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Retrospective Cost Adaptive Control with Concurrent Closed-Loop Identification of Time-Varying Nonminimum-Phase Zeros

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Abstract— **We consider retrospective cost adaptive control (RCAC) of a plant whose NMP zeros are time-dependent. The goal is to estimate the NMP zeros and replicate the estimated NMP zeros in the target model used by RCAC. This problem is challenging due to the fact that the estimates of the locations of the NMP zeros must be sufficiently accurate at each instant of time so that the target model can correctly influence the controller adaptation. We use closed-loop identification to estimate the location of the NMP zeros. In order to enhance the accuracy of the estimation, we inject an additional noise term in order to improve persistency of the control signal. Numerical examples show the feasibility and performance of the overall approach.**

I. Introduction

The reason for feedback control is uncertainty, which may entail initial conditions, disturbances, and plant dynamics. For linear plants, uncertainty in initial conditions and disturbances can be addressed by LQG theory, which provides optimal H_2 performance assuming correctly modeled dynamics and disturbance characteristics. Uncertainty in the plant dynamics can be dealt with by robust control so long as robust stabilization can be guaranteed for the prior uncertainty level, albeit at the expense of performance degradation. Adaptive control seeks to overcome the robustness/performance tradeoff by adjusting the controller coefficients based on the response of the plant during closed-loop operation. Although there is no precise definition of adaptive control, it is reasonable to think of an adaptive controller as a nonlinear controller that aims to overcome the robustness/performance tradeoff due to uncertainty in the plant dynamics and disturbance characteristics.

A fundamental question in adaptive control is determining which modeling information is essential for achieving acceptable transient and asymptotic performance. For retrospective cost adaptive control (RCAC), in the SISO case, the required modeling information consists of the sign of the leading numerator coefficient and knowledge of the nonminimum-phase (NMP) zeros, if any [3, 6, 7]. The MIMO case is more complicated and is not discussed here; for details, see [9].

The present paper focuses on the following problem: How can RCAC account for plant dynamics with unknown, time-dependent NMP zeros? The starting point for addressing this problem is the approach of [8], where the location of the NMP zeros is estimated during operation, and these estimates are replicated in the target model used to define the retrospective cost. As shown in [6], RCAC matches a specific closed-loop transfer function to the target model. Since NMP zeros are not moved by feedback, failure to replicate the NMP zeros in the target model induces RCAC to cancel these zeros by placing controller poles at those locations, which leads to instability. The approach of [8] shows that online identification of the NMP zeros is feasible, thus providing the means for avoiding unstable pole-zero cancellation.

The present paper goes beyond [8] by considering the scenario where the NMP zeros are time-dependent. In this case, the goal is to estimate the NMP zeros and then replicate the latest estimate of the NMP zeros in the target model. In order for this approach to succeed, the estimates of the NMP zeros must be sufficiently accurate so that the target model can correctly influence the controller adaptation.

In the present paper we use closed-loop identification to estimate the NMP zeros. To enhance the accuracy of the estimation, we inject an additional noise term in order to enhance the persistency of the control signal. Since this additional noise term degrades performance, the level of the noise term must be chosen to provide the required estimation accuracy while minimizing the adverse effect.

In the next section we state the adaptive control problem, and then we review RCAC. Next we present the closed-loop identification technique. Three numerical examples are considered. The first example is a second-order MP plant in which the sign of the control suddenly changes. The second example involves a second-order NMP plant in which the NMP zero is time-dependent. The third example is a fourth-order model of lateral aircraft dynamics that transition from MP to NMP. For all three examples the goal is to concurrently estimate the NMP zeros (if any) and the sign of the first Markov parameter, and use that estimate within RCAC so as to avoid unstable pole/zero cancellation.

II. RCAC Formulation

Consider the linear time-varying SIMO discrete-time system

$$x(k+1) = A(k)x(k) + B(k)u(k) + D_1 w(k), \quad (1)$$

$$y(k) = C(k)x(k) + D_2 w(k), \quad (2)$$

$$z(k) = E_1(k)x(k) + E_0 w(k), \quad (3)$$

where $k \geq 0$, $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}$ is the control input, $y(k) \in \mathbb{R}^{l_y}$ is the available measurements, $z(k) \in \mathbb{R}$ is the performance variable, and $w(k) \in \mathbb{R}^{l_w}$ is the exogenous

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signal, which can represent either a command signal to be followed, a disturbance to be rejected, or both.

