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Inertia-free Spacecraft Attitude Trajectory Tracking with Internal-Model-Based Disturbance Rejection and Almost Global Stabilization

Amit Sanyal†, Adam Fosbury, Nalin Chaturvedi and Dennis S. Bernstein

Abstract— **We derive a continuous nonlinear control law for spacecraft attitude trajectory tracking of arbitrary C^1 attitude trajectories based on rotation matrices. This formulation provides almost global stabilizability, that is, Lyapunov stability of the desired equilibrium of the error system as well as convergence from all initial states except for a subset whose complement is open and dense. This controller thus overcomes the unwinding phenomenon associated with continuous controllers based on attitude representations, such as quaternions, that are not bijective. The controller requires no inertia information and no information on constant disturbance torques. For slew maneuvers, that is, maneuvers with a setpoint command, in the absence of disturbances, the controller specializes to the continuous, nonlinear PD-type almost globally stabilizing controller of Chaturvedi, in which case the torque inputs can be arbitrarily bounded a priori.**

I. Introduction

Control of rigid spacecraft is an extensively studied problem. Despite the vast range of available techniques, however, the development of a spacecraft control system is a labor-intensive, time-consuming process. For applications in which spacecraft must be launched on short notice, it is of interest to employ control algorithms that are robust to uncertainty, such as inexact knowledge of the spacecraft mass distribution, errors in the alignment of actuators, and time delays in network-implemented feedback loops.

Spacecraft control is an inherently nonlinear problem whose natural state space involves the special orthogonal group of 3×3 rotation matrices, that is, $\mathrm{SO}(3)$. Although linear controllers can be used for maneuvers over small angles, the desire for minimum-fuel or minimum-time operation suggests that control systems that are tuned for operation on $\mathrm{SO}(3)$ are advantageous for large-angle maneuvers [2, 3]. However, the compactness of $\mathrm{SO}(3)$ presents difficulties with regard to global asymptotic stabilization, that is, Lyapunov stability of a desired equilibrium along with global convergence. To appreciate these difficulties, we can consider rotation about a fixed axis, that is, motion around a circle. By covering the unit circle with the real line with the origin 0 viewed as distinct from 2π leads to controllers that rotate the spacecraft needlessly from 2π to 0. The difficulty is due to the fact that 0 and 2π represent distinct values on the real line $\mathbb{R}$ but correspond to the same physical configuration. This unwinding phenomenon is discussed in [4].

The unwinding phenomenon suggests that global asymptotic stabilization is impossible when the controller is required to be continuous. However, if an equilibrium on $\mathrm{SO}(3)$ is required to be Lyapunov stable, then the vector field must have at least four equilibria; related issues are discussed in [5]. The mere existence of multiple equilibria precludes global asymptotic stability in the physical configuration space. In view of this impediment, the quest for continuous control signals may seem inadvisable, especially in view of the fact that many spacecraft thrusters are on-off devices, while some variable-structure control algorithms achieve robustness through high-frequency switching [6, 7]. Nevertheless, discontinuous dynamics entail special difficulties [8], and may lead to chattering in the vicinity of a discontinuity. It is thus of interest to determine which closed-loop properties can be achieved under continuous control.

A further complicating factor in spacecraft control is the choice of representation for attitude. Various attitude representations can be used, such as Euler angles, quaternions (also called Euler parameters [1]), Rodrigues parameters, modified Rodrigues parameters, direction cosine matrices, and rotation matrices (the transpose of direction cosine matrices). Difficulties arise from the fact that some representations, such as Euler angles, possess singularities and thus cannot represent all orientations, while other representations, such as quaternions, are not one-to-one. In fact, the quaternions constitute a double covering by the unit sphere $\mathrm{S}^3 \in \mathbb{R}^4$ of $\mathrm{SO}(3)$. Thus, every physical attitude is represented by two distinct quaternions. The problem with a representation that is not one-to-one is that control laws may be *inconsistent*, i.e., the same physical orientation of the spacecraft may give rise to two different control inputs, a property that is undesirable in practice. Indeed, continuous controllers can be defined in terms of the quaternion representation. Although convergence to a desired equilibrium can be achieved for every point except the remaining equilibria, Lyapunov stability fails for the desired *physical* equilibrium due to the fact that trajectories starting near the desired physical equilibrium may move very far before returning. In particular, when the quaternion representation is used, convergence to one of the two quaternion equilibria representing the desired physical orientation causes the spacecraft to exhibit unwinding in

