Faux Optimal

If you've ever tried to apply Newton's laws to a multibody system, you know that it can be difficult. To apply these laws, you must know the forces on each body, but these forces depend on the dynamics of the neighboring bodies. The forces can be determined by solving simultaneous equations, but the process is tedious.

Lagrangian dynamics, which is taught in advanced dynamics courses, makes it easy to derive the equations of motion for multibody systems. All you need is an expression for the kinetic and potential energy of each body to form the Lagrangian, which in turn yields a differential equation for the generalized coordinates. For a two-body system such as the double

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Contributors

Dimitri Jeltsema (second from left) with members of the band Rotterdam Ska-Jazz Foundation and on the drums.

Jacquelien Scherpen, with her sons Risto (left) and Timo during a hike at Pastrana, La Gomera, in the Canary Islands and presenting her inaugural lecture at the University of Groningen.

Vijay and Padma Chellaboina with their children Saankhya and Harsha.

pendulum, Lagrangian dynamics are amazingly convenient compared to Newton's laws.

For those of us who work in control, one of the interesting aspects of Lagrangian dynamics is the fact that the Lagrangian satisfies the Euler-Lagrange differential equation. This equation is identical to the first-order necessary conditions for optimality in the calculus of variations, which suggests that Lagrangian dynamics minimizes something. Such minimization is comforting since it shows that physical objects move according to an underlying rationale. We may not know *why* motion minimizes something, but at least we know that it does.

But here's the thing: It doesn't. Numerous books and even such venerable sources as Wikipedia get it wrong:

> The fundamental lemma of the calculus of variations shows that solving Lagrange's equation is equivalent to finding the path that minimizes the action functional, a quantity that is the integral of the Lagrangian over time.

A counterexample to this statement appears on page 152 of Lamberto Cesari's book *Optimization—Theory and Applications*. If you know about conjugate point theory, then you can guess how the counterexample was created.

If Lagrangian dynamics doesn't minimize the action functional, then what does it minimize? Is physical motion optimal in some sense, or does this question merely reflect our need to give reality a purpose? As far as I can tell, no one knows. But what we do know is that Lagrangian dynamics avoids dealing with the forces between bodies—the constraint forces. In other words, Lagrangian dynamics is an efficient bookkeeping technique.

Various extensions of Lagrangian dynamics are used in practice. When energy is not conserved due to damping, the Rayleigh dissipation function is needed. If external forces and torques are applied, then these terms must be incorporated. And, if friction forces are present where two or more bodies interact, then additional modification is needed.

If you work on mechanical systems, then it's likely that you know everything I've said so far. But you might not know that a version of Lagrangian dynamics applies to electrical systems, where components that capture inductance, capacitance, and resistance may be nonlinear. And this is only the tip of the iceberg.

The article "Multidomain Modeling of Nonlinear Networks and Systems" by Dimitri Jeltsema and Jacquelien Scherpen develops Lagrangian modeling techniques across mechanical, electrical, thermal, and fluid domains. A key component of their development is the concept of dynamic analogies between domains. Although these ideas are well known in our community, there are numerous subtleties. For example, how does the concept of inertial space manifest itself in electrical circuits? What is the electrical analogue of rotational dynamics? How do nonlinear electrical components correspond to nonlinear stiffness and damping?

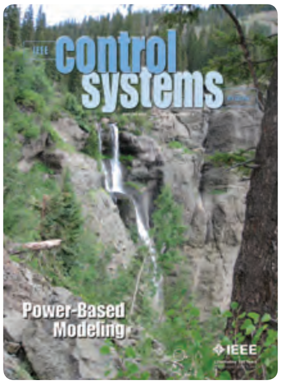
You may be surprised, as I was, to learn that kinetic and potential energy are not the full story; kinetic and potential *co*-energy are needed as well. In fact, Lagrangian and co-Lagrangian functions are used to account for nonlinear effects in all of these domains.

Rini and Prashant Mehta and their daughter Ayesha.