The control-to-performance time-domain transfer matrix at time k is defined by

$$G_{zu,k}(\mathbf{q}) \triangleq E_1(k)(\mathbf{q}I_n - A(k))^{-1}B(k), \tag{4}$$

where $\mathbf{q}$ is the forward shift operator. Furthermore, for all $i \geq 1$, the i^{th} Markov parameter at time k of $G_{zu,k}$ is defined by

$$H_i(k) \triangleq E_1(k)A(k)^{i-1}B(k). \tag{5}$$

A. Control Law

The adaptive controller is constructed as a strictly proper time-series MISO dynamic compensator of order n_{c}, with input $\xi(k) \in \mathbb{R}^{l_\xi}$, such that the control $u(k)$ is given by

$$u(k) = \sum_{i=1}^{n_c} P_i(k)u(k-i) + \sum_{i=1}^{n_c} Q_i(k)\xi(k-i), \tag{6}$$

where $P_i(k) \in \mathbb{R}$ and $Q_i(k) \in \mathbb{R}^{1\times l_\xi}$ are the controller coefficients. The transfer function of the controller from ξ to u is given by

$$G_{\mathrm{c},k}(\mathbf{q}) = \left(\mathbf{q}^{n_{\mathrm{c}}} - \mathbf{q}^{n_{\mathrm{c}}-1}P_1(k) - \cdots - P_{n_{\mathrm{c}}}(k)\right)^{-1} \cdot \left(\mathbf{q}^{n_{\mathrm{c}}-1}Q_1(k) + \cdots + Q_{n_{\mathrm{c}}}(k)\right). \tag{7}$$

The controller (6) can be expressed as

$$u(k) = \phi(k)\theta(k), \tag{8}$$

where the regressor matrix $\phi(k)$ and controller coefficient matrix $\theta(k)$ are defined by

$$\phi(k) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n_{\mathrm{c}}) \\ \xi(k-1) \\ \vdots \\ \xi(k-n_{\mathrm{c}}) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{1\times l_\theta},$$

$$\theta(k) \triangleq \mathrm{vec}\left(\begin{bmatrix} P_1(k)\cdots P_{n_{\mathrm{c}}}(k) \; Q_1(k)\cdots Q_{n_{\mathrm{c}}}(k) \end{bmatrix}\right) \in \mathbb{R}^{l_\theta},$$

where $l_\theta \triangleq n_{\mathrm{c}}(l_\xi + 1)$ and ξ are the desired feedback variables.

B. Retrospective Performance Variable

We define the retrospective control as

$$\hat{u}(k,\hat{\theta}) \triangleq \phi(k)\hat{\theta}, \tag{9}$$

where $\hat{\theta} \in \mathbb{R}^{l_\theta}$ is determined by the optimization in Section II-C. The corresponding retrospective performance variable is defined as

$$\hat{z}(k,\hat{\theta}) \triangleq z(k) + G_{\mathrm{f}}(\mathbf{q})[\hat{u}(k,\hat{\theta}) - u(k)], \tag{10}$$

where the target model $G_{\mathrm{f}}(\mathbf{q})$ is an infinite impulse response (IIR) filter written as

$$G_{\mathrm{f}}(\mathbf{q}) = \frac{\hat{N}_{n_{\mathrm{f}}}(k)\mathbf{q}^{n_{\mathrm{f}}-1} + \cdots + \hat{N}_1(k)}{\mathbf{q}^{n_{\mathrm{f}}} + D_{n_{\mathrm{f}}}\mathbf{q}^{n_{\mathrm{f}}-1} + \cdots + D_1} = \frac{\hat{N}_{\mathrm{f}}(\mathbf{q})}{D_{\mathrm{f}}(\mathbf{q})}, \tag{11}$$

where the target-model order $n_{\mathrm{f}} \geq 1$, and $\hat{N}_i, D_i \in \mathbb{R}$ for all $1 \leq i \leq n_{\mathrm{f}}$. The filter coefficient vectors in (11) for the denominator and numerator are defined by

$$D_{\mathrm{f}} \triangleq \begin{bmatrix} 1 \\ D_{n_{\mathrm{f}}} \\ \vdots \\ D_1 \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}}+1}, \; \hat{N}_{\mathrm{f}}(k) \triangleq \begin{bmatrix} \hat{N}_{n_{\mathrm{f}}}(k) \\ \vdots \\ \hat{N}_1(k) \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{f}}}, \tag{12}$$

respectively, where the numerator of the target model is updated by the closed-loop identification performed in Section III. Next, by defining $\hat{Z}(k,\hat{\theta})$, $Z(k) \in \mathbb{R}^{n_{\mathrm{f}}+1}$, $U(k) \in \mathbb{R}^{n_{\mathrm{f}}}$, and $\Phi(k) \in \mathbb{R}^{n_{\mathrm{f}}\times l_\theta}$ by

