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Wind-field Reconstruction Using Flight Data

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Abstract— **Although guidance of all aircraft is affected by wind disturbances, micro-UAVs are especially susceptible. To estimate unknown wind disturbance, we consider two illustrative scenarios for planar flight. In the first scenario, we assume that measurements of the heading angle are available, while, in the second scenario, we assume that measurements of the heading angle are not available. Since the disturbance estimation problem is nonlinear, we develop an extension of the unscented Kalman filter that provides an estimate of the unknown wind disturbance. Furthermore, we show through simulations that, when the heading angle is not measured, a kinematic ambiguity is introduced. However, when the initial heading angle is known and the subsequent heading angle is not measured, this kinematic ambiguity is resolved and accurate estimates of the wind velocity are obtained.**

I. INTRODUCTION

Small and micro air vehicles are increasingly being used to improve situational awareness by conducting surveillance, patrolling, and convoy protection [15]. These vehicles provide imagery reconnaissance capability out to five to ten miles at the company/platoon/squad level. Due to their small size, these aircraft have limited payload capacity and usually carry fixed cameras (which require accurate pointing, therefore accurate knowledge of heading) and commercial off-the-shelf autopilots (which often have poor heading measurement accuracy) [6].

Although guidance of all aircraft is affected by the atmospheric motion relative to the Earth, that is, wind, micro-UAVs are especially susceptible. Localized wind-field estimation, especially winds at low velocity, is difficult. Consequently, alternative means must be used to assess the effects of wind. Efforts in this direction include wind estimation [16, 19], and techniques for path planning in wind, for example [1, 13], which assume constant known wind fields, and [20, 22], which make use of gimbaled cameras.

In the present paper we develop a technique for using available measurements to estimate the local wind-field velocity. To do this, we use state-estimation techniques that have the ability to reconstruct exogenous disturbance signals that are not directly measured.

In the case of linear systems, early work on reconstructing exogenous signals includes input reconstruction through system inversion [14, 21], while methods using optimal filters are developed in [2, 5, 8, 23]. More recently, a technique for reconstructing unknown exogenous disturbances has been developed in [4, 17, 18] as an extension of unbiased minimum variance filtering [12].

In this paper, we extend the techniques in [18] for estimating unknown external disturbances for nonlinear systems. This technique is based on the unscented Kalman filter (UKF) [9, 10] for state estimation for nonlinear systems, which is an example of a sigma point Kalman filters (SPKF) [24]. Recent work [10, 24] illustrates the improved performance of SPKFs compared to the extended Kalman filter (EKF), which is prone to numerical problems such as initialization sensitivity, bias (divergence), and instability for strongly nonlinear systems.

The nature of the disturbance estimation (input reconstruction) problem depends on the type of measurements available. In the present paper we consider two illustrative scenarios for planar flight. In the first scenario, we assume that measurements of the heading angle are available. In this case, the estimation problem is linear, and the techniques of [4, 17, 18] are applicable. In the second scenario, we assume that measurements of the heading angle are not available. In this case, the disturbance estimation problem is nonlinear, and we therefore develop an extension of the unscented Kalman filter that provides an estimate of the unknown disturbance.

After describing the basic setting in Section 2, the two scenarios described above are developed in sections 3 and 4. For each scenario, we consider flight involving straight line and circular motion in the presence of a wind field that varies as a triangular waveform in both of its components. In the case of unknown heading angle, we show that wind field estimation requires knowledge of the initial heading angle in order to remove a kinematic ambiguity.

II. WIND-FIELD ESTIMATION

Consider the planar flight equations

$$\dot{x} = V_{\mathrm{AC/W}} \cos\psi + V_{\mathrm{W/E}} \cos\phi, \tag{2.1}$$

$$\dot{y} = V_{\mathrm{AC/W}} \sin\psi + V_{\mathrm{W/E}} \sin\phi, \tag{2.2}$$

$$\dot{\psi} = \omega, \tag{2.3}$$

where x and y are the ground coordinates of the vehicle, $V_{\mathrm{AC/W}}$ is the airspeed of the vehicle, ψ is the heading angle, ω is the steering angle rate, $V_{\mathrm{W/E}}$ is the wind speed, and ϕ is the angle of the direction of the wind as measured from the $\hat{\imath}$ axis. The magnitude and direction of velocity of the vehicle with respect to the Earth is $V_{\mathrm{AC/E}} \triangleq \sqrt{\dot{x}^2 + \dot{y}^2}$ and $\theta \triangleq \tan^{-1}\left(\frac{\dot{y}}{\dot{x}}\right)$, respectively, and note that $V_{\mathrm{AC/E}} = V_{\mathrm{AC/W}} + V_{\mathrm{W/E}}$. The relationship between the various components of

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velocities is illustrated in Figure 1. Throughout this paper, we assume that measurements of x and y are available from GPS, and that measurements of $V_{\rm AC/W}$ are available from an airspeed sensor that measures angle of attack and sideslip. We consider the problem of estimating the unknown wind speed $V_{\rm W/E}$ and angle ϕ of the wind.

