2015 American Control Conference
Palmer House Hilton
July 1-3, 2015. Chicago, IL, USA

A Subsystem Identification Technique towards Battery State of Health Monitoring under State of Charge Estimation Errors

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Abstract— **Previous work framed the battery State of Health (SoH) monitoring problem as an inaccessible subsystem identification problem and conceived an approach to monitor SoH via side reaction current density estimation when State of Charge (SoC) is perfectly known. In practice, however, SoC is only estimated, and even an SoC estimation error of less than 1% can significantly undermine the accuracy of the SoH estimation. In this paper, the development of a new inaccessible subsystem identification technique, called the Two Step Filter, is presented in a linear setting to estimate the SoC error and SoH variable simultaneously and hence allow for SoH monitoring even under SoC estimation errors. The potential of the Two Step Filter is demonstrated on a linearized battery model example. The result shows that the filter can successfully track the side reaction current density despite the presence of an SoC estimation error of 1%.**

I. INTRODUCTION

Battery State of Health (SoH) is a critical input to battery management systems for balancing the trade-off between maximizing performance and minimizing degradation. However, SoH is not a physical quantity that can be directly measured. Thus, SoH is deduced from other quantities, which this paper refers to as SoH indicators.

Based on the selection of SoH indicators, the methods to obtain SoH can be divided into two categories. Most methods track degradation through its effects such as capacity fade or rising internal resistance [1], [2], [3], [4]. The simplicity of these methods is their key advantage. However, such effects can be inaccurate in representing the SoH as they are related not only to the battery SoH, but are also influenced by environmental conditions and use patterns [5]. The second category of methods estimates various electrochemical variables as SoH indicators [6], [7], [8], [9]. The benefit of using these variables is that they can uniquely indicate the level of degradation independent of the environmental conditions and use patterns. The main challenge of using electrochemical variables as SoH indicators is that these variables are available only from invasive and/or destructive methods, so are the inputs and outputs of the battery health system that governs the dynamics of these variables. Thus, estimating these electrochemical variables from only the commonly available measurements of terminal voltage, current, and temperature presents an inaccessible subsystem identification problem [9].

This material is based upon work supported by China-US Clean Energy Research Center.

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This work belongs to the second category and builds on the concept of tracking the side reaction current density as an indicator of battery SoH as first introduced in [9]. The key motivation for tracking side reaction current density is that it is a measure for the rate of cyclable Li-ion consumption due to all degradation mechanisms that are caused by side reactions [10]. Previous work addresses the challenge of noninvasive estimation by treating the battery health subsystem as an inaccessible subsystem of the overall battery system and leveraging the Retrospective-Cost Subsystem Identification (RCSI) [11], [7], [8] technique to identify the subsystem and the side reaction current density, under the assumption that battery State of Charge (SoC) is known perfectly [9]. However, an SoC estimation error on the level of 1% is often expected in practice [12], [3], [13].

Simulation results show that the SoH identification is sensitive to SoC estimation errors [9]. RCSI can correct for state estimation errors when the system is both controllable and observable; however, due to the negligible feedback from the side reaction current density to the battery electrochemical dynamics, the battery system is not controllable. Furthermore, the fact that there is a feedthrough from the side reaction current density to the terminal voltage [9] and that a battery is a marginally stable system due to its energy storing nature makes the SoC estimation errors persistent. Hence, a new approach is needed that can track the side reaction current density under SoC estimation errors for the method to be practical.

In this paper, a new inaccessible subsystem identification technique called the Two Step Filter is introduced for SoH monitoring to overcome the problems caused by persistent SoC estimation errors. Similar to RCSI, the system is divided into two parts: the Main System represents the part of the system that is known and the Subsystem refers to the part that is unknown and to be identified. In the battery case, they correspond to the electrochemical dynamics and the health subsystem, respectively. SoC estimation error is caused by an initialization error in one of the states of the Main System. Thus, the first step in the Two Step Filter is a modification of RCSI to take into account the Main System state error. In the second step, the estimation goal is expressed as a nonlinear function of both the battery health subsystem parameter and the Main System state initialization error. Then, the Modified Extended Kalman Filter (MEKF) [14] is used to estimate the unknown subsystem parameter and the Main System state error. To illustrate the performance of the Two Step Filter, a linearized battery model example is considered.

The rest of the paper is organized as follows. Section II introduces the development of the Two Step Filter in a generic, linear framework. Section III discusses the application of the Two Step Filter to the linearized battery model. Conclusions are given in Section IV.