†corresponding author: Mechanical Engineering, University of Hawaii at Manoa, Honolulu, HI 96822 aksanyal@hawaii.edu

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the sense that initial conditions close to one quaternion equilibrium can entail a large-angle physical rotation away from and then back to the desired physical equilibrium, thereby reflecting the lack of Lyapunov stability on the physical space $\mathrm{SO}(3)$. This shortcoming arises in continuous quaternion-based controllers such as those derived in [9–11]. A discontinuous quaternion-based controller that overcomes unwinding is used in [7].

The present work is motivated by [12–15], where rotation matrices are adopted as the unique global representation of $\mathrm{SO}(3)$, and is a condensed version of the accompanying journal paper [16]. Although global asymptotic stabilization is impossible due to the inherent nature of $\mathrm{SO}(3)$ (regardless of the adopted representation), the results obtained in [12–15] possess the practically useful property of *almost global stabilization*, which means that the desired equilibrium or trajectory is Lyapunov stable and the set of initial conditions that give rise to trajectories that converge to the undesirable equilibria constitute a set of measure zero that is nowhere dense. In practice, this property is effectively equivalent to global stabilization. For numerical simulation of rigid body control schemes, it is possible to use structured numerical integration schemes, such as the Lie group variational integrator implemented in [21]. These numerical integration schemes ensure that the attitude dynamics evolve on SO(3) during numerical simulation and in practical application, without resorting to local parameterization or reprojection from $\mathbb{R}^{3\times 3}$ to $\mathrm{SO}(3)$.

The goal of the present paper is to extend the almost global stabilization controllers of [12–14], which are confined to slew maneuvers, that is, maneuvers that bring the spacecraft to rest at a desired orientation, to the problem of attitude tracking with disturbances. The results we obtain do not require knowledge of the spacecraft inertia, and thus are analogous to those given in [11]. The results given herein are distinct from those of [15], which apply to the tracking problem but use a controller that requires inertia information while allowing disturbance moments with a known a priori bound that vanish when the angular velocity vanishes.

In the present paper we consider a disturbance rejection problem involving internally or externally applied disturbance torques. The disturbances are modeled as outputs of a Lyapunov-stable exogenous system; this exogneous system can model persistent disturbances such as steps and sinusoids. To use this approach in practice, it is necessary to know all of the spectral components of the torque that may be present in each disturbance channel. We then use an internal model control approach that automatically determines whether each spectral component is present in each disturbance channel. Internal model control requires no knowledge of the amplitude or phase of each harmonic component of the disturbance [17]. Finally, the contents of this paper are based on the assumption that the spacecraft attitude and angular velocity are available for feedback and that the spacecraft is controlled by three independent torque actuators. When these assumptions are violated, the rigid body control problem becomes more complex [18–20].

II. Spacecraft Model

We model a spacecraft as a single rigid body controlled by force and torque actuators, but without internal momentum-storage devices. We consider only the rotational motion of the spacecraft with control torques applied by the force actuators. A body-fixed frame is defined for the spacecraft, whose origin is chosen to be the center of mass, and an inertial frame is specified for determining spacecraft attitude. The spacecraft equations of motion are given by Euler's equation and Poisson's equation,

$$J\dot{\omega} = (J\omega) \times \omega + Bu + z_d, \tag{1}$$
$$\dot{R} = R\omega^\times, \tag{2}$$

respectively, where $\omega \in \mathbb{R}^3$ is the angular velocity of the spacecraft frame with respect to the inertial frame resolved in the spacecraft frame, $\omega^\times$ is the cross-product matrix of ω, $J \in \mathbb{R}^{3\times 3}$ is the constant, positive-definite inertia matrix of the spacecraft relative to the center of mass resolved in the spacecraft frame. R is the proper orthogonal matrix (that is, the rotation matrix) that transforms the components of a vector resolved in the spacecraft frame into the components of the same vector resolved in the inertial frame.