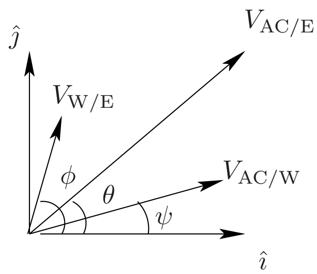

Fig. 1. Schematic of relationship between components of velocities in an Earth-fixed frame and the body-fixed frame.

III. Measured Heading Angle

We first consider the case in which the heading angle ψ is measured. In this case, we use (2.1) and (2.2) to estimate $V_{\rm W/E}$ and ϕ. By defining

$$V_{\rm W/E,x} \overset{\triangle}{=} V_{\rm W/E} \cos\phi, \tag{3.1}$$

$$V_{\rm W/E,y} \overset{\triangle}{=} V_{\rm W/E} \sin\phi, \tag{3.2}$$

it follows that (2.1), (2.2) are linear in the unknowns $V_{\rm W/E,x}$ and $V_{\rm W/E,y}$. Once estimates of the $V_{\rm W/E,x}$ and $V_{\rm W/E,y}$ are obtained, the wind speed $V_{\rm W/E}$ and angle ϕ can be obtained using the relationships

$$V_{\rm W/E} = \sqrt{V_{\rm W/E,x}^2 + V_{\rm W/E,y}^2}, \tag{3.3}$$

$$\phi = \tan^{-1}\left(\frac{V_{\rm W/E,y}}{V_{\rm W/E,x}}\right). \tag{3.4}$$

Thus the problem is stated as

Problem 1. Equations:

$$\dot{x} = V_{\rm AC/W}\cos\psi + V_{\rm W/E,x}, \tag{3.5}$$

$$\dot{y} = V_{\rm AC/W}\sin\psi + V_{\rm W/E,y}. \tag{3.6}$$

Available measurements: x, y, $V_{\rm AC/W}$, and ψ.
Unknowns: $V_{\rm W/E,x}$ and $V_{\rm W/E,y}$.

Since Problem 1 is linear in the states and linear in the unknowns $V_{\rm W/E,x}$ and $V_{\rm W/E,y}$, we can use the unbiased minimum-variance filter [18] for linear systems to estimate the states and the unknown inputs. We briefly review the Kalman filter and the unbiased minimum-variance filter.

A. Kalman Filter

For the linear stochastic discrete-time dynamic system

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + G_{k-1}w_{k-1}, \tag{3.7}$$

$$y_k = C_k x_k + v_k, \tag{3.8}$$

where $A_{k-1} \in \mathbb{R}^{n\times n}$, $B_{k-1} \in \mathbb{R}^{n\times p}$, $G_{k-1} \in \mathbb{R}^{n\times q}$, and $C_k \in \mathbb{R}^{m\times n}$ are known matrices, the state-estimation problem can be described as follows. Assume that, for all $k \geq 1$, the known data are the measurements $y_k \in \mathbb{R}^m$, the inputs $u_{k-1} \in \mathbb{R}^p$, and the statistical properties of x_0, w_{k-1} and v_k. The initial state vector $x_0 \in \mathbb{R}^n$ is assumed to be Gaussian with mean $\hat{x}_0$ and error-covariance $P_0^{xx} \triangleq {\rm E}\left[(x_0-\hat{x}_0)(x_0-\hat{x}_0)^{\rm T}\right]$. The process noise $w_{k-1} \in \mathbb{R}^q$, which represents unknown input disturbances, and the measurement noise $v_k \in \mathbb{R}^m$, concerning inaccuracies in the measurements, are assumed white, Gaussian, zero mean, and mutually independent with known covariance matrices Q_{k-1} and R_k, respectively. Next, define the cost function

$$J(x_k) \triangleq \rho(x_k|(y_1,\ldots,y_k)), \tag{3.9}$$

which is the conditional probability density function of the state vector $x_k \in \mathbb{R}^n$ given the past and present measured data $y_1,\ldots,y_k$. Under the stated assumptions, the maximization of (3.9) is the state estimation problem, while the maximizer $\hat{x}_k$ of J is the optimal state estimate.

