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A COMPARISON OF TWO ADAPTIVE ALGORITHMS FOR DISTURBANCE CANCELLATION

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Abstract. The objective of this paper is to compare two approaches to the adaptive rejection of disturbance inputs represented by a finite sum of sinusoids. The first approach is based on adapting the parameters of a finite impulse response (FIR) feedback controller using a least mean square (LMS) type algorithm to achieve asymptotic disturbance rejection. The second approach is based on using the recursive least squares (RLS) algorithm to search within the set of parametrized stabilizing controllers for a controller that leads to asymptotic disturbance rejection. The two approaches are compared in terms of their basic characteristics, as well as their performance in a simulation example.

Keywords. Adaptive control, disturbance rejection, LMS, recursive least squares.

1. INTRODUCTION

Consider a linear time-invariant system subject to a disturbance input represented by a finite sum of sinusoids, where the frequencies, amplitudes, and phases of the different sinusoids are not known. It is desired to construct an (adaptive) control system that would asymptotically reject the disturbance input. The problem is referred to as adaptive regulation.

The adaptive regulation problem described above is partly motivated by problems in the area of active noise and vibration control, where the objective is complete suppression of the effects of noise or external excitations at a particular location. In this application area, adaptive cancellation techniques have been shown to be very effective in suppressing the effects of narrow band disturbances. Among the most successful techniques of this type are those based upon least mean square (LMS) gradient approximation methods (Nelson and Elliot, 1992; Elliot *et al.*, 1987; Fuller *et al.*, 1992). Dating back to the work of Widrow and Hoff (Widrow and Hoff, 1960), these methods provide an effective technique for estimating performance gradients with respect to filter gains to adaptively cancel the effects of periodic disturbances. Another class of adaptive cancellation algorithms includes those based upon recursive least squares (RLS) techniques (Ben Amara *et al.*, 1995.; Palaniswami, 1993). These techniques are usually based upon an internal model controller structure for disturbance cancellation. By recursively modifying the controller parameters, the RLS algorithm yields control gains that adaptively cancel the disturbance signal. A recent survey of adaptive disturbance cancellation techniques can be found in (Fuller and von Flutow, 1995).

The design of an adaptive regulator involves two steps, the first of which consists of selecting a non-adaptive control system configuration. The configuration should be such that, if the disturbance input is completely known, the controller parameters can be tuned to achieve regulation. The second step involves augmenting the non-adaptive regulator with an adaptation mechanism, the purpose of which is to tune the controller parameters on-line to achieve regulation. Different (control configuration, adaptation mechanism) pairs could be identified in the literature. One of the most widely used such pairs is based on a controller in the form of a finite impulse response (FIR) filter and where the filter parameters are tuned using a Least Mean Square (LMS) type adaptation mechanism. The approach will be referred to as the FIR-LMS approach. A second approach is based on using the Youla parametrization to construct the set of parametrized stabilizing (PS) controllers. The parametrization of interest in this paper is one where the Youla parameter itself, and not the controller, is an FIR filter. The adaptation mechanism is represented by the recursive least square (RLS) algorithm with a forgetting factor, and will be used to tune the parameters of the FIR Youla parameter (filter). The approach is referred to as the PS-RLS approach.

The main objective of this paper is to present a comparison of the different properties associated with the two adaptive regulation approaches mentioned above. The comparison will be conducted at two levels. At the first level, we study the solvability of the non-adaptive regulation problem. The solvability analysis is based on characterizing the set of all regulators corresponding to each feedback configuration. The characterization is given in terms of a design parameter common to all regulators, which is the Youla parameter. Based on the derived properties of the Youla parameter, a comparative analysis of the size of the set of regulators corresponding to each design method can be given. The larger the set of regulators, the more likely other performance requirements, besides regulation, would be met. A second level of comparison involves the conditions under which each approach is guaranteed to yield regulation. The two adaptive systems are compared based on their requirements for convergence and computational load.

In order to provide a common framework for these methods, we adopt the standard two-input two output (TITO) formulation which provides a universal framework for feedback control techniques (Francis, 1987; Maciejowski, 1989) and is summarized in Section 2. After stating the adaptive disturbance cancellation problem in Section 3, we then proceed in Section 4 to present a common framework for the analysis of the non-adaptive versions of the disturbance rejection approaches discussed in this paper. The requirements for an FIR controller to be stabilizing and to achieve regulation are given in section 5. The adaptive FIR-LMS approach is then discussed in Section 6. Next, we proceed in Section 7 to review the off-line design of a PS regulator. The RLS algorithm and principal convergence results are summarized in Section 8. The performances of the LMS and RLS techniques are compared in Section 9 using simulation results for the noise cancellation problem in an acoustic duct. The behavior of the closed-loop system is simulated in the presence of a single tone disturbances. A comparison of the two adaptive approaches to disturbance rejection is given in Section 10. The comparison results indicate that the PS-RLS approach is more amenable to analysis studies and has better stability and convergence properties than the FIR-LMS approach. On the other hand, the latter has less computational requirements and is easier to implement.