Both rate (inertial) and attitude (noninertial) measurements are assumed to be available. Gyro measurements $y_{\mathrm{rate}} \in \mathbb{R}^3$ provide measurements of the angular velocity resolved in the spacecraft frame, that is,

$$y_{\mathrm{rate}} = \omega. \tag{3}$$

For simplicity, we assume that gyro measurements are available noise free and without bias. In practice, bias can be corrected by using attitude measurements. Attitude measurements can be obtained by a noninertial sensor such as a star tracker, and are assumed to be given by

$$y_{\mathrm{attitude}} = R. \tag{4}$$

When attitude measurements are given in terms of an alternative attitude representation, such as quaternions, Rodrigues's formula can be used to determine the corresponding rotation matrix. Attitude estimation on $\mathrm{SO}(3)$ is considered in [21].

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given by R follows a commanded attitude trajectory given by the possibly time-varying C^2 rotation matrix $R_{\mathrm{d}}(t)$. For $t \geq 0$, $R_{\mathrm{d}}(t)$ is given by

$$\dot{R}_{\mathrm{d}}(t) = R_{\mathrm{d}}(t)\omega_{\mathrm{d}}(t)^\times, \quad R_{\mathrm{d}}(0) = R_{\mathrm{d0}}, \tag{5}$$

where ω_{d} is the desired time-varying C^1 angular velocity. The error between $R(t)$ and $R_{\mathrm{d}}(t)$ is given in terms of the attitude-error rotation matrix

$$\tilde{R} \triangleq R_{\mathrm{d}}^{\mathrm{T}} R. \tag{6}$$

Then $\tilde{R}$ satisfies the differential equation

$$\dot{\tilde{R}} = \tilde{R}\tilde{\omega}^\times, \tag{7}$$

where the angular velocity error $\tilde{\omega}$ is defined by

$$\tilde{\omega} \triangleq \omega - \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}.$$

We rewrite (1) in terms of the angular-velocity error as

$$J\dot{\tilde{\omega}} = [J(\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}})] \times (\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}) + J(\tilde{\omega} \times \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}} - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}) + Bu + z_d. \quad (8)$$

A scalar measure of attitude error is given by the rotation angle $\theta(t)$ about an eigenaxis needed to rotate the spacecraft from its attitude $R(t)$ to the desired attitude $R_{\mathrm{d}}(t)$, which is given by [1, p. 17]

$$\theta(t) = \cos^{-1}(\tfrac{1}{2}[\mathrm{tr}\,\tilde{R}(t) - 1]). \quad (9)$$

III. Attitude Control Law

The following results are needed. The proofs of these results are given in the companion journal paper [16]. Let I denote the identity matrix, whose dimensions are given by context, and let M_{ij} denote the i, j entry of the matrix M.

Lemma III.1. *Let $A \in \mathbb{R}^{3\times3}$ be a diagonal positive-definite matrix. Then the following statements hold:*

i) *For all $i, j = 1, \ldots, 3$, $R_{ij} \in [-1, 1]$.*
ii) $\mathrm{tr}\,(A - AR) \geq 0$.
iii) $\mathrm{tr}\,(A - AR) = 0$ *if and only if $R = I$.*

For convenience we note that, if R is a rotation matrix and $x, y \in \mathbb{R}^3$, then

$$(Rx)^{\times} = Rx^{\times}R^{\mathrm{T}},$$

and, therefore,

$$R(x \times y) = (Rx) \times Ry.$$

Next we introduce the notation

$$J\omega = L(\omega)\gamma, \quad (10)$$

where $\gamma \in \mathbb{R}^6$ is defined by

$$\gamma \triangleq \begin{bmatrix} J_{11} & J_{22} & J_{33} & J_{23} & J_{13} & J_{12} \end{bmatrix}^{\mathrm{T}}$$

and

$$L(\omega) \triangleq \begin{bmatrix} \omega_1 & 0 & 0 & 0 & \omega_3 & \omega_2 \\ 0 & \omega_2 & 0 & \omega_3 & 0 & \omega_1 \\ 0 & 0 & \omega_3 & \omega_2 & \omega_1 & 0 \end{bmatrix}.$$

With this notation, (8) can be rewritten as

$$J\dot{\tilde{\omega}} = [L(\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}})\gamma]^{\times}(\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}) + L(\tilde{\omega} \times \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}} - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\gamma + Bu + z_d. \quad (11)$$