The optimal state estimate $\hat{x}_k$ is given by the Kalman filter [11], whose *forecast* step is given by

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}, \tag{3.10}$$

$$P_{k|k-1}^{xx} = A_{k-1}P_{k-1}^{xx}A_{k-1}^{\rm T} + G_{k-1}Q_{k-1}G_{k-1}^{\rm T}, \tag{3.11}$$

$$\hat{y}_{k|k-1} = C_k\hat{x}_{k|k-1}, \tag{3.12}$$

$$P_{k|k-1}^{yy} = C_kP_{k|k-1}^{xx}C_k^{\rm T} + R_k, \tag{3.13}$$

$$P_{k|k-1}^{xy} = P_{k|k-1}^{xx}C_k^{\rm T}, \tag{3.14}$$

where $P_{k|k-1}^{xx} \triangleq {\rm E}\left[(x_k-\hat{x}_{k|k-1})(x_k-\hat{x}_{k|k-1})^{\rm T}\right]$, $P_{k|k-1}^{yy} \triangleq {\rm E}\left[(y_k-\hat{y}_{k|k-1})(y_k-\hat{y}_{k|k-1})^{\rm T}\right]$, and $P_{k|k-1}^{xy} \triangleq {\rm E}\left[(x_k-\hat{x}_{k|k-1})(y_k-\hat{y}_{k|k-1})^{\rm T}\right]$, and whose *data-assimilation* step is given by

$$K_k = P_{k|k-1}^{xy}(P_{k|k-1}^{yy})^{-1}, \tag{3.15}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k(y_k - \hat{y}_{k|k-1}), \tag{3.16}$$

$$P_k^{xx} = P_{k|k-1}^{xx} - K_kP_{k|k-1}^{yy}K_k^{\rm T}, \tag{3.17}$$

where $P_k^{xx} \triangleq {\rm E}\left[(x_k-\hat{x}_k)(x_k-\hat{x}_k)^{\rm T}\right]$ is the error-covariance matrix and K_k is the Kalman gain matrix. The notation $\hat{z}_{k|k-1}$ indicates an estimate of z_k at time k based on information available up to and including time $k-1$. Likewise, $\hat{z}_k$ indicates an estimate of z at time k using information available up to and including time k. Model information is used during the forecast step, while measurement data are injected into the estimates during the data-assimilation step, specifically, (3.16).

B. Unbiased Minimum-variance Filter

Consider the system

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + H_{k-1}e_{k-1} + G_{k-1}w_{k-1}, \tag{3.18}$$

$$y_k = C_kx_k + v_k. \tag{3.19}$$

where x_k, y_k, u_{k-1}, e_{k-1}, A_{k-1}, B_{k-1}, G_{k-1} and C_k are defined as in section III-A, while $e_{k-1} \in \mathbb{R}^l$ represents the unknown input and $H_{k-1} \in \mathbb{R}^{n\times l}$ is the input matrix. We assume that A_{k-1}, B_{k-1}, C_k, D_k, and H_{k-1} are known, while e_{k-1} is unknown.

Due to the presence of the unknown non-zero-mean term $H_{k-1}e_{k-1}$, the Kalman filter estimate in Section III-A is biased in general. The optimal unbiased state estimate $\hat{x}_k$ is given by the Unbiased Minimum-Variance filter (UMV) [18], whose *forecast* step is given by (3.10) - (3.14), and whose *data-assimilation* step is given by

$$V_k \triangleq C_k H_{k-1}, \tag{3.20}$$

$$\Pi_k \triangleq (V_k^{\mathrm{T}}(P_{k|k-1}^{yy})^{-1}V_k)^{-1}V_k^{\mathrm{T}}(P_{k|k-1}^{yy})^{-1}, \tag{3.21}$$

$$L_k = H_{k-1}\Pi_k + P_{k|k-1}^{xy}(P_{k|k-1}^{yy})^{-1}(I - V_k\Pi_k), \tag{3.22}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + L_k(y_k - \hat{y}_{k|k-1}), \tag{3.23}$$

$$P_k^{xx} = P_{k|k-1}^{xx} - L_k P_{k|k-1}^{yy} L_k^{\mathrm{T}}, \tag{3.24}$$

where L_k is the UMV filter gain matrix. Finally, the estimate of the unknown signal e_{k-1} is given by

$$\hat{e}_{k-1} = H_{k-1}^{\dagger} L_k(y_k - C_k\hat{x}_{k|k-1} - D_k u_k). \tag{3.25}$$