II. The Development of the Two Step Filter

A. Problem Setup

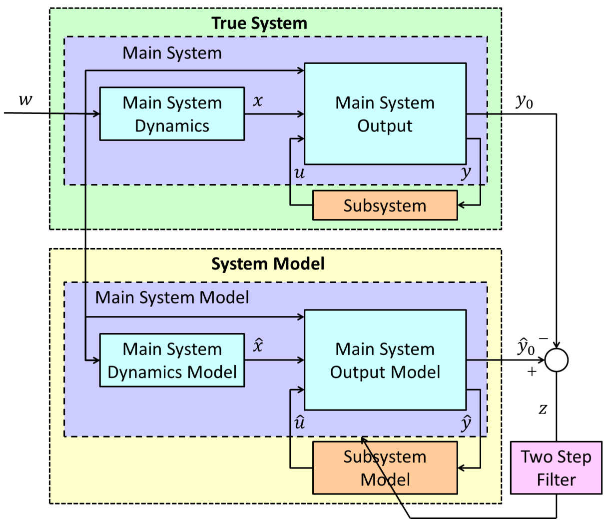


Fig. 1. The architecture for the Two Step Filter.

Fig. 1 shows the generic framework for the Two Step Filter. The True System consists of a known Main System and an unknown Subsystem. The dynamic equations and the output equations of the Main System are considered separately, because the Subsystem is in a closed-loop with only the output equations due to the assumption that the Subsystem output $u(k)$ does not affect the dynamics, but only the the system output $y_0(k)$ via direct feedthrough. The Main System Dynamics block is driven by the external excitation signal $w(k)$. The Main System Output block is coupled with the Subsystem via the variables $y(k)$ and $u(k)$.

The System Model part consists of the Main System Model and the Subsystem Model. The Main System Model block is assumed to be identical to the Main System block of the True System with the exception that there may be initialization errors in the states. The Subsystem Model block, on the other hand, has the same form as the Subsystem block in the True System, but its parameters are unknown.

Note that the input and output of the Subsystem is not directly measurable; i.e., the Subsystem is inaccessible. Instead, the output error $z(k) = \hat{y}_0(k) - y_0(k)$ is used to estimate the Subsystem parameters; hence, this is an inaccessible subsystem identification problem. The goal of the algorithm is to identify the unknown Subsystem under the presence of the Main System state estimation error $\hat{x}(k) - x(k)$.

B. The System

The equations of the Main System are

$$x(k+1) = Ax(k) + Fw(k), \tag{1}$$
$$y(k) = Cx(k) + Du(k) + Jw(k), \tag{2}$$
$$y_0(k) = E_1 x(k) + E_2 u(k) + E_3 w(k), \tag{3}$$

whereas the Subsystem is described by the equation

$$u(k) = \theta y(k), \tag{4}$$

where the parameter θ is unknown.

The features of this framework that are important for the context of this work are as follows:

1) There is no feedback from the Subsystem output, $u(k)$, into the Main System Dynamics. This architecture is motivated by the battery health problem where the health subsystem in a battery has a negligible impact on the SoC dynamics (i.e., Main System Dynamics).
2) The output $y_0(k)$ is a function of the Main System state $x(k)$ and the Subsystem output $u(k)$. This is motivated by the approximation in battery health problem that the effect of the health subsystem on the terminal voltage can be considered as a direct feedthough.
3) The Main System is marginally stable where matrix A is diagonal with at least one eigenvalue being 1. This property is due to the battery being an energy storage device.

C. Estimation Setup

It is assumed that the Main System is known and can be modeled accurately. Hence, the Main System Model is described by the following set of equations:

$$\hat{x}(k+1) = A\hat{x}(k) + Fw(k), \tag{5}$$
$$\hat{y}(k) = C\hat{x}(k) + D\hat{u}(k) + Jw(k), \tag{6}$$
$$\hat{y}_0(k) = E_1\hat{x}(k) + E_2\hat{u}(k) + E_3 w(k), \tag{7}$$

The Subsystem Model is assumed to have the same form as (4), but with an unknown parameter; i.e.,

$$\hat{u}(k) = \hat{\theta}(k)\hat{y}(k). \tag{8}$$

Therefore, the goal is to estimate the true Subsystem parameter θ with $\hat{\theta}(k)$ so that the Subsystem output $u(k)$ can be estimated.