Next, let $\hat{J} \in \mathbb{R}^{3\times3}$ denote an estimate of J, and define the inertia-estimation error

$$\tilde{J} \triangleq J - \hat{J}.$$

Letting $\hat{\gamma}, \tilde{\gamma} \in \mathbb{R}^6$ represent $\hat{J}, \tilde{J}$, respectively, it follows that

$$\tilde{\gamma} = \gamma - \hat{\gamma}. \quad (12)$$

Likewise, let $\hat{z}_d \in \mathbb{R}^3$ denote an estimate of z_d, and define the disturbance-estimation error

$$\tilde{z}_d \triangleq z_d - \hat{z}_d.$$

We now summarize the assumptions upon which the following development is based:

Assumption 1. J is constant but unknown.

Assumption 2. B is constant, nonsingular, and known.

Assumption 3. Each component of z_d is a linear combination of constant and harmonic signals, whose frequencies are known but whose amplitudes and phases are unknown.

Assumption 3 implies that z_d can be modeled as the output of an autonomous system of the form

$$\dot{d} = A_d d, \quad (13)$$

$$z_d = C_d d, \quad (14)$$

where $A_d \in \mathbb{R}^{n_d \times n_d}$ and $C_d \in \mathbb{R}^{3\times n_d}$ are known matrices and A_d is a Lyapunov-stable matrix, but $d(0)$ is unknown. Note that Assumption 3 implies that A_d can be chosen to be skew symmetric. If z_d is constant, then we set $A_d = 0$ and $C_d = I$. Let $\hat{d} \in \mathbb{R}^{n_d}$ denote an estimate of d, and define the disturbance-state estimation error

$$\tilde{d} \triangleq d - \hat{d}.$$

For $i = 1, 2, 3$, let e_i denote the ith column of the 3×3 identity matrix.

Theorem III.2. *Let K_{p} be a positive number, let $K_1 \in \mathbb{R}^{3\times3}$, let $Q \in \mathbb{R}^{6\times6}$ and $D \in \mathbb{R}^{3\times3}$ be positive definite, let $A = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal positive-definite matrix, and define*

$$S \triangleq \sum_{i=1}^{3} a_i(\tilde{R}^{\mathrm{T}}e_i) \times e_i.$$

Then the Lyapunov candidate

$$V(\tilde{\omega}, \tilde{R}, \tilde{\gamma}, \tilde{d}) \triangleq \tfrac{1}{2}(\tilde{\omega} + K_1 S)^{\mathrm{T}} J(\tilde{\omega} + K_1 S) + K_{\mathrm{p}}\mathrm{tr}\,(A - A\tilde{R}) + \tfrac{1}{2}\tilde{\gamma}^{\mathrm{T}}Q\tilde{\gamma} + \tfrac{1}{2}\tilde{d}^{\mathrm{T}}D\tilde{d} \quad (15)$$

is positive definite, that is, V is nonnegative, and $V = 0$ if and only if $\tilde{\omega} = 0$, $\tilde{R} = I$, $\tilde{\gamma} = 0$, and $\tilde{d} = 0$.

Theorem III.3. *Let K_{p} be a positive number, let $K_{\mathrm{v}} \in \mathbb{R}^{3\times3}$, let $K_1, D \in \mathbb{R}^{3\times3}$ and $Q \in \mathbb{R}^{6\times6}$ be positive definite, assume that $A_d^{\mathrm{T}}D + DA_d$ is negative semidefinite, let $A = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal positive-definite matrix, define S and V as in Theorem III.2, and let $\hat{\gamma}$ and $\hat{d}$ satisfy*

$$\dot{\hat{\gamma}} = Q^{-1}[L^{\mathrm{T}}(\omega)\omega^{\times} + L^{\mathrm{T}}(K_1\dot{S} + \tilde{\omega} \times \omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})](\tilde{\omega} + K_1 S), \quad (16)$$

where

$$\dot{S} = \sum_{i=1}^{3} a_i[(\tilde{R}^{\mathrm{T}}e_i) \times \tilde{\omega}] \times e_i, \quad (17)$$

and

$$\dot{\hat{d}} = A_d\hat{d} + D^{-1}C_d^{\mathrm{T}}(\tilde{\omega} + K_1 S), \quad (18)$$

$$\hat{z}_d = C_d\hat{d}, \quad (19)$$

and let

$$u = B^{-1}(v_1 + v_2 + v_3), \tag{20}$$

where

$$v_1 \triangleq -(\hat{J}\omega)\times\omega - \hat{J}(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}), \tag{21}$$

$$v_2 \triangleq -\hat{z}_{\mathrm{d}}, \tag{22}$$

and

$$v_3 \triangleq -K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}} S. \tag{23}$$

