

## Exquisite Coupling

If you appreciate the beauty of fine books, antique cars, and mechanical clocks, then you're a lover of vintage technology. These technologies evolved when engineering was less mathematics and computer modeling than painstaking experimentation and expert craftsmanship. In modern times, the mechanical clock has long since been overtaken by devices that are less expensive and more accurate. Yet some of the most prized and coveted objects are mechanical clocks that reflect extraordinary attention to engineering and artistic detail. Prior to the 20th century, the manufacture of precision timepieces was the pinnacle of high-tech engineering excellence. Fine horology continues this tradition using modern engineering tools and techniques.



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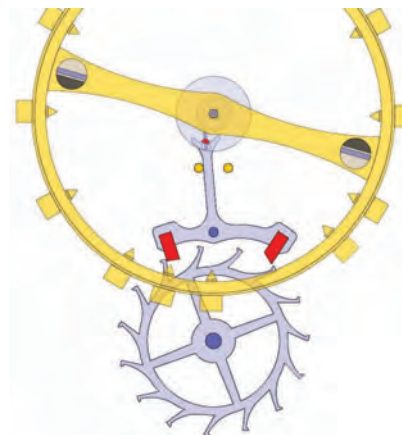
In simple terms, a mechanical clock keeps time by controlling the release of energy stored in its mainspring. The torque from the mainspring is applied to the escape wheel

by means of the gear train, while the escape wheel is alternately locked and released by the motion of the balance wheel. The intermediary between the escape wheel and the balance wheel is the pallet, a T-shaped lever, whose base pivots on an offset pin attached to the balance wheel and whose arms intermittently contact the teeth of the escape wheel.

The role of the pallet in interacting with the escape wheel and the balance wheel is remarkable. Basically, here is how it works (see Figure 1). Mounted on the tips of the arms of the pallet are a pair of pallet stones; these may be jewels or any hard substance that can withstand repeated impact and sliding. As the pallet stones contact the teeth of the escapement wheel, two distinct actions occur. At certain times, the escape wheel applies an impulsive torque to the pallet, and this torque transmits energy to the balance-wheel/hairspring subsystem, thereby compensating for the inevitable loss of energy due to damping. At other times, the pallet provides a locking torque to the escape wheel, thereby impeding its ability to rotate, temporarily bringing it to rest and limiting the release of energy from the mainspring.

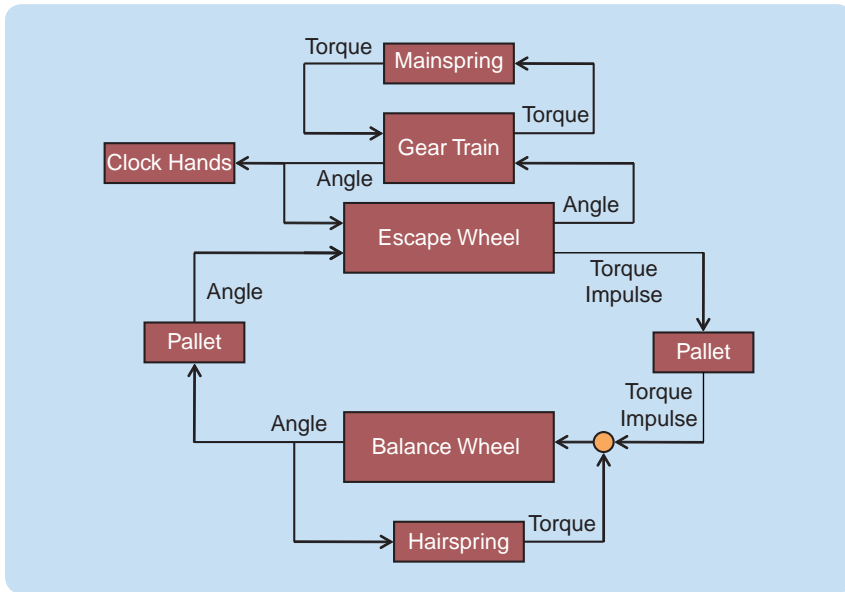
Consequently, feedback in a mechanical clock is realized by the interaction of two subsystems, one consisting of the balance wheel and hairspring, and the other consisting of the escape wheel, gear train, and

mainspring (see Figure 2). The balance wheel and hairspring comprise a rotational mass-spring system, whose natural frequency depends on the hairspring stiffness and wheel inertia. This subsystem has unavoidable damping and thus constitutes a