C. Results: Wind Estimation with Measured Heading Angle

The steering angle is chosen to be alternating sequences of zeros and ones, which represents the aircraft flying alternately in a straight line and in circles. The wind-velocity component profiles are chosen to be triangular waveforms. Figure 2 shows the flight path in the absence of wind disturbance, while Figure 3 shows the flight path in the presence of the wind disturbance.

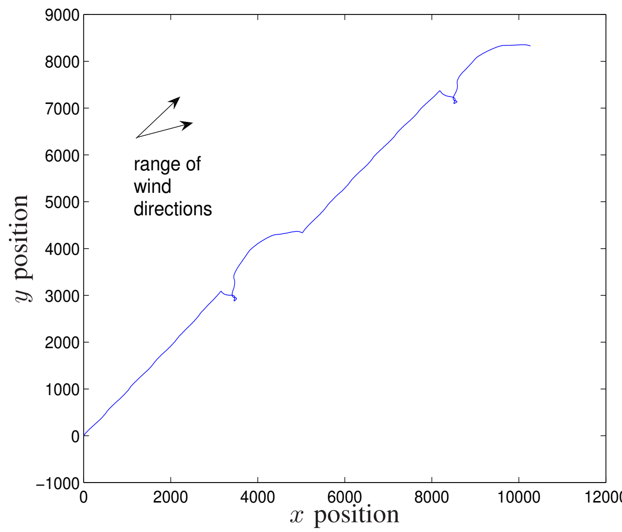

Fig. 3. Flight path of the aircraft in the presence of wind disturbance. The steering angle is an alternating sequence of zeros and ones, which represents the aircraft flying in a straight lines and in circles alternately. The two arrows show the extremities of the wind direction, which is a time-varying triangular waveform.

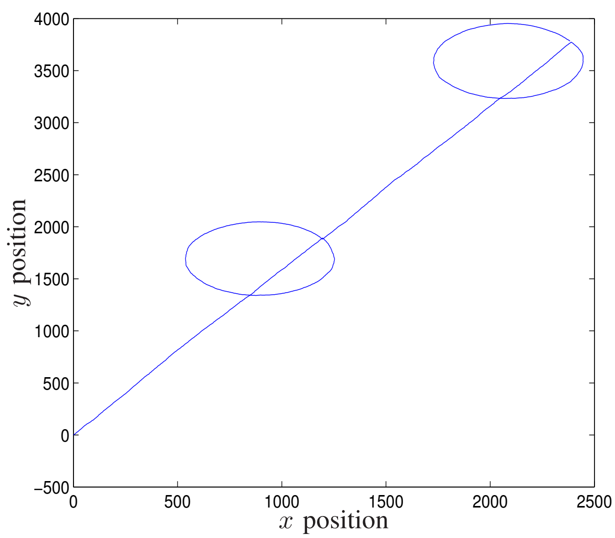

Fig. 2. Flight path of the aircraft in the absence of wind disturbance. The steering angle is an alternating sequence of zeros and ones, which represents the aircraft flying in a straight lines and in circles alternately.

Since Problem 1 is linear in the unknown wind-velocity components, we apply the UMV filter (3.20)-(3.24) and (3.25) to estimate the states and unknown inputs, respectively. Figure 4 compares the actual flight path and their estimates using the Kalman filter and the UMV filter. Figure 5 shows a magnified version of the time interval from 32 sec to 48 sec of Figure 4. Although measurements of x and y positions are available, the state estimates using the UMV filter are seen to be better than the state estimates using the Kalman filter. Finally, Figure 6 shows the actual wind velocity components and their estimates from (3.25) for both the UMV filter and the Kalman filter, while Figure 7 shows a zoomed in portion of the interval between 32 sec and 48 sec from Figure 6.

In practice, although measurements of the heading angle ψ are available, they are often unreliable due to the size and cost restrictions of the sensors on a micro-UAV. Hence, we next consider the case in which the heading angle ψ is unknown.