Then,

$$\dot{V}(\tilde{\omega},\tilde{R},\tilde{\gamma},\tilde{d}) = -(\tilde{\omega}+K_1S)^{\mathrm{T}}K_{\mathrm{v}}(\tilde{\omega}+K_1S) - K_{\mathrm{p}}S^{\mathrm{T}}K_1S + \tfrac{1}{2}\tilde{d}^{\mathrm{T}}(A_d^{\mathrm{T}}D + DA_d)\tilde{d}. \tag{24}$$

If A_d is chosen to be skew symmetric, then choosing D to be a multiple of the identity implies that $A_d^{\mathrm{T}}D + DA_d = 0$, and thus is negative semidefinite. Equations (18) and (19), which generate an estimate of the disturbance, is based on an internal model of the disturbance dynamics. Internal model control theory provides asymptotic tracking and disturbance rejection without knowledge of the amplitude or phase, but requires knowledge of the spectral content of the exosystem. For details, see [17] and the references therein.

The proofs of the following results are given in [16].

Lemma III.4. *Define S as in Theorem III.2, and assume that a_1, a_2, a_3 are positive and distinct. If $S = 0$, then $\tilde{R} \in \mathcal{R}$, where*

$$\mathcal{R} \triangleq \{I, \mathrm{diag}(1,-1,-1), \mathrm{diag}(-1,1,-1), \mathrm{diag}(-1,-1,1)\}.$$

Lemma III.5 below is given by Lemma 2 in [15], and is necessary to prove almost global stability of the tracking scheme.

Lemma III.5. *Let $K_{\mathrm{p}}, K_{\mathrm{v}}, K_1, D, Q, \hat{\gamma}, \hat{d}$, and u be as in Theorem 2, let $0 < a_1 < a_2 < a_3$, and let A_d be skew symmetric. Then the closed-loop system* (17)-(19) *and* (8) *has four disjoint equilibrium manifolds in $\mathbb{R}^3\times\mathrm{SO}(3)\times\mathbb{R}^6\times\mathbb{R}^3$ given by*

$$\mathcal{E}_i = \Big\{(\tilde{\omega},\tilde{R},\tilde{\gamma},\tilde{d}) \in \mathbb{R}^3\times\mathrm{SO}(3)\times\mathbb{R}^6\times\mathbb{R}^3 : \tilde{R} = \mathcal{R}_i, \tilde{\omega}\equiv 0,\ (\tilde{\gamma},\tilde{d}) \in \mathcal{Q}_i\Big\}, \tag{25}$$

where, for all $i \in \{0,1,2,3\}$, $\mathcal{Q}_i$ is the closed subset of $\mathbb{R}^6\times\mathbb{R}^3$ defined by

$$\mathcal{Q}_i \triangleq \{(\tilde{\gamma},\tilde{d}) \in \mathbb{R}^6\times\mathbb{R}^3 : [L(\mathcal{R}_i^{\mathrm{T}}\omega_{\mathrm{d}})\tilde{\gamma}]^{\times}(\mathcal{R}_i^{\mathrm{T}}\omega_{\mathrm{d}}) - L(\mathcal{R}_i^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\tilde{\gamma} + C_d\tilde{d} = 0,\ \dot{\tilde{\gamma}} = 0,\ \dot{\tilde{d}} = A_d\tilde{d}\}. \tag{26}$$

Furthermore, the equilibrium manifold $(\tilde{\omega},\tilde{R},(\tilde{\gamma},\tilde{d})) = (0, I, \mathcal{Q}_0)$ of the closed-loop system given by (17)-(19) *and (8) is locally asymptotically stable, and the remaining equilibrium manifolds given by $(0,\mathcal{R}_i,\mathcal{Q}_i)$, for $i \in \{1,2,3\}$ are unstable. Finally, the set of all initial conditions converging to these equilibrium manifolds forms a lower dimensional submanifold of $\mathbb{R}^3\times\mathrm{SO}(3)\times\mathbb{R}^6\times\mathbb{R}^3$.*

The following result now follows from Theorem III.3, Lemma III.4 and Lemma III.5.