Estimating θ and $u(k)$ with $\hat{\theta}$ using the structure of Fig. 1 is a challenge due to two reasons:

1) Due to the features 1) and 3) above, the main system state error $\hat{x}(k) - x(k)$ is persistent, which affects the estimation of $u(k)$.
2) Due to the feature 2), the difference $z(k) = \hat{y}_0(k) - y_0(k)$ can be caused by either the difference between $\hat{u}(k)$ and $u(k)$ or the state difference $\hat{x}(k) - x(k)$. The unique determination of $u(k)$ from only the measurement of $y_0(k)$ is not possible given that $x(k)$ is not measured.

These difficulties make the estimation of θ using RCSI challenging under the presence of main system state error. Hence, a new approach is described below.

D. The Two Step Filter

Assume that $A \in R^{n\times n}$ is organized as

$$A = \begin{bmatrix} I_m & 0 \\ 0 & \Lambda \end{bmatrix} \tag{9}$$

where I_m represents the identity matrix of dimension m ($m \leq n$) and

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{n-m} \end{bmatrix} \tag{10}$$

with $\lambda_i < 1$ for all i.

The main system state vector is $x = \begin{bmatrix} x_1(k) & \cdots & x_n(k) \end{bmatrix}^{\mathrm{T}}$ where the states $x_1, x_2, \cdots, x_m$ are such that their initialization errors are persistent. Define the persistent state vectors $\zeta(k)$ and $\hat{\zeta}(k)$ as

$$\zeta \triangleq \begin{bmatrix} x_1 & \cdots & x_m \end{bmatrix}^{\mathrm{T}}, \tag{11}$$

$$\hat{\zeta} \triangleq \begin{bmatrix} \hat{x}_1 & \cdots & \hat{x}_m \end{bmatrix}^{\mathrm{T}}. \tag{12}$$

Let d represent the constant main system state difference vector,

$$d \triangleq \hat{\zeta}(k) - \zeta(k) = \begin{bmatrix} \hat{x}_1(k) - x_1(k) \\ \vdots \\ \hat{x}_m(k) - x_m(k) \end{bmatrix} \in \mathbf{R}^m. \tag{13}$$

Let $\mathcal{E}_1$ and $\mathcal{C}$ consist of the elements of E_1 and C, respectively, that correspond to the persistent states:

$$\mathcal{E}_1 \triangleq \begin{bmatrix} E_{1,1} & \cdots & E_{1,m} \end{bmatrix} \in \mathbf{R}^{l_{y_0}\times m}, \tag{14}$$

$$\mathcal{C} \triangleq \begin{bmatrix} C_1 & \cdots & C_m \end{bmatrix} \in \mathbf{R}^{l_y\times m}. \tag{15}$$

where l_{y_0} and l_y are the length of y_0 and y, respectively.

1) The First Step: Assume that the initial estimation errors in all the asymptotically stable main system states diminish when time step is larger than a constant T. For any $k > T$, the error $z(k)$ is expressed as a function of $u(k)$, $\hat{u}(k)$ and d:

$$\begin{aligned} z(k) &= \hat{y}_0(k) - y_0(k) \\ &= \mathcal{E}_1\left[\hat{\zeta}(k) - \zeta(k)\right] + E_2\left[\hat{u}(k) - u(k)\right] \\ &= \mathcal{E}_1 d + E_2\left[\hat{u}(k) - u(k)\right]. \end{aligned} \tag{16}$$

Next, a cost function is formulated:

$$J(u^s(k)) = z^{s\mathrm{T}}(k) R_z z^s(k) + u^{s\mathrm{T}}(k) R_u u^s(k), \tag{17}$$

where R_z and R_u are tunable positive semi-definite weights; the substituted $z(k)$, $z^s(k)$, is defined such that $\hat{u}(k)$ in $z(k)$ is replaced by any substitute $u^s(k)$,

$$z^s(k) \triangleq z(k) - \mathcal{E}_1 d - E_2\hat{u}(k) + E_2 u^s(k). \tag{18}$$

The optimal u, $u^*(k)$, is defined to be the minimizer of $J(u^s(k))$. When $R_u = 0$, the minimizer of $J(u^s(k))$ also minimizes $z^s(k)$ given that the effect of d in $z(k)$ is not compensated by $u^*(k)$. Let $z^*(k)$ denote the $z^s(k)$ that corresponds to $u^*(k)$; i.e.,

$$z^*(k) = z(k) - \mathcal{E}_1 d - E_2\hat{u}(k) + E_2 u^*(k). \tag{19}$$

Substitute (18) into (17) and find $u^*(k)$:

$$\begin{aligned} \left.\frac{\partial J}{\partial u^s}\right|_{u^s = u^*(k)} &= 2u^{*\mathrm{T}}(k)\left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right) + \\ & 2\left(z(k) - \mathcal{E}_1 d - E_2\hat{u}(k)\right)^{\mathrm{T}} R_z E_2 = 0. \end{aligned} \tag{20}$$

Note that $z(k) \in \mathbf{R}^{l_{y_0}}$ and $E_2{}^{\mathrm{T}} R_z E_2 + R_u$ is symmetric.