IV. Heading Angle not Measured

We now assume that measurements of the heading angle ψ are not available. Since ψ must be estimated, we consider the complete equations (2.1) - (2.3). Thus the problem can be stated as

Problem 2.

$$\dot{x} = V_{\mathrm{AC/W}}\cos\psi + V_{\mathrm{W/E,x}}, \tag{4.1}$$

$$\dot{y} = V_{\mathrm{AC/W}}\sin\psi + V_{\mathrm{W/E,y}}, \tag{4.2}$$

$$\dot{\psi} = \omega. \tag{4.3}$$

Available measurements: x, y, $V_{\mathrm{AC/W}}$, and ω.
Unknowns: ψ, $V_{\mathrm{W/E,x}}$, and $V_{\mathrm{W/E,y}}$.

In this case, since ψ is not measured the state equations are nonlinear. We thus require a filter for nonlinear systems.

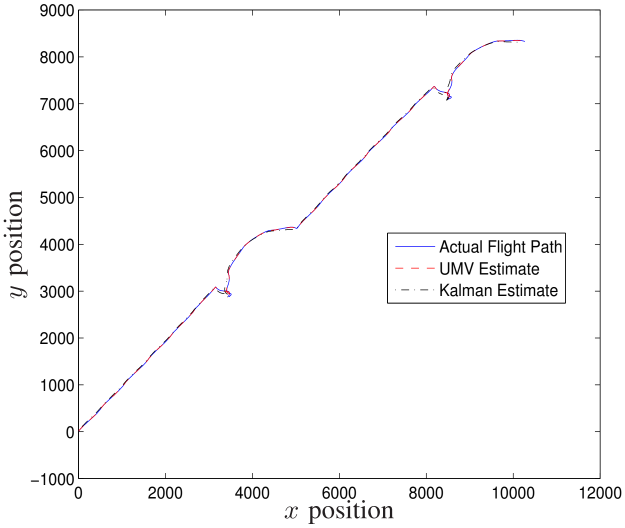

Fig. 4. Actual flight path and estimate of the flight path using the Kalman filter and the unbiased minimum-variance filter in the presence of an unknown wind disturbance.

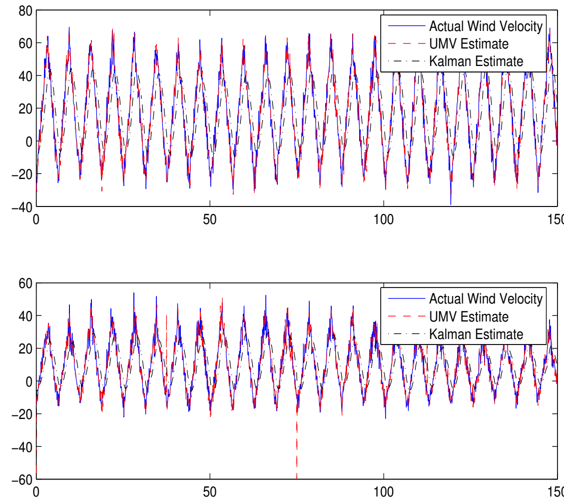

Fig. 6. Actual wind velocity and filter estimate when measurements of the heading angle are available.

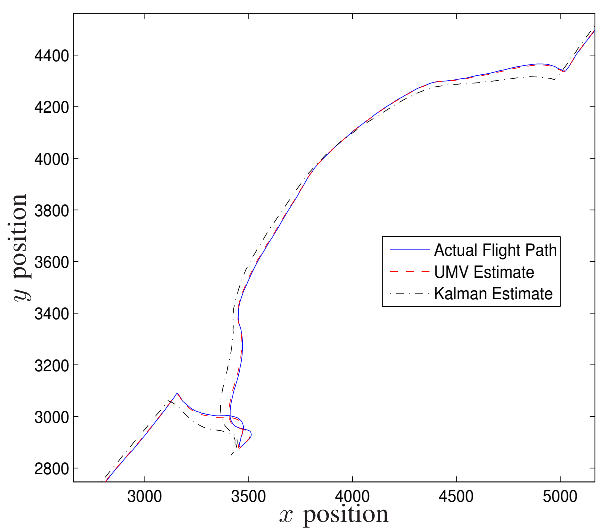

Fig. 5. Actual flight path and estimate of the flight path using the Kalman filter and the unbiased minimum-variance filter in the presence of an unknown wind disturbance.