Theorem III.6. *Let the assumptions of Lemma 4 hold. Then, there exists an invariant subset $\mathcal{M}$ in $\mathbb{R}^3\times\mathrm{SO}(3)\times\mathbb{R}^6\times\mathbb{R}^3$ whose complement is open and dense and is such that, for all initial conditions $(\tilde{\omega}(0),\tilde{R}(0),\tilde{\gamma}(0),\tilde{d}(0)) \notin \mathcal{M}$, the solution of the closed-loop system consisting of (7), (11), (17), and (18) satisfies $\tilde{R}(t)\to I$, $\tilde{\omega}(t)\to 0$ as $t\to\infty$.*

Theorem III.6 does not imply that the estimates of the inertia-matrix entries or the estimates of the disturbance components converge to their true values. As discussed in [11, 22], convergence of the estimates of the inertia-matrix entries depends on the persistence of the command signals.

IV. Specialization to Slew Maneuvers

We now specialize the results of Section 3 to the case of slew maneuvers, where the objective is to bring the spacecraft to rest with a specified attitude. Hence we assume that R_{d} is constant, and we set $\omega_{\mathrm{d}} = 0$ in Theorem III.2.

In the special case in which the disturbance is zero, the controller given in Theorem III.2 simplifies considerably. In this case, it is not necessary to include an estimate of the unknown inertia. Specifically, set $K_1 = 0$ and define V as

$$V(\omega,\tilde{R}) \triangleq \tfrac{1}{2}\omega^{\mathrm{T}}J\omega + K_{\mathrm{p}}\mathrm{tr}\,(A - A\tilde{R}). \tag{27}$$

Taking u to be the control law

$$u = -B^{-1}(K_{\mathrm{v}}\omega + K_{\mathrm{p}}S) \tag{28}$$

yields

$$\dot{V}(\omega,\tilde{R}) = -\omega^{\mathrm{T}}K_{\mathrm{v}}\omega, \tag{29}$$

which implies almost global stabilization of the constant desired configuration R_{d}. This control law, which is given in [12], achieves zero steady-state error for setpoint commands without integral action and without knowledge of J.

The following result shows that, for slew maneuvers without disturbances, it is possible to arbitrarily bound the level of torque about each axis. Let $\sigma_{\max}(M)$ and $\sigma_{\min}(M)$ denote, respectively, the maximum and minimum singular values of a matrix M. Furthermore, let $\|x\|_\infty$ denote the largest absolute value of the components of the vector x.

Proposition IV.1. *Let α and β be positive numbers, let $A = \mathrm{diag}(a_1,a_2,a_3)$ be a diagonal positive-definite matrix with distinct diagonal entries, and let K_{p} and $K_{\mathrm{v}} = K_{\mathrm{v}}(\omega)$ be given by*

$$K_{\mathrm{p}} = \frac{\alpha}{\mathrm{tr}\,A} \tag{30}$$

and

$$K_{\mathrm{v}} = \beta\begin{bmatrix} \frac{1}{1+|\omega_1|} & 0 & 0 \\ 0 & \frac{1}{1+|\omega_2|} & 0 \\ 0 & 0 & \frac{1}{1+|\omega_3|} \end{bmatrix}. \tag{31}$$

Furthermore, assume that $d = 0$. Then, for all $t \geq 0$, the control torque given by (28) satisfies

$$\|u(t)\|_\infty \leq \frac{\alpha + \beta}{\sigma_{\min}(B)}. \tag{32}$$

Numerical simulation results for a slew maneuver of a satellite facing a constant disturbance torque, actuator misalignment and time delay in actuation, are given in the companion journal paper [16].