$$\hat{Z}(k,\hat{\theta}) \triangleq \begin{bmatrix} \hat{z}(k,\hat{\theta}) \\ \vdots \\ \hat{z}(k-n_{\mathrm{f}}+1,\hat{\theta}) \end{bmatrix}, \; Z(k) \triangleq \begin{bmatrix} z(k) \\ \vdots \\ z(k-n_{\mathrm{f}}+1) \end{bmatrix},$$

$$U(k) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n_{\mathrm{f}}) \end{bmatrix}, \quad \Phi(k) \triangleq \begin{bmatrix} \phi(k-1) \\ \vdots \\ \phi(k-n_{\mathrm{f}}) \end{bmatrix},$$

and

$$\hat{z}_{\mathrm{f}}(k,\hat{\theta}) \triangleq D_{\mathrm{f}}^{\mathrm{T}}\hat{Z}(k,\hat{\theta}), \tag{13}$$

$$z_{\mathrm{f}}(k) \triangleq D_{\mathrm{f}}^{\mathrm{T}}Z(k), \tag{14}$$

$$\phi_{\mathrm{f}}(k) \triangleq \hat{N}_{\mathrm{f}}^{\mathrm{T}}(k)\Phi(k), \tag{15}$$

$$u_{\mathrm{f}}(k) \triangleq \hat{N}_{\mathrm{f}}^{\mathrm{T}}(k)U(k), \tag{16}$$

we can rewrite (10) as

$$\hat{z}_{\mathrm{f}}(k,\hat{\theta}) = z_{\mathrm{f}}(k) + [\phi_{\mathrm{f}}(k)\hat{\theta} - u_{\mathrm{f}}(k)]. \tag{17}$$

The target model G_{f} sets target locations for the closed-loop poles [6]. Since G_{f} is an IIR target model, the closed-loop pole locations are attracted to the roots of $D_{\mathrm{f}}(\mathbf{q})$. Also, in [6], the numerator of the ideal G_{f} is chosen to include the NMP zeros of the plant. This feature ensures that the NMP zeros of the plant are not canceled by the poles of G_{c}. In the present paper, we replicate estimates of the first nonzero Markov parameter and all NMP zeros obtained from the closed-loop identification in the numerator of the target model. The target model having "free parameters" in the numerator is similar to that proposed in [2], where the authors tune the numerator of the reference model to ensure no unstable pole-zero cancellations.

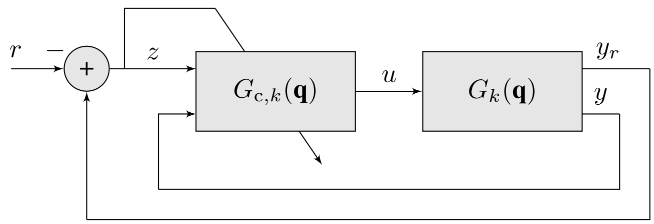

Fig. 1: The adaptive command-following problem.

C. Retrospective Cost

Consider the retrospective cost function

$$J(k,\hat{\theta}) = \sum_{i=1}^{k} \lambda^{k-i} \Big[(\hat{z}_{\mathrm{f}}(i,\hat{\theta}) R_z \hat{z}_{\mathrm{f}}(i,\hat{\theta}) + (\phi(i)\hat{\theta})^{\mathrm{T}} R_u (\phi(i)\hat{\theta}) + (\hat{\theta} - \theta(i-1))^{\mathrm{T}} R_\delta (\hat{\theta} - \theta(i-1)) \Big], \quad (18)$$

where $\lambda \in (0,1]$ is the forgetting factor, $R_z, R_u \in \mathbb{R}$ and $R_\delta \in \mathbb{R}^{l_\theta \times l_\theta}$ are positive-definite weighting matrices. The unique global minimizer $\hat{\theta}$ of (18) satisfies

$$\hat{\theta} = -A_\theta(k)^{-1} b_\theta(k), \quad (19)$$

where

$$\begin{aligned} A_\theta(k) &= \sum_{i=1}^{k} \lambda^{k-i} \left[\phi_{\mathrm{f}}(i)^{\mathrm{T}} R_z \phi_{\mathrm{f}}(i) + \phi(i)^{\mathrm{T}} R_u \phi(i) + R_\delta \right], \\ &= \lambda A_\theta(k-1) + \phi_{\mathrm{f}}(k)^{\mathrm{T}} R_z \phi_{\mathrm{f}}(k) + \phi(k)^{\mathrm{T}} R_u \phi(k) + R_\delta, \\ b_\theta(k) &= \sum_{i=1}^{k} \lambda^{k-i} \left[\phi_{\mathrm{f}}(i)^{\mathrm{T}} R_z (z_{\mathrm{f}}(i) - u_{\mathrm{f}}(i) - R_\delta \theta(i-1) \right], \\ &= \lambda^{k-i} b_\theta(k-1) + \phi_{\mathrm{f}}(k)^{\mathrm{T}} R_z \psi_{\mathrm{f}}(k) - R_\delta \theta(k-1), \\ \psi_{\mathrm{f}}(k) &= z_{\mathrm{f}}(k) - u_{\mathrm{f}}(k). \end{aligned}$$

Note that $A_\theta(k) \in \mathbb{R}^{l_\theta \times l_\theta}$, $b_\theta(k) \in \mathbb{R}^{l_\theta}$ and $A_\theta(0) = R_\delta$.

