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Adaptive Control of Uncertain Hammerstein Systems with Non-monotonic Input Nonlinearities Using Auxiliary Nonlinearities

Jin Yan and Dennis S. Bernstein

***Abstract*— We extend retrospective cost adaptive control (RCAC) with auxiliary nonlinearities to command following for uncertain Hammerstein systems with non-monotonic input nonlinearities. We assume that only one Markov parameter of the linear plant is known and that the non-monotonic input nonlinearity is uncertain. Auxiliary nonlinearities are used within RCAC to account for the non-monotonic input nonlinearity. The required modeling information for the input nonlinearity includes the intervals of monotonicity as well as values of the nonlinearity that determine overlapping segments of the range of the nonlinearity within each interval of monotonicity.**

I. INTRODUCTION

A *Hammerstein system* consists of linear dynamics preceded by an input nonlinearity [1–6]. This nonlinearity may represent the properties of an actuator, such as saturation to reflect magnitude restrictions on the control input, deadzone to represent actuator stiction, or a signum function to represent on-off operation. The ability to invert the nonlinearity is often precluded in practice by the fact that the nonlinearity may be neither one-to-one nor onto, and it may also be uncertain.

If the input nonlinearity is uncertain, then adaptive control may be useful for learning the characteristics of the nonlinearity online and compensating for the distortion that it introduces. Adaptive inversion control of Hammerstein systems with uncertain input nonlinearities and linear dynamics is considered in [7–10]. In contrast, the retrospective-cost adaptive control (RCAC) approach applied to Hammerstein systems in [11] makes no attempt to invert the input nonlinearity. This approach is applicable to linear plants that are possibly MIMO, nonminimum phase (NMP), and unstable [12–18]. RCAC relies on knowledge of Markov parameters and, for NMP open-loop-unstable plants, estimates of the NMP zeros. This information can be obtained from either analytical modeling or system identification [19].

RCAC was applied to a command-following problem involving SISO Hammerstein systems in [11], where the input nonlinearity was assumed to be monotonic. If the input nonlinearity is nonincreasing, then an auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}$ was used to create a composite nonlinearity that is nondecreasing, thus preserving the signs of the Markov parameters of the linearized system. An additional auxiliary saturation nonlinearity $\mathcal{N}_{\rm sat}$, which is used to tune the transient response of the closed-loop system, may depend on estimates of the range of the input nonlinearity and the gain of the linear dynamics.

The authors are with the Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109-2140.

In [7–9], the input nonlinearities are assumed to be piecewise linear. The approach of [11] does not impose this restriction. Numerical examples involving cubic, deadzone, saturation, and on-off input nonlinearities are presented in [11].

In the present paper we consider the more challenging case in which the input nonlinearity is non-monotonic. To address this problem, we utilize an auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}$ and an auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}$, which, together with $\mathcal{N}_{\rm r}$ and $\mathcal{N}_{\rm sat}$, create a composite nonlinearity that is nondecreasing. The modeling information required for this approach consists of the range of the input nonlinearity within each interval of monotonicity. In addition, it is necessary to ensure that the composite nonlinearity is globally nondecreasing rather than only piecewise nondecreasing. This property is achieved by using a blocking nonlinearity that removes portions of the nonlinearity so that the overall nonlinearity is globally nondecreasing. The modeling information required by this approach is precisely the information needed to construct an appropriate blocking nonlinearity. No other modeling information is needed.

The contents of the paper are as follows. In Section II, we describe the Hammerstein command-following problem. In Section III, we first apply an extension of RCAC using auxiliary nonlinearities to the Hammerstein command-following problem with non-monotonic input nonlinearities. In Section IV, we present illustrative examples on the construction of the auxiliary nonlinearities. Numerical simulation results are presented in Section V, and conclusions are given in Section VI.

II. HAMMERSTEIN COMMAND-FOLLOWING PROBLEM

Consider the SISO discrete-time Hammerstein system

$$x(k+1) = Ax(k) + B\mathcal{N}(u(k)) + D_1 w(k), \qquad (1)$$

$$y(k) = Cx(k), \qquad (2)$$

where $x(k) \in \mathbb{R}^n$, $u(k)$, $y(k) \in \mathbb{R}$, $w(k) \in \mathbb{R}^d$, $\mathcal{N} : \mathbb{R} \to \mathbb{R}$, and $k \geq 0$. We consider the Hammerstein command-following problem with the performance variable

$$z(k) = y(k) - r(k), \qquad (3)$$

where $z(k)$, $r(k) \in \mathbb{R}$. The goal is to develop an adaptive output feedback controller that minimizes the command-following error z with minimal modeling information about the dynamics, disturbance w, and input nonlinearity $\mathcal{N}$. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ are not available. A block diagram for (1)-(3) is shown in Figure 1.