The solution to (20) is

$$\begin{aligned} u^*(k) = &-\left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \\ &\left[z(k) - \mathcal{E}_1 d - E_2\hat{u}(k)\right]. \end{aligned} \tag{21}$$

The terms that can be constructed from the measurable signal $z(k)$ and estimated signal $\hat{u}(k)$ are lumped into the variable $\tilde{u}(k)$:

$$\begin{aligned} \tilde{u}(k) \triangleq &-\left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \\ &\left[z(k) - E_2\hat{u}(k)\right]. \end{aligned} \tag{22}$$

Therefore,

$$\tilde{u}(k) = u^*(k) - \left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \mathcal{E}_1 d. \tag{23}$$

The goal of the first step is to calculate $\tilde{u}(k)$ from $z(k)$ and $\hat{u}(k)$ as in (22).

2) The Second Step: Ideally it is desired that $z^*(k)$ converges to zero. By substituting (16) into (19), it can be shown that $u^*(k) = u(k)$ when $z^*(k) = 0$. Therefore, $u^*(k)$ in (23) can be constructed as

$$u^*(k) = \theta y(k). \tag{24}$$

For $k > T$, express $y(k)$ using the estimated signals $\hat{y}(k)$ and $\hat{u}(k)$:

$$\begin{aligned} y(k) &= Cx(k) + Du(k) + Jw(k) \\ &= \hat{y}(k) - \mathcal{C}d - D\left(\hat{u}(k) - u(k)\right) \\ &= \left[\hat{y}(k) - D\hat{u}(k)\right] - \mathcal{C}d + D\theta y(k). \end{aligned} \tag{25}$$

Therefore,

$$\begin{aligned} y(k) = &\left(I_{l_y} - D\theta\right)^{-1}\left[\hat{y}(k) - D\hat{u}(k)\right] \\ &- \left(I_{l_y} - D\theta\right)^{-1}\mathcal{C}d. \end{aligned} \tag{26}$$

Substitute (24) and (26) into (23) to obtain

$$\begin{aligned} \tilde{u}(k) = &\,\theta\left(I_{l_y} - D\theta\right)^{-1}\left(\hat{y}(k) - D\hat{u}(k)\right) \\ &- \theta\left(I_{l_y} - D\theta\right)^{-1}\mathcal{C}d \\ &- \left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \mathcal{E}_1 d. \end{aligned} \tag{27}$$

(27) constructs $\tilde{u}(k)$ with the unknown parameters θ and d. Now, any nonlinear estimation method can be used to estimate θ and d from $\tilde{u}(k)$ calculated in the first step. In this paper, the Modified Extended Kalman Filter (MEKF) [14] is used in the second step to estimate θ and d.

When the subsystem is SISO and only one of the eigenvalues of A is 1, the MEKF algorithm for (27) is as presented below.

Define the parameter vector as $\omega \triangleq \begin{bmatrix} \theta & d \end{bmatrix}^{\mathrm{T}}$. The estimation of the parameter vector is defined to be $\hat{\omega}(k) \triangleq \begin{bmatrix} \hat{\theta}(k) & \hat{d}(k) \end{bmatrix}^{\mathrm{T}}$. Let $P(k)$ be the covariance matrix of $\hat{\omega}(k)$. Then,

$$\hat{\omega}(k+1) = \hat{\omega}(k) + K(k)\left[\tilde{u}(k) - \hat{\tilde{u}}(\hat{\omega}(k))\right], \tag{28}$$

$$P(k+1) = (\alpha + 1)\left[P(k) - K(k)H(k)P(k) + Q(k)\right], \tag{29}$$

where

$$\hat{\tilde{u}}(\hat{\omega}(k)) = \frac{\hat{\theta}(k)}{1 - D\hat{\theta}(k)}(\hat{y}(k) - D\hat{u}(k)) - \mathcal{C}\frac{\hat{\theta}(k)\hat{d}(k)}{1 - D\hat{\theta}(k)} - \left(E_2{}^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \mathcal{E}_1 \hat{d}(k), \tag{30}$$