III. Closed-Loop Plant Identification

We consider the adaptive command-following problem shown in Fig. 1, where

$$G_k = \begin{bmatrix} G_{y_r u,k} \\ G_{yu,k} \end{bmatrix}. \quad (20)$$

This problem is a special case of (1) – (3), where E_1 is a row of C that produces the output y_r, $D_1 = D_2 = 0$, and $E_0 = -1$. The exogenous signal w is the command r, and $z = y_r - r$.

The objective is to identify $G_{y_r u,k}$, which is the transfer function from the controller input u to the output y_r. Estimates of the first nonzero Markov parameter and all NMP zeros of $G_{y_r u,k}$ are used to update the numerator coefficients of the target model G_{f}.

Consider the input-output model of $G_{y_r u,k}$

$$y_r(k) = \sum_{j=1}^{n} a_j(k) u(k-j) - \sum_{j=1}^{n} b_j(k) y_r(k-j), \quad (21)$$

where n is the plant order. In order to determine the coefficients $a_j(k), b_j(k)$ in (21), we use moving window least squares optimization. The input-ouput model (21) can be expressed as [4]

$$y_r(k) = \phi_{\mathrm{m}}(k) \theta_{\mathrm{m}}(k), \quad (22)$$

where the regressor matrix $\phi_{\mathrm{m}}(k)$ and the model coefficient matrix $\theta_{\mathrm{m}}(k)$ are defined as

$$\phi_{\mathrm{m}}(k) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n) \\ -y_r(k-1) \\ \vdots \\ -y_r(k-n) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{1 \times 2n}, \quad (23)$$

$$\theta_{\mathrm{m}}(k) \triangleq \begin{bmatrix} a_1(k) \cdots a_n(k) \ b_1(k) \cdots b_n(k) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{2n}.$$

Let the window size of the least squares optimization be $p \geq 2n$. Then, the least-squares solution for the coefficients of the input-output model is

$$\theta_{\mathrm{m}}(k) = A_{\mathrm{m}}^{-1}(k) b_{\mathrm{m}}(k), \quad (24)$$

where

$$A_{\mathrm{m}}(k) \triangleq \Phi_{\mathrm{m}}(k)^{\mathrm{T}} \Phi_{\mathrm{m}}(k) \in \mathbb{R}^{2n \times 2n}, \quad (25)$$

$$b_{\mathrm{m}}(k) \triangleq \Phi_{\mathrm{m}}(k)^{\mathrm{T}} \begin{bmatrix} y_r(k) \\ \vdots \\ y_r(k-p) \end{bmatrix} \in \mathbb{R}^{2n}, \quad (26)$$

$$\Phi_{\mathrm{m}}(k) = \begin{bmatrix} \phi_{\mathrm{m}}(k) \\ \vdots \\ \phi_{\mathrm{m}}(k-p) \end{bmatrix} \in \mathbb{R}^{p \times 2n}. \quad (27)$$

Note that (24) requires that $A_{\mathrm{m}}(k)$ be positive definite and thus the input u must be persistently exciting. The coefficients, $a_j(k)$, produce the numerator of the estimated transfer function $G_{y_r,u,k}$. The leading numerator coefficient and NMP zeros (if any) are extracted from this estimate and is used to update the target model numerator coefficients $\hat{N}_i$ at each time step k.

IV. Numerical Examples

For the numerical examples presented in this section, we introduce a known perturbation signal v to the control input as shown in Fig 2. This signal ensures that the plant $G_{y_r u,k}$ is persistently excited and thus enhances the speed and accuracy of the closed-loop identification.

We assume that the first nonzero Markov parameter and the relative degree of the initial system are known.