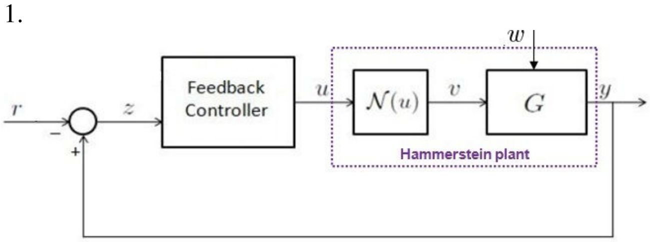


Fig. 1. Adaptive command-following problem for a Hammerstein plant with a non-monotonic input nonlinearity $\mathcal{N}$. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ and $w(k)$ are not available.

III. ADAPTIVE CONTROL FOR THE HAMMERSTEIN COMMAND-FOLLOWING PROBLEM

For the Hammerstein command-following problem, we assume that G is uncertain except for an estimate of a single nonzero Markov parameter. The non-monotonic input nonlinearity $\mathcal{N}$ is also uncertain.

To account for the presence of the input nonlinearity $\mathcal{N}$, the RCAC controller in Figure 2 uses four auxiliary nonlinearities, $\mathcal{N}_{\rm sat}$, $\mathcal{N}_{\rm b}$, $\mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$ to account for the presence of the input nonlinearity $\mathcal{N}$ in Figure 2. The construction of $\mathcal{N}_{\rm sat}, \mathcal{N}_{\rm b}, \mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$ is described in Section IV. The RCAC controller is given in [11].

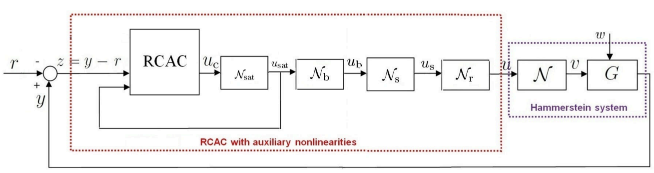


Fig. 2. Hammerstein command-following problem with the RCAC adaptive controller and auxiliary nonlinearities $\mathcal{N}_{\rm sat}$, $\mathcal{N}_{\rm b}$, $\mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$.

IV. AUXILIARY NONLINEARITIES

In this section, we construct the auxiliary nonlinearities $\mathcal{N}_{\rm sat}, \mathcal{N}_{\rm b}, \mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$ in Figure 2 along with the required model information. $\mathcal{N}_{\rm sat}$ modifies $u_{\rm c}$ to obtain the regressor input $u_{\rm sat}$, while $\mathcal{N}_{\rm b}$, $\mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$ modify $u_{\rm sat}$ to produce the Hammerstein plant input u. The auxiliary nonlinearities $\mathcal{N}_{\rm b}$, $\mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$ are chosen such that the composite input nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing. To avoid unnecessary complications, we assume that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is redefined at points of discontinuity to render it piecewise right continuous.

For the Hammerstein command-following problem, we assume that G is uncertain except for an estimate of a single nonzero Markov parameter. The input nonlinearity $\mathcal{N}$ is also uncertain, as described below.

A. Auxiliary Saturation Nonlinearity $\mathcal{N}_{\rm sat}$

The auxiliary saturation nonlinearity $\mathcal{N}_{\rm sat}$ is defined to be the saturation function ${\rm sat}_{p,q}$ given by

$$\mathcal{N}_{\rm sat}(u_{\rm c}) = {\rm sat}_{p,q}(u_{\rm c}) = \begin{cases} p, & \text{if} \quad u_{\rm c} < p, \\ u_{\rm c}, & \text{if} \quad p \le u_{\rm c} \le q, \\ q, & \text{if} \quad u_{\rm c} > q, \end{cases} \quad (4)$$

where the real numbers p and q are the lower and upper saturation levels, respectively. For minimum-phase plants, the auxiliary nonlinearity $\mathcal{N}_{\rm sat}$ is not needed, and thus, in this case, the saturation levels p and q are chosen to be large negative and positive numbers, respectively. For NMP plants, the saturation levels are used to tune the transient behavior. In addition, the saturation levels are chosen to provide a sufficiently large range of the control input to follow the command r. These values depend on the range of the input nonlinearity $\mathcal{N}$ as well as the gain of the linear system G at frequencies in the spectra of r and w.

B. Auxiliary Reflection Nonlinearity $\mathcal{N}_{\rm r}$

If the input nonlinearity $\mathcal{N}$ is not monotonic, then the auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}$ is used to create a composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ that is piecewise nondecreasing. To construct $\mathcal{N}_{\rm r}$, we assume that the intervals of monotonicity of the input nonlinearity $\mathcal{N}$ are known, as described below.