$$H(k) = \left.\frac{\partial \tilde{u}(\omega)}{\partial \omega}\right|_{\omega=\hat{\omega}(k)} = \begin{bmatrix} \frac{1}{\left(1-D\hat{\theta}(k)\right)^2}\left(\hat{y}(k) - D\hat{u}(k) - \mathcal{C}\hat{d}(k)\right) \\ -\frac{\mathcal{C}\hat{\theta}(k)}{1-D\hat{\theta}(k)} - \left(E_2^{\mathrm{T}} R_z E_2 + R_u\right)^{-1} E_2{}^{\mathrm{T}} R_z \mathcal{E}_1 \end{bmatrix}^{\mathrm{T}}, \tag{31}$$

$$K(k) = P(k) H^{\mathrm{T}}(k)\left(H(k) P(k) H^{\mathrm{T}}(k) + R_t(k)\right)^{-1}. \tag{32}$$

$Q(k)$ is the process noise covariance matrix. $R_t(k)$ is the output noise covariance matrix. $\alpha \in [0,1]$ acts like a forgetting factor with $\alpha + 1 = \frac{1}{\lambda}$ where λ is the forgetting factor.

E. Estimation of the Subsystem Output

Because there is a persistent difference between $\hat{x}(k)$ and $x(k)$, the input to the subsystem model $\hat{y}(k)$ will not converge to the true subsystem input $y(k)$. Therefore, even when $\hat{\theta}(k)$ converges to θ, the output of the subsystem model $\hat{u}(k)$ will not converge to the subsystem output $u(k)$ unless a correction based on $\hat{d}(k)$ is introduced.

To estimate the subsystem output $u(k)$, $\hat{y}(k)$ should be corrected with $\hat{d}(k)$. Let $\hat{u}'(k)$ and $\hat{y}'(k)$ denote the corrected estimates of the subsystem output $u(k)$ and subsystem input $y(k)$, respectively, which can be derived as follows.

The difference between $\hat{y}(k)$ and $y(k)$ is given as

$$\hat{y}(k) - y(k) = \mathcal{C}d + D(\hat{u}(k) - u(k)) = \mathcal{C}d + D\left(\hat{\theta}(k)\hat{y}(k) - \theta y(k)\right) \tag{33}$$

Solving for $y(k)$ yields

$$y(k) = \left(I_{l_y} - D\theta\right)^{-1}\left[\left(I_{l_y} - D\hat{\theta}(k)\right)\hat{y}(k) - \mathcal{C}d\right]. \tag{34}$$

Replacing d and θ with their estimates $\hat{d}$ and $\hat{\theta}$, we obtain

$$\hat{y}'(k) = \hat{y}(k) - \left(I_{l_y} - D\hat{\theta}(k)\right)^{-1}\mathcal{C}\hat{d}(k). \tag{35}$$

Then,

$$\hat{u}'(k) = \hat{\theta}\hat{y}'(k) = \hat{\theta}(k)\left[\hat{y}(k) - \left(I_{l_y} - D\hat{\theta}(k)\right)^{-1}\mathcal{C}\hat{d}(k)\right]. \tag{36}$$

III. Linearized Battery Model Example

In this section, the Two Step Filter is used in a linearized battery model example to estimate the subsystem parameter and the persistent main system state error, which pertain to the battery SoH and SoC estimation error, respectively. The subsystem parameter is also estimated with RCSI. The advantage of the Two Step Filter is highlighted with a comparison between the estimation results of the Two Step Filter and those of RCSI.

The battery model used in this section is the linearization of an electrochemical-based battery model around a particular operating point. Both the electrochemical-based battery model and its linearized version have the same form as the True System in Fig. 1.