Example IV.1: *Asymptotically stable, minimum-phase (MP) plant with an unknown sudden change in sign of the first nonzero Markov parameter.* Consider the asymptotically stable, minimum-phase plant

$$G_{y_r u}(\mathbf{q}) = \frac{0.5(\mathbf{q} - 0.3)}{(\mathbf{q} - 0.1)(\mathbf{q} - 0.6)}. \quad (28)$$

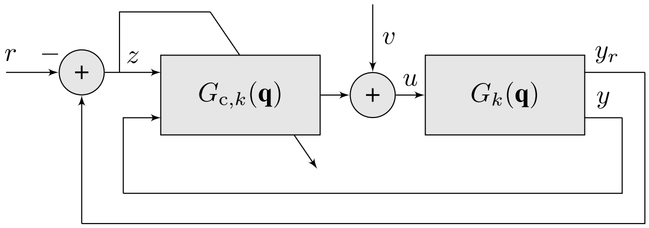

Fig. 2: The adaptive command-following problem with external control perturbation v.

For this example, we consider an unknown sudden change in sign of the first nonzero Markov parameter 0.5 at $k = 1200$ to -0.2. The feedback signal used in the controller is z, and thus $\xi(k) = z(k)$ and no feedback signal y is used. Let the controller order $n_c = 10$, $\lambda = 0.9993$, $R_\delta = 0.1 I_{l_\theta}$, $R_z = 1.0$, and $R_u(k) = 1 \times 10^{-4} z(k)^2$. The target model is chosen to have order $n_\mathrm{f} = 2$, where the desired closed-loop poles are chosen to be 0.1 and 0.2, and thus, $D_\mathrm{f}^\mathrm{T} = [1 \ -0.3 \ 0.02]$. The numerator of the target model is initialized at $\hat{N}_\mathrm{f}(0)^\mathrm{T} = [0.5 \ 0]$. The estimated coefficients of G are initialized to $\theta_\mathrm{m}(0) = [0 \ 0 \ 0.5 \ 0]^\mathrm{T}$, with a least-squares window size $p = 20$. The external control perturbation signal is chosen to be white noise with $v(k) \sim \mathcal{N}(0, 0.001)$. Figs. 3 and 4 show the results of the adaptive command following problem. Fig. 3 shows the command r and response y_r as well as the control u. Notice that at $k = 1200$, the output y_r deviates from the command due to the sudden change in the sign of the first Markov parameter but recovers command following after 40 time steps. Similarly, as expected, the control input changes sign to accommodate the sudden transition. Fig. 4 shows the controller and numerator of the target model coefficients. Notice that at $k = 1200$, the controller coefficients adapt to the sudden change in the sign of the Markov parameter. Likewise, the numerator coefficients of the target model correctly identify the change in sign with the correct magnitude -0.2. ■

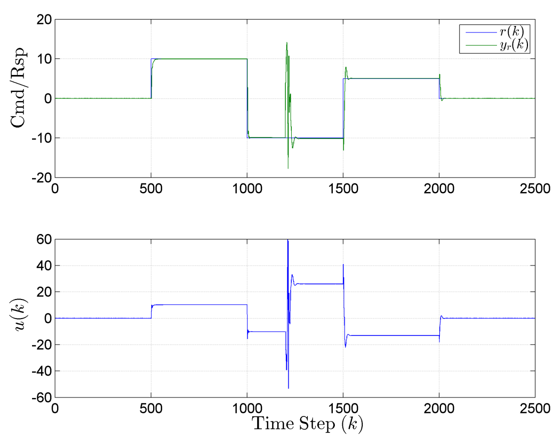

Fig. 3: Example IV.1: Command/response and control output. The upper plot shows the command/response, while the lower plot shows the control. Notice that at $k = 1200$, the response y_r deviates from the reference command due to the sudden change in the sign of the Markov parameter but regains tracking after 40 time steps. Similarly, the control input correctly changes sign to accommodate for the sudden transition.

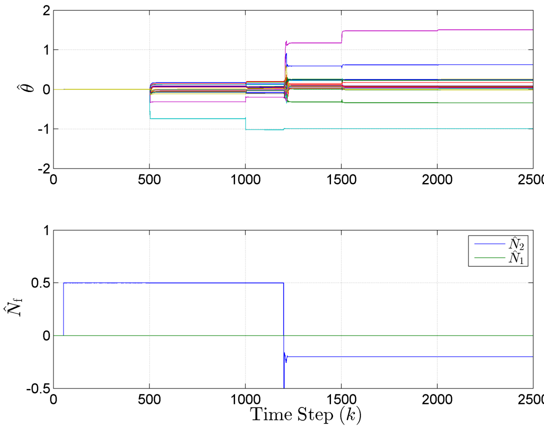

Fig. 4: Example IV.1: Controller and filter coefficients. The upper plot shows the controller coefficients, while the lower plot shows the numerator coefficients of the target model. Notice that at $k = 1200$, the controller coefficients adapt to the sudden change in the sign of the Markov parameter. Similarly, the numerator coefficients of the target model correctly estimate the change in the sign with the correct magnitude.