A. State Estimation for Nonlinear Systems

Consider the nonlinear stochastic discrete-time dynamic system

$$x_k = f_{k-1}(x_{k-1},\ u_{k-1},\ w_{k-1}), \tag{4.4}$$

$$y_k = h_k(x_k) + v_k, \tag{4.5}$$

where $f_{k-1} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^n$ and $h_k : \mathbb{R}^n \to \mathbb{R}^m$ are, respectively, the process and observation models. The objective of the state-estimation problem is, for all $k \geq 1$, to maximize (3.9). However, the solution to this problem is complicated [3] by the fact that, for nonlinear systems, $\rho(x_k|(y_1, \ldots, y_k))$ is not completely characterized by its first and second-order moments. We thus use an approximation based on the classical Kalman filter to provide a suboptimal solution to the nonlinear case.

B. Unscented Kalman Filter

First, for nonlinear systems, we consider the unscented Kalman filter (UKF) [9] to provide a suboptimal solution to the state-estimation problem. Instead of analytically linearizing (4.4)-(4.5) and using (3.10)-(3.17), UKF employs the unscented transform (UT) [10], which approximates the posterior mean $\hat{y} \in \mathbb{R}^m$ and covariance $P^{yy} \in \mathbb{R}^{m\times m}$ of a random vector y obtained from the nonlinear transformation $y = h(x)$, where x is a prior random vector whose mean $\hat{x} \in \mathbb{R}^n$ and covariance $P^{xx} \in \mathbb{R}^{n\times n}$ are assumed known. UT yields the actual mean $\hat{y}$ and the actual covariance P^{yy} if $h = h_1 + h_2$, where h_1 is linear and h_2 is quadratic [10]. Otherwise, $\hat{y}_k$ is a *pseudo mean* and P^{yy} is a *pseudo covariance*.

UT is based on a set of deterministically chosen vectors known as sigma points. To capture the mean $\hat{x}^{\mathrm{a}}_{k-1}$ of the augmented prior state vector

$$x^{\mathrm{a}}_{k-1} \triangleq \begin{bmatrix} x_{k-1} \\ w_{k-1} \end{bmatrix}, \tag{4.6}$$

where $x^{\mathrm{a}}_{k-1} \in \mathbb{R}^{n_{\mathrm{a}}}$ and $n_{\mathrm{a}} \triangleq n+q$, as well as the augmented prior error covariance

$$P^{xx\mathrm{a}}_{k-1} \triangleq \begin{bmatrix} P^{xx}_{k-1|k-2} & 0_{n\times q} \\ 0_{q\times n} & Q_{k-1} \end{bmatrix}, \tag{4.7}$$

the sigma-point matrix $\mathcal{X}_{k-1} \in \mathbb{R}^{n_\mathrm{a} \times (2n_\mathrm{a}+1)}$ is chosen as

$$\begin{cases} \mathrm{col}_0(\mathcal{X}_{k-1}) & \triangleq \hat{x}^\mathrm{a}_{k-1}, \\ \mathrm{col}_i(\mathcal{X}_{k-1}) & \triangleq \hat{x}^\mathrm{a}_{k-1} \\ & \quad + \sqrt{(n_\mathrm{a}+\lambda)}\, \mathrm{col}_i \left[\left(P^{xx\mathrm{a}}_{k-1}\right)^{1/2} \right], \\ & \quad i = 1, \ldots, n_\mathrm{a}, \\ \mathrm{col}_{i+n_\mathrm{a}}(\mathcal{X}_{k-1}) & \triangleq \hat{x}^\mathrm{a}_{k-1} \\ & \quad - \sqrt{(n_\mathrm{a}+\lambda)}\, \mathrm{col}_i \left[\left(P^{xx\mathrm{a}}_{k-1}\right)^{1/2} \right], \\ & \quad i = 1, \ldots, n_\mathrm{a}, \end{cases}$$

with weights

$$\begin{cases} \gamma_0^{(m)} \triangleq \dfrac{\lambda}{n_\mathrm{a}+\lambda}, \\ \gamma_0^{(c)} \triangleq \dfrac{\lambda}{n_\mathrm{a}+\lambda} + 1 - \alpha^2 + \beta, \\ \gamma_i^{(m)} \triangleq \gamma_i^{(c)} \triangleq \gamma_{i+n_\mathrm{a}}^{(m)} \triangleq \gamma_{i+n_\mathrm{a}}^{(c)} \triangleq \dfrac{1}{2(n_\mathrm{a}+\lambda)}, \\ \qquad i = 1, \ldots, n_\mathrm{a}, \end{cases}$$