The nonlinear battery model is based on [9], [6] and is summarized here briefly. The main system states in the battery model follow linear dynamics:

$$x(k+1) = Ax(k) + FI(k), \tag{37}$$

where $x(k) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^2$ is the main system state, and $I(k)$ is the input current. A is diagonal with the eigenvalue associated with the first state being 1. There is no term containing the subsystem output $u(k)$ in (37). Hence, the initialization error in the first state is persistent. This state is used for the calculation of the solid-electrolyte interphase concentration as follows:

$$c_{se,n}(k) = C_{se,n}x(k) + D_{se,n}I(k), \tag{38}$$

$$c_{se,p}(k) = \frac{1}{\epsilon_p L_p A_p}\left[n_{Li} - \epsilon_n L_n A_n c_{se,n}(k)\right], \tag{39}$$

where $c_{se,n}(k)$ and $c_{se,p}(k)$ are the solid-electrolyte interphase concentration in the anode and cathode, respectively. $c_{se,n}(k)$ is a measurement of battery SoC; therefore, the persistent error in the initialization of the first state leads to a persistent error in SoC.

The output of the system is the terminal voltage $V(k)$ and is given by

$$\begin{aligned} V(k) = {} & U_{\mathrm{ref},p}(c_{se,p}(k)) - U_{\mathrm{ref},n}(c_{se,n}(k)) \\ & + \frac{RT}{\alpha F}\ln\Big(\xi_p(c_{se,p}(k), I(k)) + \sqrt{\xi_p^2(c_{se,p}(k), I(k)) + 1}\Big) \\ & - \frac{RT}{\alpha F}\ln\Big(\xi_n(c_{se,n}(k), I(k)) + \sqrt{\xi_n^2(c_{se,n}(k), I(k)) + 1}\Big) - R_{int}I(k). \end{aligned} \tag{40}$$

The input $y(k)$ to the health subsystem is

$$y(k) = e^{\frac{RT}{\alpha F}\left[\eta_n(c_{se,n}(k), I(k), J_{sd}(k)) + U_{\mathrm{ref},n}(c_{se,n}(k))\right]}. \tag{41}$$

The subsystem has the following linear static form:

$$u(k) = -i_{0,sd}a_{s,n}e^{\frac{\alpha F}{RT}U_{\mathrm{ref},sd}}y(k). \tag{42}$$

The details of the battery model and the nomenclature are omitted here due to limited space, but can be found in [9], [6].

Define $w(k) = I(k)$, $y_0(k) = V(k)$ and $u(k) = J_{sd}(k)$. Then, the nonlinear model can be written as

$$x(k+1) = Ax(k) + Fw(k), \tag{43}$$

$$y(k) = f(x(k), u(k), w(k)), \tag{44}$$

$$y_0(k) = g(x(k), u(k), w(k)), \tag{45}$$

$$u(k) = \theta y(k). \tag{46}$$

Compared with the model in [9], this model contains no feedback of $u(k)$ in the main system dynamics (43). This change is justified due the negligible influence of $u(k)$ on the states compared with the influence of the external input $w(k)$. This change makes the model suitable for the Two Step Filter.

The battery model (43) - (46) can be linearized around different points for given SoC $(x(k))$ and current $(w(k))$,

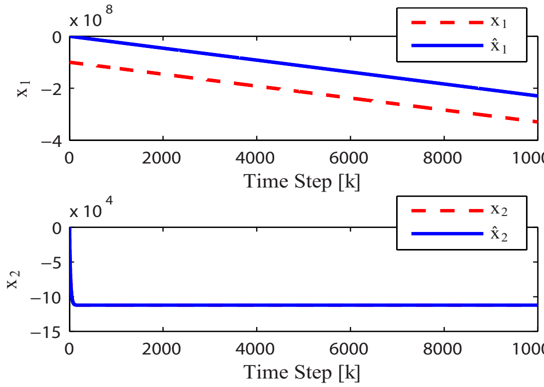


Fig. 2. The main system states in the true system and the system model.

leading to a linear system with the form of (1) - (4). Here, as an example, the battery model is linearized around the point $SoC = 0.7$ with $0.1C$ constant charge current. The resulting parameters are:

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0 \\ 0 & 0.96 \end{bmatrix}, \\ F &= \begin{bmatrix} 9.98\times 10^4 & 2.13\times 10^4 \end{bmatrix}, \\ C &= \begin{bmatrix} 5.41\times 10^{-10} & 5.65\times 10^{-9} \end{bmatrix}, \\ D &= \left[1.21\times 10^{-5}\right], J = [1.35], \\ E_1 &= \begin{bmatrix} -5.10\times 10^{-12} & -5.33\times 10^{-11} \end{bmatrix}, \\ E_2 &= \left[1.19\times 10^{-7}\right], E_3 = \left[-1.38\times 10^{-2}\right], \\ \theta &= -1862.16. \end{aligned}$$

The Main System state is initialized at $x(1) = \begin{bmatrix} -1\times 10^8 & -60 \end{bmatrix}^{\mathrm{T}}$, which corresponds to a 1% error in the SoC with respect to the operating point of $SoC = 0.7$. The Main System Model state is initialized at $\hat{x}(1) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$ for both the Two Step Filter and RCSI. The initial error in the first state is persistent, thus $d = \hat{x}_1(1) - x_1(1) = 1\times 10^8$. Note that there is also an initial error in the second state. However, the second state is asymptotically stable and the error diminishes with time. Fig. 2 presents both the true Main System state $x(k)$ and the Main System Model state $\hat{x}(k)$. It confirms that only the error in the first state is persistent.