V. Spin Maneuver Example

We consider a spin maneuver with J given by

$$J = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{kg-m}^2, \tag{33}$$

whose principal moments of inertia are 1.4947, 3.7997, and 5.2056 kg-m^2, $B = I$, and with the spacecraft initially at rest with $R = I$. The specified attitude is given by $R_{\rm d}(0) = I$ with desired constant angular velocity

$$\omega_{\rm d} = \begin{bmatrix} 0.5 & -0.5 & -0.3 \end{bmatrix}^{\rm T} \text{rad/sec},$$

and the disturbance is chosen to be the constant torque

$$d = \begin{bmatrix} 0.7 & -0.3 & 0 \end{bmatrix}^{\rm T} \text{N-m}.$$

We choose $A = \text{diag}(1, 2, 3)$, $\alpha = \beta = 1$, $K_1 = D = I$, and $Q = I$. Figures 1–6 show, respectively, the attitude errors, angular velocity components, torque inputs, torque input norm, inertia-estimate errors, and disturbance-estimate errors. Note that the spin command consists of a specified time history of rotation about a specified body axis aligned in a specified inertial direction. Figure 5 shows that the disturbance estimates do not converge to the true values, although this has no effect on asymptotic tracking.

In the companion journal paper [16], we give numerical results showing asymptotic tracking of a spin maneuver with a harmonic disturbance of known frequency.

VI. Conclusions and Future Research

Almost global stabilizability, that is, Lyapunov stability with almost global convergence, of spacecraft tracking is feasible without inertia information and with continuous feedback. In addition, asymptotic rejection of harmonic disturbances (including constant disturbances as a special case) is possible with knowledge of the disturbance spectrum but without knowledge of either the amplitude or phase. These results have practical advantages relative to prior controllers that i) require exact or approximate inertia information or ii) are based on attitude parameterizations such as quaternions that require discontinuous control laws, fail to be physically consistent, that is, specify different control torques for the same physical orientation, or suffer from unwinding.

A key problem that this paper does not fully resolve is that of torque saturation. Although an approximate saturation technique provides a simple technique for reducing the torque during transients, it is desirable to extend this technique to cases in which sufficient torque is not available to follow the desired trajectory or reject the ambient disturbances. In addition, the problem of determining persistent inputs that guarantee convergence of the inertia estimates to their true values may be of interest in some applications. Finally, almost global stabilization for momentum-bias spacecraft is of interest.

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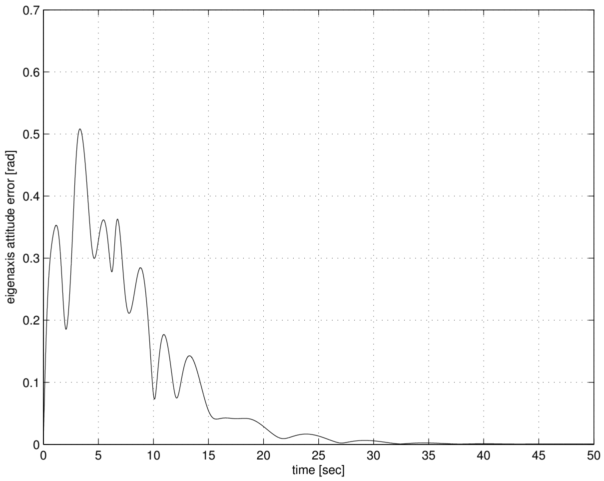

Fig. 1. Eigenaxis attitude errors for the spin maneuver.

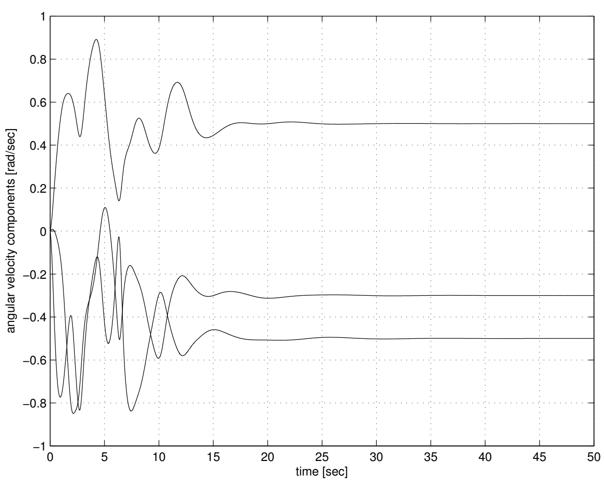

Fig. 2. Angular-velocity components for the spin maneuver.