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**FIGURE 1** The energy stored in the mainspring is provided to the escape wheel by means of a constant torque. The balance wheel oscillates due to its connection to the hairspring. As the escape wheel rotates, it provides a torque impulse to the balance wheel, thereby compensating for the energy lost due to damping. As the balance wheel oscillates, it provides a locking torque to the escape wheel, which limits the rate at which energy is released from the mainspring. The impulse torque applied by the escape wheel to the balance wheel as well as the locking torque applied by the balance wheel to the escape wheel are transmitted by the pallet, a T-shaped lever that alternately locks and releases the escape wheel. The base of the pallet pivots on an offset pin attached to the balance wheel. The pair of pallet stones attached to the arms of the pallet alternate between conveying impulse torques from the escape wheel to the balance wheel and locking the escape arm to limit the release of energy from the mainspring.



**FIGURE 2** At the heart of a mechanical clock is a feedback loop in which the escape wheel interacts with the balance wheel. The mediator between the escape wheel and the balance wheel is the pallet, which performs multiple functions, including locking and releasing the escape wheel as well as conveying energy from the mainspring to the balance wheel.



**FIGURE 3** Breguet tourbillon. An offset of the center of mass of the balance wheel from its axis of rotation causes the time-keeping rate of the watch to vary as the tilt angle of the watch changes. By continually rotating the entire balance wheel, hairspring, and escape-wheel assembly about the balance wheel's axis of rotation, this rate bias is averaged, and thus the timekeeping rate becomes independent of the tilt angle. (Image courtesy of Montres Breguet SA.)

damped oscillator. The escape wheel is driven by the energy stored in the mainspring through the gear train. Finally, the rotation of the gear train drives the hands of the clock.

The net effect of the intermittent motion of the escape wheel is to regulate the flow of energy from the mainspring. By analogy, the pallet operates like a faucet that releases a single drop at a time, where the energy from the water pressure automatically reshuts the valve after each drop is released. The output of the clock—that is, the velocity of the hands—is quantized, alternating between moving (1) and not moving (0). Counting the number of ones provides the time. In effect, the mechanical clock has digital output.

The interaction between the pallet stones and the escape wheel includes impacts, sliding contact, and periods of no contact; therefore, the closed-loop system has

hybrid—combined continuous and discrete—dynamics. These dynamics give rise to a limit cycle whose period of oscillation depends on the parameters of the subsystems. The more robust the period of this limit cycle is to the parameters of the mechanism—not to mention temperature, dirt, and wear—the more accurate the watch is over the long term.

A clock is not a servomechanism, however, since no reference signal is provided for the clock to follow. Instead, a mechanical clock operates in a stable limit cycle, a mode of

oscillation that owes its existence to feedback.