In Section IV-C and IV-D below we restrict $\mathcal{N}_{\rm s}$ and $\mathcal{N}_{\rm b}$ so that $\mathcal{N}_{\rm s} : [p,q] \to [p,q]$ and $\mathcal{N}_{\rm b} : [p,q] \to [p,q]$. With this construction, we need to consider only $u_{\rm s} \in [p,q]$. Therefore, let $I_1, I_2, \ldots$ be the smallest number of intervals of monotonicity of $\mathcal{N}$ that are a partition of the interval $[p,q]$. If $\mathcal{N}$ is nondecreasing on I_i, then $\mathcal{N}_{\rm r}(u_{\rm s}) \stackrel{\triangle}{=} u_{\rm s}$ for all $u_{\rm s} \in I_i$. Alternatively, if $\mathcal{N}$ is nonincreasing on $I_i = [p_i, q_i)$, then $\mathcal{N}_{\rm r}(u_{\rm s}) \stackrel{\triangle}{=} p_i + q_i - u_{\rm s} \in I_i$ for all $u_{\rm s} \in I_i$. Finally, if $\mathcal{N}$ is constant on I_i, then either choice can be used. Thus, $\mathcal{N}_{\rm r}$ is a piecewise-linear function that reflects $\mathcal{N}$ about $u_{\rm s} = \frac{p_i+q_i}{2}$ within each interval of monotonicity so that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is nondecreasing on I_i, and thus $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing on I. Let $\mathcal{R}_I(f)$ denote the range of the function f with arguments in I.

Proposition 4.1: Assume that $\mathcal{N}_{\rm r}$ is constructed by the above rule. Then the following statements hold:

i) $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing on $[p,q]$.
ii) $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N})$.

Example 4.1: Consider the nonincreasing input nonlinearity $\mathcal{N}(u) = -{\rm sat}_{-1,1}(u-5)$ shown in Figure 3(a). Let $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} + 10$ for all $u_{\rm s} \in [3,7]$ according to Proposition 4.1. Figure 3(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is nondecreasing on $I \stackrel{\triangle}{=} [-2,2]$. Note that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1,1]$. ■

Example 4.2: Consider the non-monotonic input nonlinearity $\mathcal{N}(u) = |u-5|$ shown in Figure 4(a). Let $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} + 6$ for all $u_{\rm s} \in [1,5)$ and $\mathcal{N}_{\rm r}(u_{\rm s}) = u_{\rm s}$ otherwise according to Proposition 4.1. Figure 4(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing but not globally nondecreasing on $I \stackrel{\triangle}{=} [1,9]$, and that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [0,4]$. ■

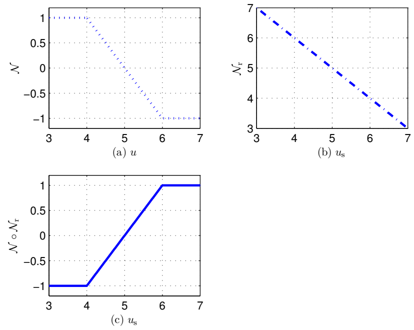


Fig. 3. Example 4.1. (a) Input nonlinearity $\mathcal{N}(u) = -\text{sat}_{-1,1}(u-5)$. (b) Auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} + 10$ for $u_{\rm s} \in [3, 7]$. (c) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is nondecreasing on $I \stackrel{\triangle}{=} [3, 7]$ and $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1, 1]$.

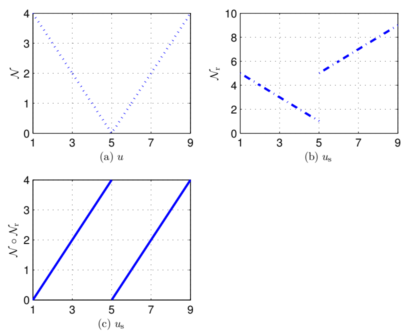


Fig. 4. Example 4.2 (a) Non-monotonic input nonlinearity $\mathcal{N}(u) = -|u - 5|$. (b) Auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} + 6$ for $u_{\rm s} \in [1, 5)$ and $\mathcal{N}_{\rm r}(u_{\rm s}) = u_{\rm s}$ otherwise. (c) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing but not globally nondecreasing on $I \stackrel{\triangle}{=} [1, 9]$, and that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [0, 4]$.

Example 4.3: Consider the non-monotonic input nonlinearity

$$\mathcal{N}(u) = \begin{cases} -\frac{1}{2}u, & \text{if } u \leq 0, \\ u - 1, & \text{if } u > 0, \end{cases} \tag{5}$$

shown in Figure 5(a). Let $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} - 2$ for all $u_{\rm s} \in [-2, 0)$ and $\mathcal{N}_{\rm r}(u_{\rm s}) = u_{\rm s}$ otherwise according to Proposition 4.1. Figure 5(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing but not globally nondecreasing on $I \stackrel{\triangle}{=} [-2, 1]$, and that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1, 1]$. ■

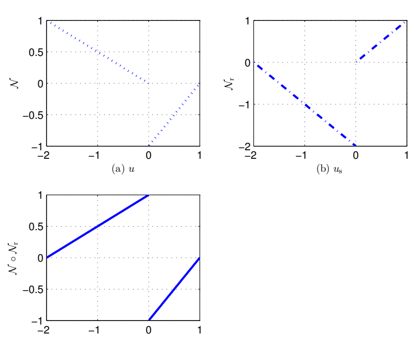


Fig. 5. Example 4.3 (a) Non-monotonic input nonlinearity (5). (b) Auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} - 2$ for $u_{\rm s} \in [-2, 0)$ and $\mathcal{N}_{\rm r}(u_{\rm s}) = u_{\rm s}$ otherwise. (c) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing but not globally nondecreasing on $I \stackrel{\triangle}{=} [-2, 0]$, and that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1, 1]$.