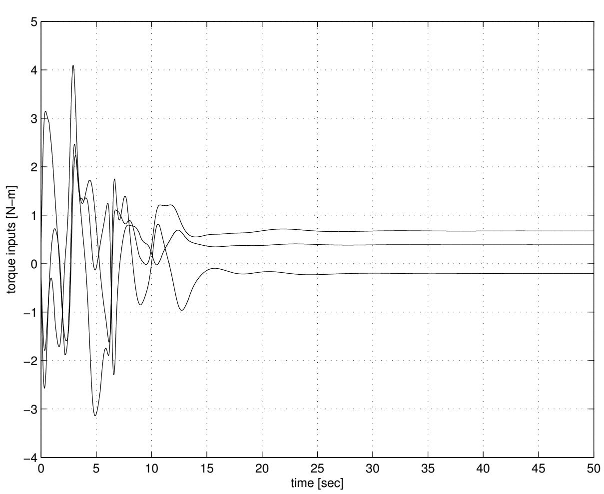

Fig. 3. Torque inputs for the spin maneuver.

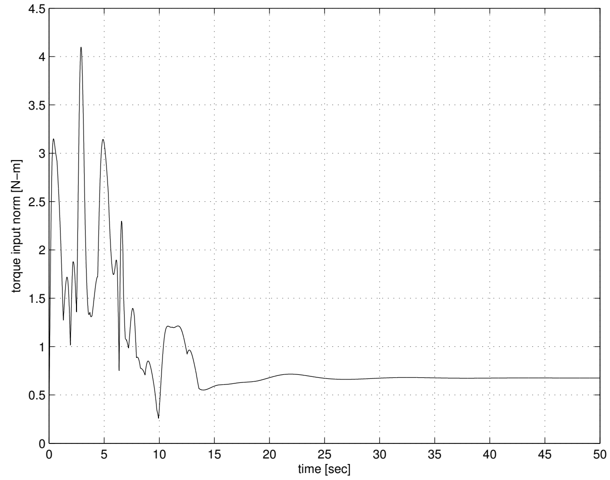

Fig. 4. Torque input norm for the spin maneuver.

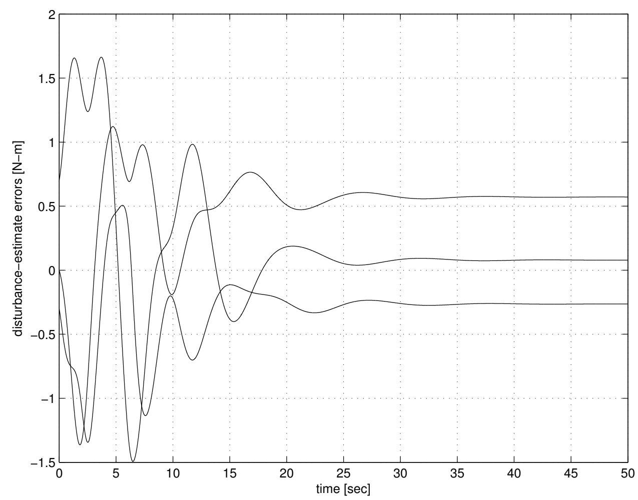

Fig. 5. Disturbance-estimate errors for the spin maneuver.

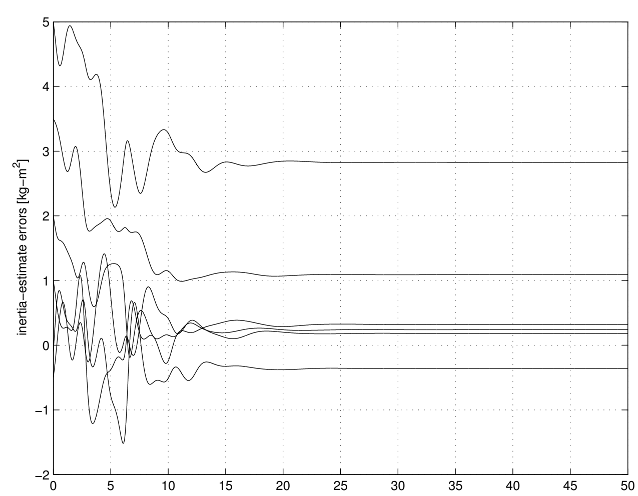

Fig. 6. Inertia-estimate errors for the spin maneuver.