Example IV.2: *Asymptotically stable, nonminimum-phase (NMP) plant with unknown ramp transition in the location of the NMP zero.* Consider the asymptotically stable, nonminimum-phase plant

$$G_{y_r u}(\mathbf{q}) = \frac{(\mathbf{q} - 1.3)}{(\mathbf{q} - 0.1)(\mathbf{q} - 0.6)}, \tag{29}$$

For this example, we consider an unknown ramp transition of the location of the NMP zero from 1.3 at $k = 1000$ to 2.0 at $k = 1500$. The goal is to follow the command $r(k) = 5\sin(k/18)$. The feedback signal used in the controller is z, and thus $\xi(k) = z(k)$ and no feedback signal y is used. Let the controller order $n_c = 15$, $\lambda = 0.9993$, $R_\delta = 1 \times 10^{-3} I_{l_\theta}$, $R_z = 1.0$, and $R_u(k) = 1 \times 10^{-4} z(k)^2$. The target model is chosen to have order $n_\mathrm{f} = 2$, where the desired closed loop poles are chosen to be 0.1 and 0.2, and thus, $D_\mathrm{f}^\mathrm{T} = [1 \ -0.3 \ 0.02]$. The numerator of the target model is initialized at $\hat{N}_\mathrm{f}(0)^\mathrm{T} = [1.0 \ -1.3]$. The estimated plant coefficients of $G_{y_r u}$ are initialized to $\theta_\mathrm{m}(0) = [0 \ 0 \ 1.0 \ -1.3]^\mathrm{T}$, with a least-squares window size $p = 6$. The external control perturbation signal is chosen to be white noise with $v(k) \sim \mathcal{N}(0, 0.01)$. Figs. 5 and 6 show the results of the adaptive command following problem. Fig. 5 shows the command r and output y_r as well as the control u. Notice that from $k = 1000$ and $k = 1500$, the response develops a small error in following the harmonic command due to the ramp transition of the NMP zero location from 1.3 to 2.0. Similarly, the control input reaches a lower magnitude to accommodate the ramp transition. Fig. 4 shows the controller and numerator coefficients of the target model. Notice that at $k = 1000$, the controller coefficients adapt to the ramp transition of the NMP zero location. Likewise, the numerator coefficients of the target model correctly estimate the location of the NMP zero. ■

Example IV.3: *Control of the lateral dynamics of an aircraft with a time-varying transition to NMP dynamics.*

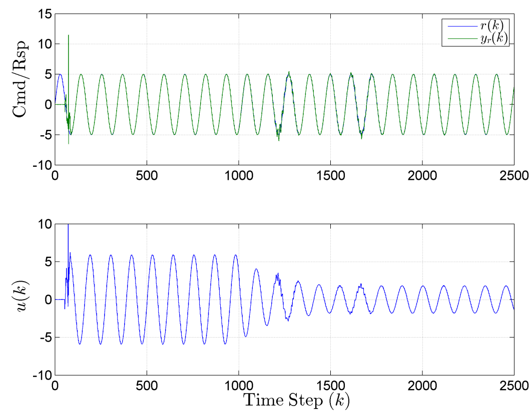

Fig. 5: Example IV.2: Command/response and control output. The upper plot shows the command/response while the lower plot shows the control. Notice that from $k = 1000$ to $k = 1500$, the output y_r develops minor tracking error of the reference command due to the ramp transition of the NMP zero location. Similarly, the control ramps to a lower magnitude to accommodate for transition.

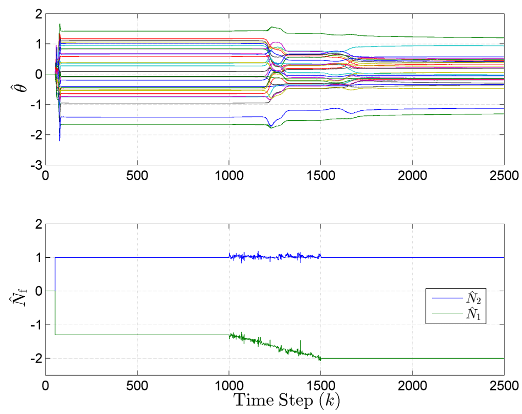

Fig. 6: Example IV.2: Controller and filter coefficients. The upper plot shows the controller coefficients, while the lower plot shows the numerator coefficients of the target model. Notice that at $k = 1000$, the controller coefficients adapt to the ramp transition of the NMP zero location. Similarly, the numerator coefficients of the target model correctly identifies the transition and ramps to the correct value of -2.0.