C. Auxiliary Sorting Nonlinearity $\mathcal{N}_{\rm s}$

As illustrated by Example 4.2 and Example 4.3, $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing but not globally nondecreasing. In order to construct a composite input nonlinearity that is globally nondecreasing, we introduce the auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}$ and auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}$. The auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}$ sorts portions of the piecewise nondecreasing nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ to create a composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ so that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing. $\mathcal{N}_{\rm b}$ is discussed in Section IV-D. To construct $\mathcal{N}_{\rm s}$, we assume that the range of $\mathcal{N} \circ \mathcal{N}_{\rm r}$ within each interval of monotonicity is known. No further modeling information about $\mathcal{N}$ is needed.

Let $\mathcal{N}_{\rm s}$ be the piecewise right-continuous affine function defined as follows. Let $I_1 = [p_1, q_1), I_2 = [p_2, q_2), \ldots$ be the smallest number of intervals of monotonicity of $\mathcal{N}$ that are a partition of the interval $[p, q]$. If $\mathcal{R}_{I_i}(\mathcal{N} \circ \mathcal{N}_{\rm r}) \subset \mathcal{R}_{I_j}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ for all $i \neq j$ or $(\mathcal{N} \circ \mathcal{N}_{\rm r})(q_i) \leq (\mathcal{N} \circ \mathcal{N}_{\rm r})(q_j)$, where $q_i < q_j$, then $\mathcal{N}_{\rm s}(u_{\rm b}) \stackrel{\triangle}{=} u_{\rm b}$ for all $u_{\rm b} \in I_i \cup I_j = [p_i, q_i) \cup [p_j, q_j)$, and thus $\mathcal{N}_{\rm s}$ is not needed. Alternatively, if $\mathcal{R}_{I_i}(\mathcal{N} \circ \mathcal{N}_{\rm r}) \not\subseteq \mathcal{R}_{I_j}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ for all $i \neq j$ and $(\mathcal{N} \circ \mathcal{N}_{\rm r})(q_i) > (\mathcal{N} \circ \mathcal{N}_{\rm r})(q_j)$, where $q_i < q_j$, then $\mathcal{N}_{\rm s}(u_{\rm b}) \stackrel{\triangle}{=} \frac{1}{q_i - p_i}\left[(q_j - p_j)u_{\rm b} + p_j q_i - p_i q_j\right] \in I_j$ for all $u_{\rm b} \in I_i$ and $\mathcal{N}_{\rm s}(u_{\rm b}) \stackrel{\triangle}{=} \frac{1}{q_j - p_j}\left[(q_i - p_i)u_{\rm b} + p_i q_j - p_j q_i\right] \in I_i$ for all $u_{\rm b} \in I_j$.

Proposition 4.2: Assume that $\mathcal{N}_{\rm s}$ is constructed by the above rule. Then the following statements hold:

i) $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[p, q]$.
ii) $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = \mathcal{R}_I(\mathcal{N})$.

Example 4.4: Consider the case where $\mathcal{R}_{[-2,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r}) \cap \mathcal{R}_{[0,1]}(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \varnothing$ as shown in Figure 6(a). We assume that values of $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0)$ and $(\mathcal{N} \circ \mathcal{N}_{\rm r})(1)$ are known. In particular, $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0) = 1 > (\mathcal{N} \circ \mathcal{N}_{\rm r})(1) = 0$. We thus choose $\mathcal{N}_{\rm s}(u_{\rm b}) = 0.5u_{\rm b} + 1$ for $u_{\rm b} \in [-2, 0)$ and $\mathcal{N}_{\rm s}(u_{\rm b}) = 2u_{\rm b} - 2$ for $u_{\rm b} \in [0, 1]$ as shown in Figure 6(b). Note that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing on $[-2, 1]$. Figure 6(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-2, 1]$. ■

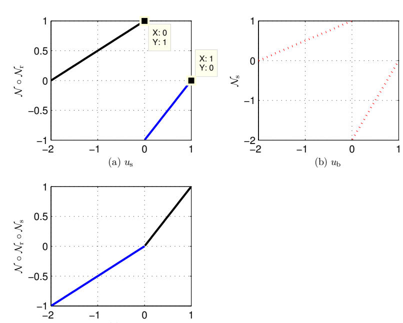


Fig. 6. Example 4.4. In this example, $\mathcal{R}_{[-2,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r}) \cap \mathcal{R}_{[0,1]}(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \varnothing$. (a) Nondecreasing composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$. Note that $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0) = 1 > (\mathcal{N} \circ \mathcal{N}_{\rm r})(1) = 0$. (b) Auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}(u_{\rm b}) = 0.5u_{\rm b} + 1$ for $u_{\rm b} \in [-2, 0)$ and $\mathcal{N}_{\rm s}(u_{\rm b}) = 2u_{\rm b} - 2$ for $u_{\rm b} \in [0, 1]$. (c) The composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$.