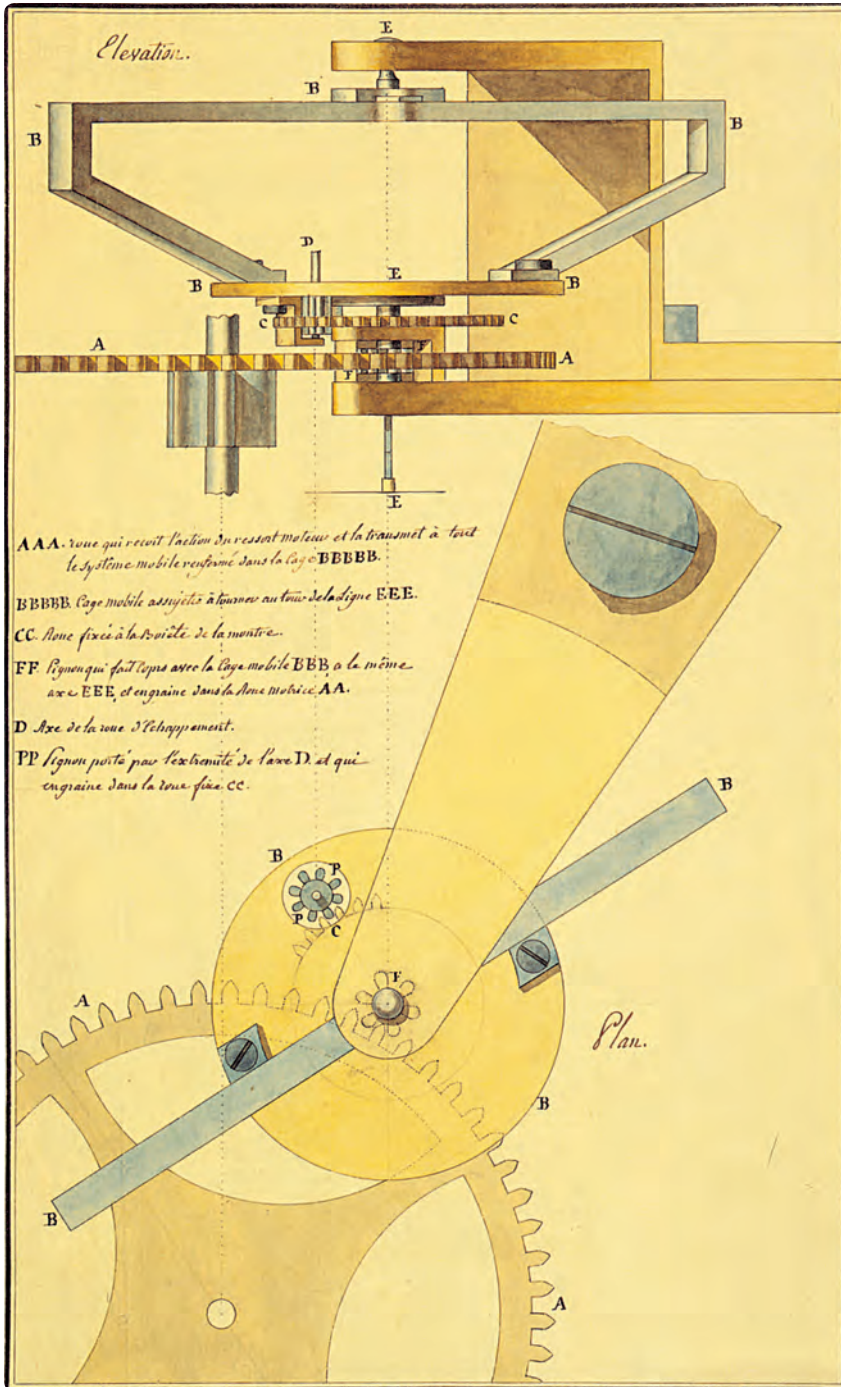
### VIVE BREGUET

In 1795, the French-Swiss watchmaker Abraham Louis Breguet had an innovative idea for resetting watches, which, due to their size, were less accurate than table clocks. Breguet designed a table clock, called a *pendule sympathétique*, with an interface to a wristwatch. By attaching the wristwatch to the table clock, the wristwatch would be automatically synchronized with the table clock within a few hours. In his book *The Origins of Feedback Control*, Otto Mayr describes the operation of this device as exploiting “true feedback control.” Unlike a clock, which depends on the period of an emergent limit cycle, the basis of Breguet’s *pendule sympathétique* is a servomechanism. Breguet Depuis 1775 still manufactures these devices.



From left: Rick Middleton, Roberto Tempo, and Dennis Bernstein.





**FIGURE 4** This watercolor illustration is part of the tourbillon patent filed in 1801 at Institut National de La Propriété Industrielle, Paris, by Abraham Louis Breguet. (Image courtesy of Montres Breguet SA.)

Breguet also invented the tourbillon (see Figure 3), whose operation was analyzed in the June 2010 issue in the article “The Tourbillon and How It Works,” by Mark Denny. The tourbillon is intended to improve the precision of a wristwatch, although not

necessarily its accuracy. In reality, the center of mass of a balance wheel does not reside at exactly its axis of rotation. Consequently, the period of oscillation of the balance wheel/hairspring subsystem depends on the tilt angle of the watch. Breguet’s innovation



**FIGURE 5** The Breguet Classique Grande Complication wristwatch, in platinum with sapphire crystal, features twin rotating tourbillons. The tourbillons are coupled by means of differential gears and mounted on a rotating center plate, which completes one revolution every 12 hours. The differential conveys the tourbillons’ mean rate to the hands of the watch. The rate of the watch is thus the mean rate of the two tourbillons. (Image courtesy of Montres Breguet SA.)

was to use the intermittent advances of the escape wheel to rotate the entire balance wheel, hairspring, pallet, and escape wheel assembly (and thus the clock hands), thereby averaging the bias due to the offset of the center of mass and tilt angle of the watch (see Figure 4). The latest innovation of Breguet Depuis 1775 is a dual-tourbillon watch, which averages the rate of two tourbillons (see Figure 5).

## CLOCKS AND CONTROL

A mechanical clock uses feedback, but not for command following or disturbance rejection. Instead, feedback provides a stable limit cycle that arises from hybrid dynamics and whose frequency of oscillation regulates the flow of stored energy. The rate of this energy release is set by the watchmaker to optimize the accuracy of the timepiece. The technology may be vintage, but the principles are timeless.

Dennis S. Bernstein

