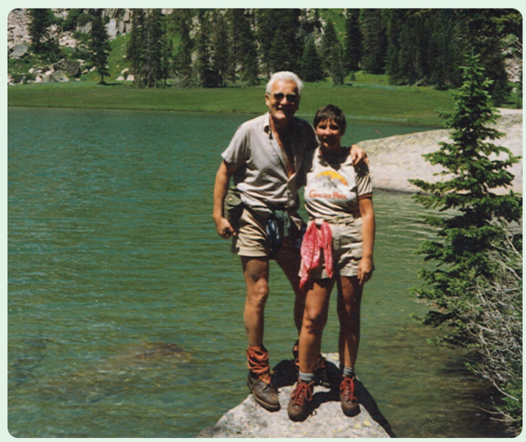

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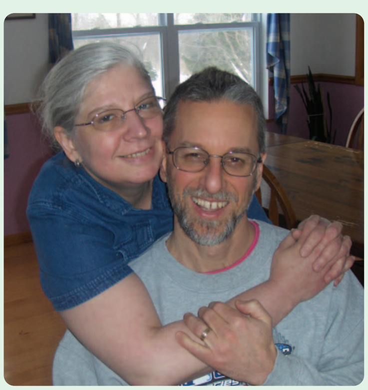

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To open the door to true discontinuity is jarring; a jump from one moment to the next strikes us as impossible. Yet, it sometimes seems that it is the discontinuities that matter the most in the long run. Research ideas progress monotonically and incrementally through a sequence of developmental stages until unexpected evidence shatters the smooth progression of ideas. It happened in high temperature superconductivity and on September 11.

**In this issue of *IEEE Control Systems Magazine* we continue our series of special articles on modeling.**


Control theory tells us that optimal controls are often discontinuous. Optimality insists that control values jump instantly. Fortunately, technology often obliges. On-off is a good model of how many devices operate, dimmer switches notwithstanding.

Even without optimality, much of control exploits discontinuity. Sliding mode control pushes the idea to an extreme through chattering, which may seem inelegant but not to Seurat.

In this issue of *IEEE Control Systems Magazine* (*CSM*) we continue our series of special articles on modeling. The first two installments, which appeared in the April and December 2007 issues, focused on physical quantities such as energy and power, as well as system properties such as causality. In contrast, this third article focuses on a single mathematical modeling topic. That topic is discontinuous differential equations.

By a discontinuous differential equation, we mean an ordinary differential equation with a right-hand side—aka the vector field—that is a discontinuous function of the state. The state of a system of this type has a derivative that can jump. Surprisingly, in many cases of practical interest, the state itself is continuous despite the discontinuity of the vector field. Solutions of this type are the focus of the article.

The mathematics of discontinuous differential equations are intricate. The classical text by Fillipov is the seminal work, but it is not easy going. Fortunately, Jorge Cortés, who has contributed several features to *CSM*, agreed to write a tutorial article on the subject. Jorge has done a valuable service to our community by providing an accessible account of the main ideas in discontinuous differential equations.

What is fascinating about discontinuous differential equations is that there is no universally accepted notion of a solution, and different notions of solution may have different properties. As you read Jorge's article, ask yourself the following questions: Since a discontinuous differential equation can possess different solutions depending on the notion of solution that is adopted, on which grounds do I distinguish the solution that is physically meaningful? Do I follow my intuition about how I believe the world works, or is a physical experiment needed to determine the appropriate notion? Without the classical comfort of Lipschitz continuity, we are forced to reconcile mathematical ambiguity with physical reality. This is what modeling is about.

Among the columns that this issue of *CSM* brings you, we recognize and honor 49 IEEE Control Systems Society members who reach the 25-year mark in 2008. Congratulations and thanks! Our "Expert" columnist for this issue is Associate Editor Carl Knospe, who explains magnetic bearing technology, a quintessential control application. Amir Chaghajerdi explains the fascinating details behind optical disks, which store data and movies. For "People in Control," we speak with Stanislaw Tarasiewicz, who works on a wide range of process and biomedical control problems, and we quiz Michael Polis and Zongli Lin on their plans for book reviews. Mike joins us from *IEEE Transactions on Automatic Control*, as we transfer book review responsibility from *IEEE Transactions on Automatic Control* to *CSM*. We also have an historical article by Petar Kokotovic on his journey from Yugoslavia to the United States as an emergent control practitioner, researcher, and educator. I view this article as the second in a sequence of historical reminiscences, of which the first was authored by Fahmida Chowdhury in the August 2006 issue.

This issue also brings you four book reviews and a report on the 46th CDC in New Orleans. We remember with sadness John Zaborszky, who touched many in our field. And we close with another control crossword by Dan Simon.

Looking toward the future, we have several special issues in progress on topics such as friction, hysteresis, active automotive safety, and, of course, Kalman filtering. We are always looking for your contributions—after all this is *your* magazine—either short or long. Please contact me at dsbaero@umich.edu at any time. Better yet, speak to me in person at the 2008 ACC in Seattle!

**Dennis S. Bernstein**