where $\mathrm{col}_i\left[(\cdot)^{1/2}\right]$ is the ith column of the Cholesky square root, $0 < \alpha \leq 1$, $\beta \geq 0$, $\kappa \geq 0$, and $\lambda \triangleq \alpha^2(\kappa + n_\mathrm{a}) - n_\mathrm{a}$. We set $\alpha = 1$ and $\kappa = 0$ [7] such that $\lambda = 0$ [9] and set $\beta = 2$ [7]. Alternative schemes for choosing sigma points are given in [9].

The UKF *forecast* equations are given by

$$\mathcal{X}_{k-1} = \left[\hat{x}^\mathrm{a}_{k-1} \quad \hat{x}^\mathrm{a}_{k-1} 1_{1\times n_\mathrm{a}} + \sqrt{(n_\mathrm{a}+\lambda)}\left(P^{xx\mathrm{a}}_{k-1}\right)^{1/2} \quad \hat{x}^\mathrm{a}_{k-1} 1_{1\times n_\mathrm{a}} - \sqrt{(n_\mathrm{a}+\lambda)}\left(P^{xx\mathrm{a}}_{k-1}\right)^{1/2}\right], \tag{4.8}$$

$$\mathrm{col}_i(\mathcal{X}^x_{k|k-1}) = f_{k-1}(\mathrm{col}_i(\mathcal{X}^x_{k-1}),\ u_{k-1},\ \mathrm{col}_i(\mathcal{X}^w_{k-1})), \quad i = 0, \ldots, 2n_\mathrm{a}, \tag{4.9}$$

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2n_\mathrm{a}} \gamma_i^{(m)} \mathrm{col}_i(\mathcal{X}^x_{k|k-1}), \tag{4.10}$$

$$P^{xx}_{k|k-1} = \sum_{i=0}^{2n_\mathrm{a}} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{X}^x_{k|k-1}) - \hat{x}_{k|k-1}][\mathrm{col}_i(\mathcal{X}^x_{k|k-1}) - \hat{x}_{k|k-1}]^\mathrm{T}, \tag{4.11}$$

$$\mathrm{col}_i(\mathcal{Y}_{k|k-1}) = h_k(\mathrm{col}_i(\mathcal{X}^x_{k|k-1})), \quad i = 0, \ldots, 2n_\mathrm{a}, \tag{4.12}$$

$$\hat{y}_{k|k-1} = \sum_{i=0}^{2n_\mathrm{a}} \gamma_i^{(m)} \mathrm{col}_i(\mathcal{Y}_{k|k-1}), \tag{4.13}$$

$$P^{yy}_{k|k-1} = \sum_{i=0}^{2n_\mathrm{a}} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}][\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}]^\mathrm{T} + R_k, \tag{4.14}$$

$$P^{xy}_{k|k-1} = \sum_{i=0}^{2n_\mathrm{a}} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{X}^x_{k|k-1}) - \hat{x}_{k|k-1}][\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}]^\mathrm{T}, \tag{4.15}$$

where $\begin{bmatrix} \mathcal{X}^x_{k-1} \\ \mathcal{X}^w_{k-1} \end{bmatrix} \triangleq \mathcal{X}_{k-1}$, $\mathcal{X}^x_{k-1} \in \mathbb{R}^{n\times(2n_\mathrm{a}+1)}$, and $\mathcal{X}^w_{k-1} \in \mathbb{R}^{q\times(2n_\mathrm{a}+1)}$. The UKF *data-assimilation* equations are given by (3.15)-(3.17).

C. Unbiased Minimum-variance Unscented Filter

Next, for nonlinear systems with unknown inputs, we consider an extension of the UKF along the lines of the linear UMV filter. Thus, to obtain the pseudo mean and the pseudo error covariances we use the unscented transform, and to estimate the states and unknown inputs, we use the expressions derived for the UMV filter. Thus, the *forecast* equations for the unbiased minimum-variance unscented (UMVU) filter are given by (4.8) - (4.15). The *data-assimilation* equations for the UMVU filter are given by (3.20) - (3.24).