We consider the problem of controlling an aircraft with time-varying dynamics presented in [1, 5]. Specifically, the dynamics are first expressed in terms of an asymptotically stable, MP nominal plant which, at an unknown time and in an unknown manner, transitions to an asymptotically stable, NMP off-nominal plant. The discretized nominal and off-nominal plants with sample time $T_s = 0.1$ sec are given by

$$A_{\text{nom}} = \begin{bmatrix} 0.9553 & 0.0265 & -0.0934 & 0.0039 \\ -2.5210 & 0.9680 & 0.1310 & -0.0050 \\ 0.0551 & 0.0045 & 0.9708 & 0.0001 \\ -0.1282 & 0.0989 & -0.0369 & 1.0004 \end{bmatrix},$$

$$A_{\text{off}} = \begin{bmatrix} 0.8482 & 0.0255 & -0.0900 & 0.0038 \\ -10.3212 & 0.8598 & 0.5152 & -0.0210 \\ -0.0186 & 0.0041 & 0.9723 & 0.0001 \\ -0.5304 & 0.0953 & -0.0239 & 0.9999 \end{bmatrix},$$

$$B_{\text{nom}} = \begin{bmatrix} 0.0034 \\ 0.2492 \\ -0.0017 \\ 0.0126 \end{bmatrix}, \qquad B_{\text{off}} = \begin{bmatrix} 0.0036 \\ 0.2390 \\ -0.0061 \\ 0.0124 \end{bmatrix},$$

The control input is the elevon deflection, and the states x are $[\beta\ P\ R\ \phi_{\text{roll}}]^{\text{T}}$, which are sideslip angle, roll rate, yaw rate, and roll angle, respectively.

The transition from the nominal to the off-nominal plant occurs at a constant rate. Specifically, the dynamics vary linearly from $(A_{\text{nom}}, B_{\text{nom}})$ to $(A_{\text{off}}, B_{\text{off}})$ within 100 sec. The goal is to follow a roll reference command in the presence of the time-varying dynamics. Specifically, the relevant transfer function is from the control input u to the roll output ϕ_{roll}, and thus $E_1(k) = [0\ 0\ 0\ 1]$ for all k. The control-to-performance transfer function at time k is defined as

$$G_{\phi_{\text{roll}}u,k}(\mathbf{q}) = E_1(k)(\mathbf{q}I_n - A(k))^{-1}B(k). \qquad (30)$$

Fig. 7 shows the transition of the poles from the nominal to the off-nominal plant. Note that the dynamics are pointwise asymptotically stable throughout the transition. Fig. 8 shows

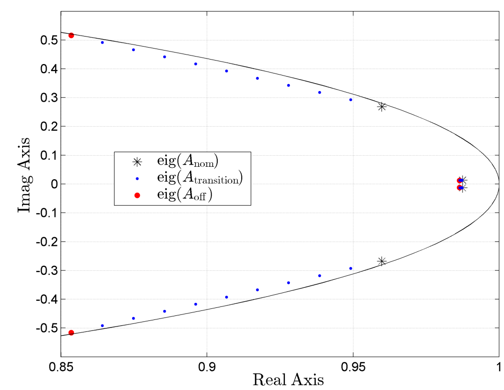

Fig. 7: Example IV.3: The eigenvalue locations of the A matrix before, during, and after the transition from the nominal to the off-nominal plant.

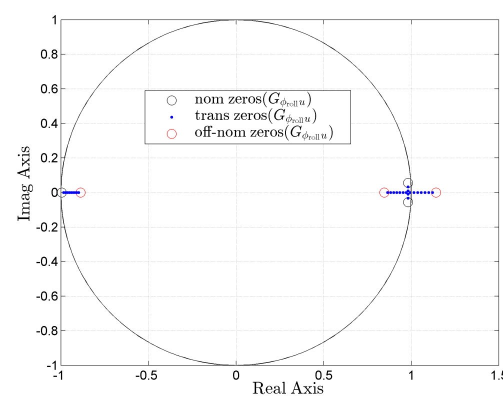

Fig. 8: Example IV.3: The zero locations of $G_{\phi_{\text{roll}}u,k}$ before, during, and after the transition from the nominal to the off-nominal plant.

the transition of the zeros of $G_{\phi_{\text{roll}}u,k}$ from the nominal to the off-nominal plant. Note that the zeros transition from MP to NMP.