Example 4.5: Consider the case where range of $\mathcal{N} \circ \mathcal{N}_{\rm r}$ on subintervals of its domain has partially overlapping intervals as shown in Figure 7(a), where neither $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ nor $\mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ is contained in the other set. We assume that values of $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0)$ and $(\mathcal{N} \circ \mathcal{N}_{\rm r})(5)$ are known. In particular, $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0) = 4 > (\mathcal{N} \circ \mathcal{N}_{\rm r})(5) = 1$, we thus choose $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b} + 5$ for $u_{\rm b} \in [-5, 0)$ and $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b} - 5$ for $u_{\rm b} \in [0, 5]$ as shown in Figure 7(b). Note that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-5, 5]$, and Figure 7(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-5, 5]$. ■

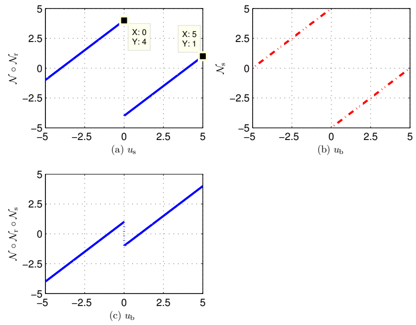


Fig. 7. Example 4.5. In this example, range of $\mathcal{N} \circ \mathcal{N}_{\rm r}$ on subintervals of its domain has partially overlapping intervals, where neither $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ nor $\mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r})$ is contained in the other set. Note that $(\mathcal{N} \circ \mathcal{N}_{\rm r})(0) = 4 > (\mathcal{N} \circ \mathcal{N}_{\rm r})(5) = 1$. (a) Piecewise nondecreasing composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ with partially overlapping intervals. (b) Auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b} + 5$ for $u_{\rm b} \in [-5, 0)$ and $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b} - 5$ for $u_{\rm b} \in [0, 5]$. (c) The composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-5, 5]$.

D. Auxiliary Blocking Nonlinearity $\mathcal{N}_{\rm b}$

As shown in Proposition 4.2 and illustrated by Example 4.5, $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing. In order to construct a composite input nonlinearity that is globally nondecreasing, we introduce the auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}$. To construct $\mathcal{N}_{\rm b}$, we assume that the range of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ within each interval of monotonicity is known. If, in addition, these ranges are partially overlapping, then selected intermediate values of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ must also be known. No further modeling information about $\mathcal{N}$ is needed.

Let $\mathcal{N}_{\rm b}$ be the piecewise right-continuous affine function defined as follows. Let $I_1, I_2, \ldots$ be the smallest number of intervals of monotonicity of $\mathcal{N}$ that are also a partition of the interval $[p, q]$. If $\mathcal{R}_{I_i}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \cap \mathcal{R}_{I_j}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = \varnothing$ for all $i \neq j$, then we choose $\mathcal{N}_{\rm b}(u_{\rm sat}) \stackrel{\triangle}{=} u_{\rm sat}$ for all $u_{\rm sat} \in I_j \cup I_j$. Alternatively, if $\mathcal{R}_{I_i}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \cap \mathcal{R}_{I_j}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \neq \varnothing$ and $\mathcal{R}_{I_i}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subsetneq \mathcal{R}_{I_j}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$, we block the overlapping segments as shown by the following examples.

Example 4.6: Consider the case where range of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ on subintervals of its domain has partially overlapping intervals, where neither $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ nor $\mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ is contained in the other set. In particular, as shown in Figure 8(a), $\mathcal{R}_{[-2,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = \mathcal{R}_{[0,2]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$. In this case, we assume that intermediate values of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ are known. In particular, knowledge of $(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})_{[-5,0]}(0) = 1$ is sufficient to construct $\mathcal{N}_{\rm b}$. We choose $\mathcal{N}_{\rm b}(u_{\rm sat}) = -2$ for $u_{\rm sat} \in [-2, 0)$ and $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$ otherwise. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-5, 5]$, and Figure 8(b) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $[-5, 5]$. ■

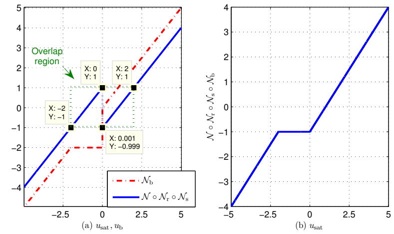


Fig. 8. Example 4.6. In this example, range of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ on subintervals of its domain has partially overlapping intervals, where neither $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ nor $\mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ is contained in the other set. (a) Piecewise nondecreasing composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ with partially overlapping intervals, where $\mathcal{R}_{[-2,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = \mathcal{R}_{[0,2]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ and the auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$. (b) The composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $[-5, 5]$.