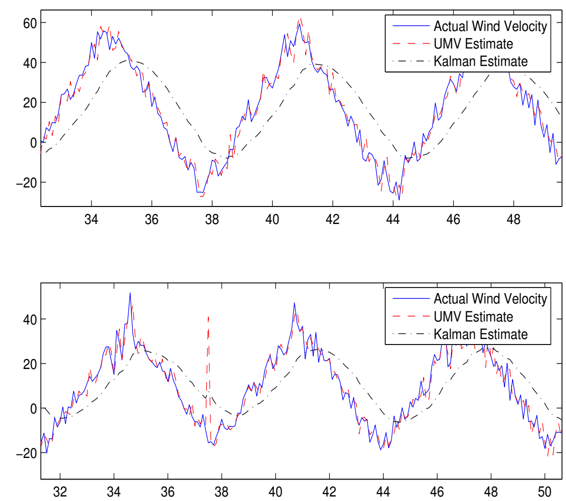

Fig. 7. Actual wind velocity and filter estimate when measurements of the heading angle are available.

D. Results: Wind Estimation with Heading Angle not Measured

To estimate the states and the unknown inputs in Problem 2, we use the UMVU filter described above. We use the same simulation parameters as in the known heading case. Figure 8 shows the actual wind velocity components and their estimates obtained from the UMVU filter.

As can be seen from Figure 8, the estimates of the wind velocity do not match the actual wind velocity. This is due to the fact that there is a kinematic ambiguity because of the combined effect of unknown heading angle and unknown wind velocity. This kinematic ambiguity is resolved by assuming that the initial heading angle is known. This is a reasonable assumption in practice since many small and micro UAV's are launched from catapults. When the initial heading angle is assumed to be known, but the subsequent heading is not measured, the estimates of the wind velocity components using the UMVU filter are shown in Figure 9.

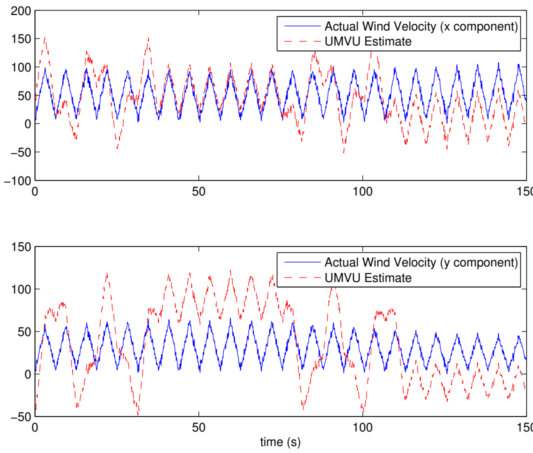

Fig. 8. Actual wind velocity and filter estimate when the heading angle is not measured. Due to a kinematic ambiguity, accurate estimates of the wind are not obtained.

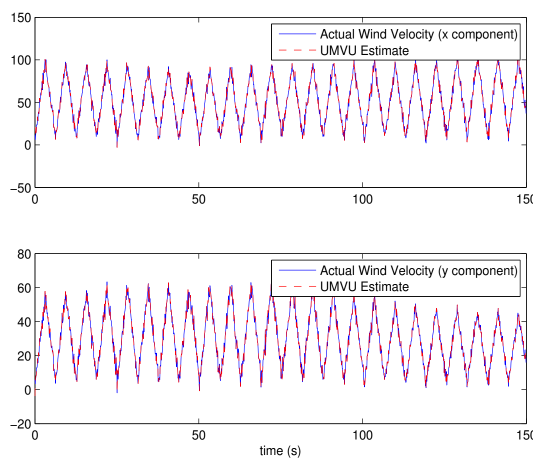

Fig. 9. Actual wind velocity and filter estimate when the heading angle is not measured. When the initial heading is assumed to be known, the kinematic ambiguity is resolved and accurate estimates of the wind disturbances are obtained.

V. CONCLUSIONS

To estimate unknown wind disturbances, we considered two illustrative scenarios for planar flight. In the first scenario, we assumed that measurements of the heading angle are available. In this case, since the estimation problem is linear, we applied techniques of [18] to estimate the wind disturbance. In the second scenario, we assumed that measurements of the heading angle were not available. In the second scenario, since the disturbance estimation problem is nonlinear, we developed an extension of the unscented Kalman filter that provided an estimate of the unknown wind disturbance. When the heading angle is not measured, a kinematic ambiguity was introduced. However, when the initial heading angle was known and the subsequent heading angle was not measured, this kinematic ambiguity was resolved and accurate estimates of the wind disturbance were obtained.

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