Sanjay and Reshma Bhat with their daughters Arpita (left) and Archita.

This article provides a comprehensive introduction to these deep and powerful ideas.

As its title suggests, "Multidomain Modeling of Nonlinear Networks and Systems" is the latest installment of the *IEEE Control Systems Magazine* (*CSM*) series on modeling. Previous articles in this series include in-depth treatments of bond graphs, behaviors, discontinuous dynamics, the double-gimbal torus, and hybrid dynamics. This series is based on the firm belief that modeling is a crucial component of control. In fact, modeling based on rigorous mathematics as practiced by our community can provide a crucial bridge from theory to applications that can benefit all areas of science and technology.



The second feature of this issue, which is also part of the modeling series, should be of interest to readers of this magazine for whom chemistry is not their cup of tea. I remember finding it hard to grasp chemistry because of what seemed to be a collection of "rules" with endless exceptions and special cases. What chemistry seemed to lack was something concrete and believable—like differential equations! The article "Modeling and Analysis of Mass-Action Kinetics" by Vijay Chellaboina, Sanjay Bhat, Wassim Haddad, and myself shows that chemical reactions are indeed governed by differential equations, called the reaction kinetics.

Each state of the reaction kinetics represents a concentration of a species involved in a reaction. Consequently, the states of the reaction kinetics are nonnegative. Furthermore, as long as no mass is added or removed from the chemical reactions, mass is conserved, and the total mass of the system—which is linear in the states—provides a Lyapunov function for proving stability.

Stability for chemical reactions cannot be asymptotic for the simple reason that, for a reaction that converges to steady-state concentrations, the limiting concentrations depend, in general, on the initial concentrations. In other words, the amount of water you obtain depends on the initial amounts of hydrogen and oxygen. The relevant notion of stability in this case is *semistability*. Semistability plays a role in applications such as rigid body dynamics with friction, where a sliding body comes to rest at a displacement determined by its initial velocity.

In addition to these modeling features, this issue brings you an "Ask the Experts" column by Prashant Mehta, who discusses the challenges and rewards of controlling combustion. In this issue we recognize, as we do each year, "quarter-century" IEEE Control Systems Society (CSS) members. Congratulations to all of you who have reached this milestone!

This month's "Technical Committees" department is devoted to the Technical Committee on Industrial Process Control. If you're not a member of any technical committee, then I strongly recommend that you join one (I did). There is no charge, no obligation, and much to be gained. Feel free to contact CSS Vice President for Technical Activities Shuzhi Sam Ge at elegesz@nus.edu.

For "People in Control," we speak with Dorothée Normand-Cyrot and Tarek Sobh, who discuss their views of control research and education.

In addition to two book reviews, Mike Polis and Zongli Lin, *IEEE CSM* associate editors for book reviews, inaugurate a section on book announcements within the "Bookshelf" column. The large number of books that are published and the delay in obtaining book reviews has convinced Mike and Zongli that it is important to inform readers of newly published and updated books. The plan is to announce the publication of books of interest to *IEEE CSM* readers by giving the book title, authors, basic book data, and a brief description of the contents, including the intended audience. This section will be in addition to the usual book reviews, and no book will be announced until the associate editors examine a review copy.

Finally, as usual, we end this issue with "Random Inputs." Please see the Call for Artists in the June issue, which invites you to submit photographs of your artistic renderings of the CSS logo. I look forward to receiving pictures of your creations.

Dennis S. Bernstein

The Sandwich Method for General Principles

To present an abstract idea—a general formula, a general law, a theorem—a good way is to present in order:

- An easy example illustrating the principle
- The general statement and explanation of the principle
- A harder example using the principle

This corresponds to the way people think. It's much easier to understand an abstract idea if you can test it out on a simple example. Then once you've understood the principle, it's fun to see its application to a more complex example that was too difficult to handle before.

—Arthur P. Mattuck, *The Torch or the Firehose, A Guide to Section Teaching*, M.I.T., 1981, 1995, p. 21.