Example 4.7: Consider the case $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subset \mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ as shown in Figure 9(a). In particular, $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = [-2, 3]$ and $\mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = [-5, 5]$. We let $\mathcal{N}_{\rm b}(u_{\rm sat}) = -5$ for all $u_{\rm sat} \in [-5, 0)$ and $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$ for all $u_{\rm sat} \in [0, 5]$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-5, 5]$ and Figure 9(b) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $[-5, 5]$. ■

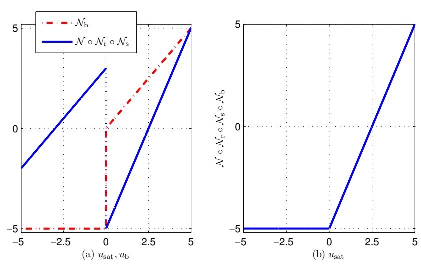


Fig. 9. Example 4.7. In this example, $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subset \mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$. (a) Piecewise nondecreasing composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$, where $\mathcal{R}_{[-5,0]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subset \mathcal{R}_{[0,5]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$ and the auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}(u_{\rm sat}) = -5$ for $u_{\rm sat} \in [-5, 0)$ and $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$ for $u_{\rm sat} \in [0, 5]$. (b) The composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $[-5, 5]$.

Proposition 4.3: Assume that $\mathcal{N}_{\rm b}$ is constructed by the above rule. Then the following statements hold:

i) $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $I \stackrel{\triangle}{=} [p, q]$.
ii) $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}) = \mathcal{R}_I(\mathcal{N})$.

E. Examples Illustrating the Construction of $\mathcal{N}_{\rm b}$, $\mathcal{N}_{\rm s}$, and $\mathcal{N}_{\rm r}$

Example 4.8: Consider the non-monotonic input nonlinearity

$$\mathcal{N}(u) = \begin{cases} -\mathrm{sat}_{-0.5,0.5} u, & \text{if } u < 2, \\ 0.5u - 2, & \text{if } 2 \leq u < 4, \\ 0, & \text{if } u \geq 4, \end{cases} \qquad (6)$$

which is shown in Figure 10(a). Let $\mathcal{N}_{\rm sat}(u_{\rm c}) = \mathrm{sat}_{p,q}(u_{\rm c})$, where $p = -2$ and $q = 6$. According to Propositions 4.1, 4.2, and 4.3, let

$$\mathcal{N}_{\rm r}(u_{\rm s}) = \begin{cases} -u_{\rm s}, & \text{if } -2 \le u_{\rm s} < 2, \\ u_{\rm s}, & \text{if } 2 \le u_{\rm s} \le 6, \end{cases} \tag{7}$$

$$\mathcal{N}_{\rm s}(u_{\rm b}) = \begin{cases} u_{\rm b} + 4, & \text{if } -2 \le u_{\rm b} < 2, \\ u_{\rm b} - 4, & \text{if } 2 \le u_{\rm b} \le 6, \end{cases} \tag{8}$$

and

$$\mathcal{N}_{\rm b}(u_{\rm sat}) = \begin{cases} 4, & \text{if } 2 \le u_{\rm sat} < 4, \\ u_{\rm sat}, & \text{otherwise}. \end{cases} \tag{9}$$

Figure 10(b) shows the auxiliary nonlinearities $\mathcal{N}_{\rm r}$ and $\mathcal{N}_{\rm s}$. Figures 10(c) and 10(d) show that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$ and $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ are piecewise nondecreasing on $I \overset{\triangle}{=} [-2, 6]$. Figures 10(e) shows auxiliary blocking nonlinearities $\mathcal{N}_{\rm b}$ and Figures 10(f) shows the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on I. Note that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}) = \mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) = \mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1, 0.5]$. ■

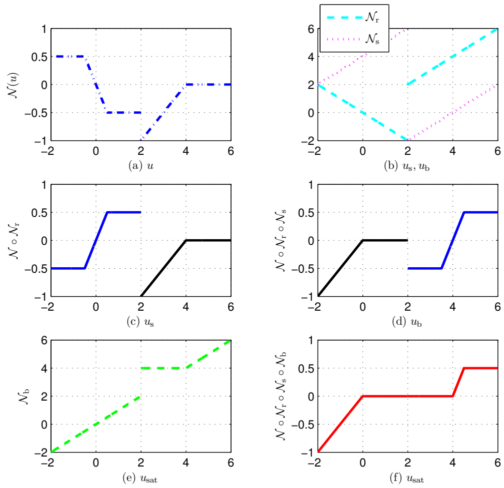


Fig. 10. Example 4.8. (a) Input nonlinearity $\mathcal{N}(u)$ given by (6). (b) The auxiliary reflection nonlinearity $\mathcal{N}_{\rm r}$ given by (7) for $u_{\rm s} \in [-2, 6]$ and the auxiliary sorting nonlinearity $\mathcal{N}_{\rm s}$ given by (8) for $u_{\rm b} \in [-2, 6]$. (c) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r}$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r}$ is piecewise nondecreasing on $[-2, 6]$. (d) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$. Note that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing on $[-2, 6]$. (e) The auxiliary blocking nonlinearity $\mathcal{N}_{\rm b}$ given by (9) for $u_{\rm sat} \in [-2, 6]$. (f) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing on $[-2, 6]$, and $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}) = \mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm b}) = \mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_{\rm r}) = \mathcal{R}_I(\mathcal{N}) = [-1, 0.5]$.