Next, we introduce an additional state, namely, the integral of the performance variable defined as

$$z_{\text{int}}(k) \triangleq z(k) + z_{\text{int}}(k-1). \tag{31}$$

For the regressor of $G_{\text{c},k}$, let

$$\xi(k) = \begin{bmatrix} z(k) \\ z_{\text{int}}(k) \\ P(k) \end{bmatrix}. \tag{32}$$

Let the controller order $n_\text{c} = 15$, $\lambda = 0.9993$, $R_\delta = 50 I_{l_\theta}$, $R_z = 1.0$, and $R_u(k) = 5 \times 10^{-7} z(k)^2$. The target model is chosen to have order $n_\text{f} = 2$, where the desired closed-loop poles are chosen to be 0.1 and 0.98, and thus, $D_\text{f}^\text{T} = [1\ -1.08\ 0.098]$. The numerator of the target model is initialized at $\hat{N}_\text{f}(0)^\text{T} = [0.0126\ 0]$. The estimated coefficients of $G_{y_r,u}$ are initialized to $\theta_\text{m}(0) = [0\ 0\ 0\ 0\ 0.0126\ 0\ 0\ 0]^\text{T}$, with a least-squares window size $p = 40$. The external control perturbation signal is chosen to be white noise with $v(k) \sim \mathcal{N}(0, 0.01)$.

Figs. 9 and 10 show the response for the roll command following problem. The transition from nominal to off-nominal dynamics occurs from $t = 200$ sec to $t = 300$ sec and the plant becomes NMP at $t \geq 232.7$ sec. Fig. 9 shows the command $r(t)$ and the output $\phi_\text{roll}(t)$ as well as the control $u(t)$. Notice that from $t = 200$ sec to $t = 300$ sec, the response deviates from the command due to the transition in the dynamics. Similarly, the controller adjusts to accommodate the changing dynamics. Fig. 10 shows the controller and numerator coefficients of the target model. Notice that at $t = 100$ sec, the controller coefficients adapt to the changing dynamics and overcome the transition to NMP. Likewise, the numerator coefficients of the target model correctly identifies the transition and estimates the NMP zero. ■

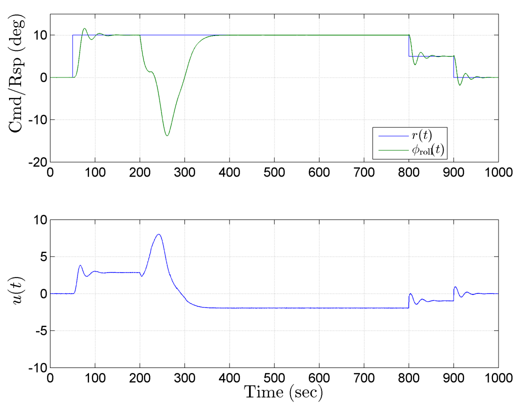

Fig. 9: Command/response and control output for Example IV.3. The upper plot shows the command/response, while the lower plot shows the control. Notice that from $t = 100$ sec to $t = 150$ sec, the output ϕ_roll deviates from the command due to the transition in the dynamics. Similarly, the controller adjusts to accommodate the changing dynamics.

V. Conclusions

We considered retrospective cost adaptive control (RCAC) of a plant whose NMP zeros are time-dependent. The goal was to estimate the NMP zeros and then replicate the latest estimates of the NMP zeros in the target model. This was done using concurrent closed-loop identification. An additional control perturbation was injected into the loop to enhance persistency and improve the accuracy of the closed-loop identification. Numerical results involving NMP second-order and fourth-order plants showed that the overall approach can both estimate and account for the time-dependent NMP zeros. For all examples, the onset and time-dependence of the changing plant dynamics were assumed to be unknown to the adaptive controller. The price paid for this approach is the performance degradation due to the control perturbation. Future research will quantify this degradation relative to the ability to estimate and account for time-dependent plant dynamics.

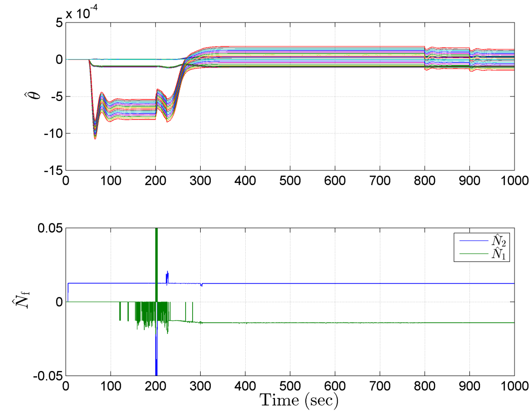

Fig. 10: Controller and filter coefficients for Example IV.3. The upper plot shows the controller coefficients, while the lower plot shows the numerator coefficients of the target model. Notice that at $t = 100$ sec, the controller coefficients adapt to the changing dynamics and overcome the transition to NMP. Similarly, the numerator coefficients of the target model correctly identify the transition and produces the correct NMP zero.

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