Therefore in the Two Step Filter, the true parameter vector is $\omega = \begin{bmatrix} \theta & d \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} -1862.16 & 1\times 10^8 \end{bmatrix}^{\mathrm{T}}$. The estimated parameter vector with the Two Step Filter is initialized at $\hat{\omega}(1) = \begin{bmatrix} \hat{\theta}(1) & \hat{d}(1) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$. The parameters in the Two Step Filter are set as follows: the weights in the cost function of the first step are set to be $R_z = I_1$ and $R_u = 0$; the initial covariance matrix is set to be $P_0 = 10 I_2$; the parameters in the MEKF are set to be $Q(k) = 10^{-2} I_2$, $R_t(k) = 10^2 I_1$ and $\alpha = 0.01$.

The details and definitions of RCSI parameters can be found in [9]. RCSI estimates only the subsystem parameter θ with $\hat{\theta}_{\mathrm{RCSI}}$, which is also initialized at $\hat{\theta}_{\mathrm{RCSI}}(1) = 0$. The parameters in RCSI are set as follows: the weights of the cost function are $R_z = I_1$ and $R_u = 0$; the Markov parameter is set to be $H_0 = E_2$; initial covariance matrix is set to be $P_0 = 10^2 I_2$; the parameters in Kalman Filter are set to be $Q = I_1$, $R_k = 0.5 I_1$ and $R_1 = 0$.

Assuming a nominal battery capacity of 2.3Ah, a constant charge current of 0.1C corresponds to an external input of $w(k) = -0.23$.

To quantify the accuracy of the estimation, the following relative estimation errors with the Two Step Filter are defined:

$$\Delta\theta(k) \triangleq \frac{\hat{\theta}(k) - \theta}{\theta} \times 100\%, \tag{47}$$

$$\Delta d(k) \triangleq \frac{\hat{d}(k) - d}{d} \times 100\%, \tag{48}$$

$$\Delta u(k) \triangleq \frac{\hat{u}'(k) - u(k)}{u(k)} \times 100\%. \tag{49}$$

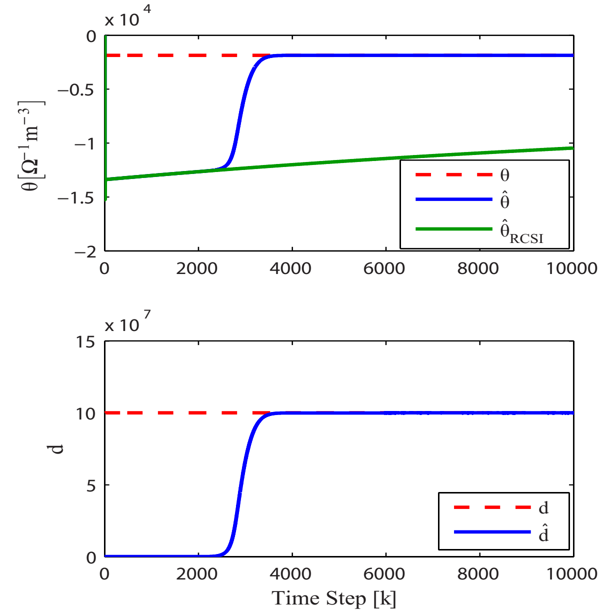
The relative estimation errors for the subsystem parameter and the subsystem output with RCSI are defined similarly and denoted by $\Delta\theta_{\mathrm{RCSI}}$ and Δu_{RCSI}, respectively.