Knowledge of the intervals of monotonicity of $\mathcal{N}$, the ranges of $\mathcal{N} \circ \mathcal{N}_{\rm r}$ and $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ within each interval of monotonicity, and selected intermediate values of $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ in the case of partially overlapping interval ranges is needed to modify the controller output $u_{\rm sat}$ so that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is globally nondecreasing. It thus follows that $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ preserves the signs of the Markov parameters of the linearized Hammerstein system.

V. NUMERICAL EXAMPLES

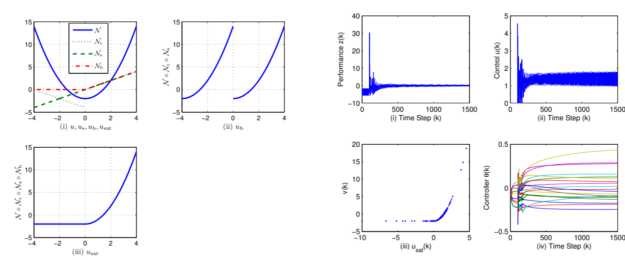
In all examples, we assume that at least one nonzero Markov parameter of G is known. For convenience, each example is constructed such that the first nonzero Markov parameter $H_d = 1$, where d is the relative degree of G. RCAC generates a control signal $u_{\rm c}(k)$ that attempts to minimize the performance $z(k)$ in the presence of the input nonlinearity $\mathcal{N}$. In all cases, we initialize the adaptive controller to be zero, that is, $\theta(0) = 0$. We let $\lambda = 1$ for all examples.

Example 5.1: We consider the asymptotically stable, nonminimum-phase plant

$$G(\mathbf{z}) = \frac{\mathbf{z} - 1.2}{\mathbf{z}^2 + 0.3\mathbf{z} - 0.1}, \tag{10}$$

with the quadratic input nonlinearity $\mathcal{N}(u) = u^2 - 2$, which is neither one-to-one nor onto and satisfies $\mathcal{N}(0) = -2$. Note that $d = 1$ and $H_d = 1$. As shown in Figure 11(a.i), since $\mathcal{N}(u)$ is not monotonic and G is nonminimum-phase, we choose $\mathcal{N}_{\rm sat}(u_{\rm c}) = {\rm sat}_{p,q}(u_{\rm c})$, where $p = -4$ and $q = 4$ in (4), let $\mathcal{N}_{\rm r}(u_{\rm b}) = -u_{\rm b} - 4$ for $u_{\rm b} \in [-4, 0]$ and $\mathcal{N}_{\rm r}(u_{\rm b}) = u_{\rm b}$ otherwise, and select $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b}$ so that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is piecewise nondecreasing in Figure 11(a.ii). Knowledge of only the monotonicity of $\mathcal{N}$ is used to choose $\mathcal{N}_{\rm r}$. To construct $\mathcal{N}_{\rm b}$, note that the piecewise nondecreasing composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ satisfies $\mathcal{R}_{[-4,0)}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subset \mathcal{R}_{[0,4]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$, which is not partially overlapping. Therefore, no additional information about $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}$ is needed. We let $\mathcal{N}_{\rm b}(u_{\rm sat}) = 0$ for $u_{\rm sat} \in [-4, 0)$ and $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$ otherwise. Figure 11(a.iii) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is nondecreasing.

We consider the single-tone sinusoidal command $r(k) = \sin \Omega_1 k$, where $\Omega_1 = \pi/5$ rad/sample, and the disturbance $w(k) = 0.5 \sin(\frac{\pi}{2} k)$. We let $n_{\rm c} = 10$, $P_0 = 0.01 I_{2n_{\rm c}}$, $\eta_0 = 0.1$, and $\tilde{\mathcal{H}} = H_1$. Figure 11(b) shows the time history of z with the input nonlinearity and disturbance present and RCAC is able to follow the command. ■



(a) (b)

Fig. 11. Example 5.1. (a.i) shows the quadratic input nonlinearity $\mathcal{N}(u) = u^2 - 2$ and the auxiliary nonlinearities $\mathcal{N}_{\rm b}$ and $\mathcal{N}_{\rm r}$. (a.ii) shows the piecewise nondecreasing input nonlinearity $\mathcal{R}_{[-4,0)}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s}) \subset \mathcal{R}_{[0,4]}(\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s})$, which is not partially overlapping. (a.iii) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_{\rm r} \circ \mathcal{N}_{\rm s} \circ \mathcal{N}_{\rm b}$ is nondecreasing. (b) shows the closed-loop response of the stable minimum-phase plant G given by (10) with the sinusoidal command $r(k) = \sin(0.2\pi k)$ and disturbance $w(k) = 0.5 \sin(\frac{\pi}{2} k)$.

Example 5.2: We consider the asymptotically stable, minimum-phase plant

$$G(\mathbf{z}) = \frac{1}{\mathbf{z} - 0.5} \tag{11}$$

with the non-monotonic input nonlinearity

$$\mathcal{N}(u) = \cos(2u), \tag{12}$$

which is neither one-to-one nor onto and satisfies $\mathcal{N}(0) = 1$. Note that $d = 1$ and $H_d = 1$. As shown in Figure 12(b), $\mathcal{N}(u)$ is increasing for all $u \in \bigcup_{n\in\mathbb{Z}} \left((n-\frac{1}{2})\pi, n\pi\right)$, and decreasing for all $u \in \bigcup_{n\in\mathbb{Z}} \left(n\pi, (n+\frac{1}{2})\pi\right)$. We thus choose $\mathcal{N}_{\rm sat}(u_{\rm c}) = {\rm sat}_{p,q}(u_{\rm c})$, where $p = -10^6$ and $q = 10^6$ in (4), let $\mathcal{N}_{\rm r}(u_{\rm s}) = u_{\rm s}$ in the intervals where $\mathcal{N}$ is increasing, and $\mathcal{N}_{\rm r}(u_{\rm s}) = -u_{\rm s} + (2n+1/2)\pi$ in the intervals where $\mathcal{N}$ is decreasing, and select $\mathcal{N}_{\rm s}(u_{\rm b}) = u_{\rm b}$. The composite nonlinearity $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}$ is piecewise nondecreasing in Figure 12(e). Knowledge of only the monotonicity intervals of $\mathcal{N}$ is used to choose $\mathcal{N}_{\rm r}$. To construct $\mathcal{N}_{\rm b}$, note that the piecewise nondecreasing composite nonlinearity $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}$ satisfies $\mathcal{R}_{I_i}(\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}) \subset \mathcal{R}_{I_j}(\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s})$ for all $i \neq j$. Therefore, no additional information of $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}$ is needed. We let $\mathcal{N}_{\rm b}(u_{\rm sat}) = 0$ for $u_{\rm sat} < 0$, $\mathcal{N}_{\rm b}(u_{\rm sat}) = \pi/2$ for $u_{\rm sat} > \pi/2$ and $\mathcal{N}_{\rm b}(u_{\rm sat}) = u_{\rm sat}$ otherwise. Figure 12(f) shows that the composite nonlinearity $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}\circ\mathcal{N}_{\rm b}$ is nondecreasing.

We consider the single-tone sinusoidal command $r(k) = \sin\Omega_1 k$, where $\Omega_1 = \pi/5$ rad/sample. We let $n_{\rm c} = 10$, $P_0 = 0.1 I_{2n_{\rm c}}$, $\eta_0 = 0$, and $\tilde{\mathcal{H}} = H_1$. Figure 12(a) shows the time history of the performance z with the input nonlinearity present and z approaches zero in about 500 time steps. Figure 12(b) shows the input nonlinearity $\mathcal{N}$, (c) and (d) show the auxiliary nonlinearity $\mathcal{N}_{\rm r}$ and $\mathcal{N}_{\rm b}$. ■

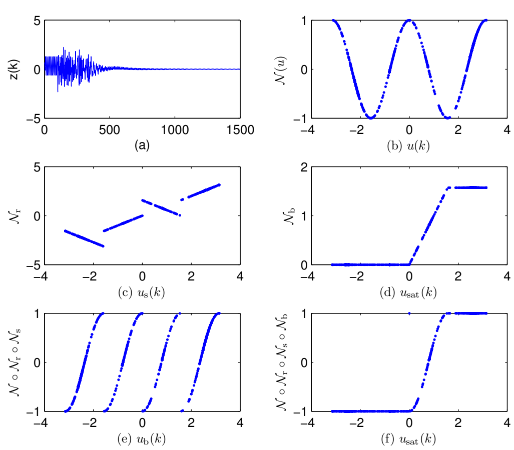


Fig. 12. Example 5.2. (a) shows that RCAC follows the sinusoidal command for the Hammerstein system. (b) shows the input nonlinearity $\mathcal{N}$, (c) and (d) show the auxiliary nonlinearities $\mathcal{N}_{\rm r}$ and $\mathcal{N}_{\rm b}$, (e) shows that the composite nonlinearity $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}$ is piecewise increasing, and (f) shows that the composite nonlinearity $\mathcal{N}\circ\mathcal{N}_{\rm r}\circ\mathcal{N}_{\rm s}\circ\mathcal{N}_{\rm b}$ is nondecreasing.

VI. CONCLUSIONS

Retrospective cost adaptive control (RCAC) was applied to a command-following problem for Hammerstein systems with non-monotonic input nonlinearities. RCAC was used with limited modeling information. In particular, RCAC uses knowledge of the first nonzero Markov parameter of the linear dynamics. To handle the effect of the non-monotonic nonlinearity, RCAC was augmented by auxiliary nonlinearities chosen based on the monotonicity properties of the input nonlinearity. The auxiliary nonlinearities combine with the input nonlinearity to form a composite nonlinearity that is globally nondecreasing. Simulation results show that RCAC is able to follow the commands for the Hammerstein systems with unknown disturbance when the composite input nonlinearity is globally nondecreasing.

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