Fig. 3 (a) presents the estimation results of $\omega = \begin{bmatrix} \theta & d \end{bmatrix}^{\mathrm{T}}$. It shows that the Two Step Filter estimations $\hat{\theta}(k)$ and $\hat{d}$ can be far away from their true values when k is small, but converge to θ and d when $k > 2000$. The RCSI estimation $\hat{\theta}_{\mathrm{RCSI}}$ under the same main system state estimation error is also illustrated. $\hat{\theta}$ and $\hat{\theta}_{\mathrm{RCSI}}$ are close before $k = 2000$, but $\hat{\theta}_{\mathrm{RCSI}}$ does not converge to the true θ after $k > 2000$. Fig. 3 (b) presents the estimation of the subsystem output $u(k) = J_{sd}(k)$. The Two Step Filter estimation $\hat{u}'(k)$ can track the change in $u(k)$ after 2000 steps, while a constant bias is present between the RCSI estimation $\hat{u}_{\mathrm{RCSI}}$ and u. Fig. 3 (c) shows the relative estimation errors of the Two Step Filter when $k > 3500$ and those of RCSI when $k > 10$. It can be observed that $|\Delta\theta(k)|$, $|\Delta d(k)|$ and $|\Delta u(k)|$ are all smaller than 0.5% when $k > 3500$ for the Two Step Filter, while both $\Delta\theta_{\mathrm{RCSI}}$ and Δu_{RCSI} are larger than 450% for RCSI.

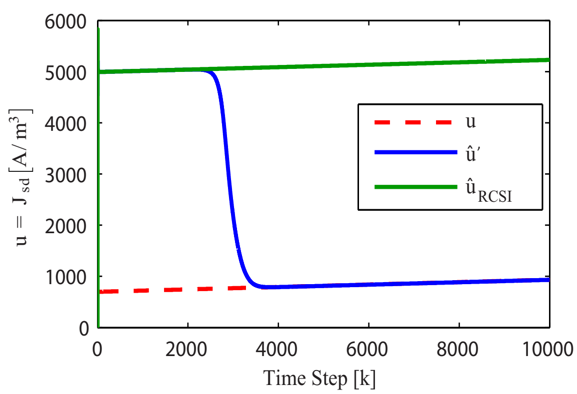
IV. Conclusion

In this paper, a new subsystem identification algorithm, the Two Step Filter, is developed. The Two Step Filter can estimate both the subsystem parameter and the persistent main system state estimation error simultaneously for marginally stable systems as exemplified by the SoH monitoring problem in batteries when SoC estimation errors are present.

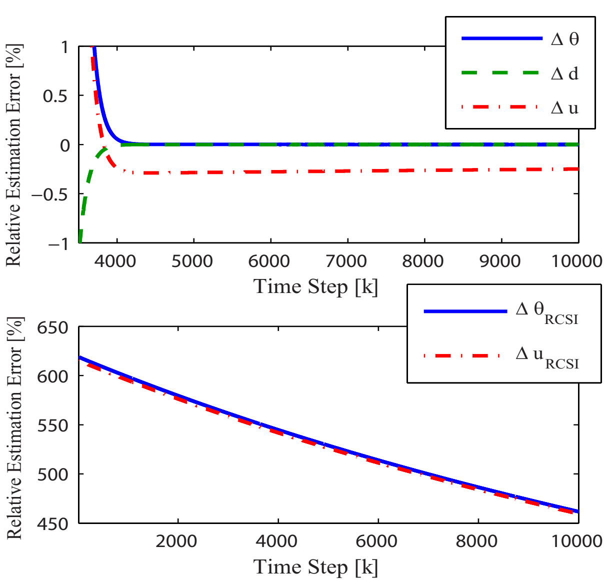
A linearized battery model example is presented as an application of the Two Step Filter. The simulation results show that with the Two Step Filter the relative estimation errors of the subsystem parameter, main system state error and the subsystem output are all within $\pm 0.5\%$ after 3500 steps, while the relative estimation errors with RCSI are larger than 450% for the same example under the same persistent Main System state estimation errors. The example demonstrates that the Two Step Filter can not only significantly improve the accuracy of subsystem estimation under the presence of persistent Main System state estimation errors, but also provide an estimation of these errors.



(a) Estimation of θ and d.



(b) Estimation of the side reaction current density, $u(k)$.



(c) Relative estimation errors.

Fig. 3. The estimation results in the linearized battery model.

In this paper, MEKF is used in the second step as the nonlinear estimation method. One alternative estimation method to MEKF is the Unscented Kalman Filter (UKF) [15], which will be included in a future study.

The results in this paper show the potential of the Two Step Filter in accurately identifying the battery health subsystem under the influence of the SoC estimation error. Future work will focus on the application of the Two Step Filter to the nonlinear battery model.

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