2. THE TITO STANDARD PROBLEM FORMULATION

As a motivating example, we consider the noise cancellation problem in an acoustic duct. The acoustic duct can be modeled as a TITO system by considering a finite number of modes (Hong *et al.*, 1995) and defining the inputs and outputs as shown in Figure 1. The transfer

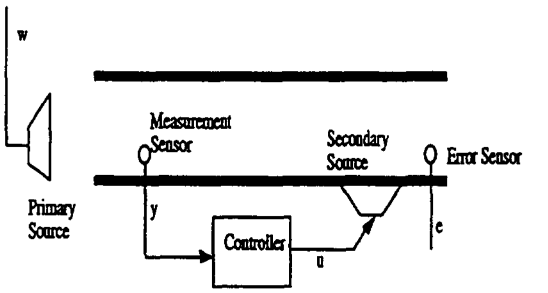

Fig. 1. Duct example.

functions can be interpreted as G_{we} from the primary source (disturbance w) to the error (performance e), G_{ue} from the secondary source (control input u) to the error e, G_{wy} from the primary source w to the reference (measurement y) and G_{uy} from the secondary source u to the reference y. The controlled system can be expressed in state space form as

$$x(k+1) = Ax(k) + B_w w(k) + B_u u(k), \quad (1)$$

$$e(k) = C_e x(k) + D_{we} w(k) + D_{ue} u(k), \quad (2)$$

$$y(k) = C_y x(k) + D_{wy} w(k) + D_{uy} u(k), \quad (3)$$

The state equations (1) - (3) give rise to four transfer functions, namely, G_{we} (from w to e), G_{ue} (from u to e), G_{wy} (from w to y) and the *plant* G_{uy} (from u to y). From equations (1)-(3), it follows that $e = G_{we}w+G_{ue}u$ and $y = G_{wy}w + G_{uy}u$. We introduce the notation

$$\mathbf{G} = \begin{bmatrix} G_{we} & G_{ue} \\ G_{wy} & G_{uy} \end{bmatrix} \sim \left[\begin{array}{c|c} A & B_w \; B_u \\ \hline C_e & D_{we} \; D_{ue} \\ C_y & D_{wy} \; D_{uy} \end{array}\right] \quad (4)$$

to denote a state space realization of a system with transfer matrix **G**. A block disgram of the closed loop system is shown in Figure 2.

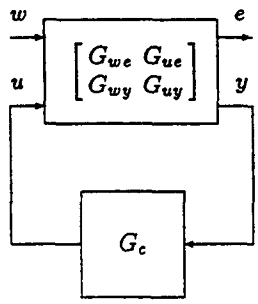

Fig. 2. Block diagram of the closed-loop system.

In the next section, we discuss the augmentation of the system shown in Figure 2 with an adaptation mechanism in order to reject a particular class of disturbance inputs.

3. THE ADAPTIVE DISTURBANCE CANCELLATION PROBLEM

Throughout the rest of the paper, it is assumed that the disturbance is a linear combination of sinusoids of the form

$$w(k) = \sum_{i=1}^{k_0} c_i \cos(\omega_i k + \phi_i) \quad (5)$$

with unknown frequencies ω_i, amplitudes c_i, and phases ϕ_i, $0 \leq i \leq k_0$. The purpose of the adaptation is to tune a particular parametrization of a feedback controller G_c in order to achieve asymptotic disturbance rejection. A block diagram of the adaptive closed-loop system is shown in Figure 3.

Two approaches to the adaptive rejection of sinusoidal disturbance inputs are considered in this paper. The first approach, representing the conventional approach, is based on adapting the parameters of a finite impulse response (FIR) feedback controller using the least mean squares (LMS) algorithm to achieve asymptotic disturbance rejection (FIR-LMS approach). The second approach is based on considering the set of parametrized stabilizing (PS) controllers for the plant, and using the recursive least squares (RLS) algorithm to search within this set for a controller that leads to asymptotic disturbance rejection (PS-RLS approach). Since the problem

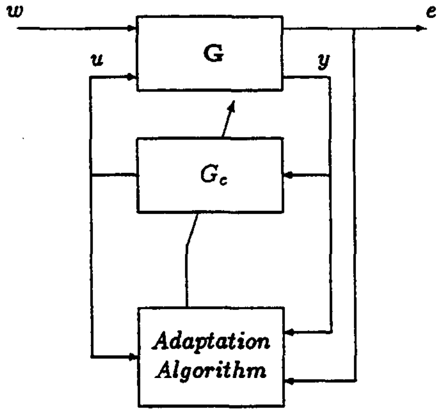

Fig. 3. Block diagram of the adaptive closed-loop system.

under consideration is complete disturbance rejection, and not simply attenuation of the disturbance effects, it is desired to first determine conditions under which the existence of feedback FIR and PS regulators is guaranteed. Assuming the existence conditions are satisfied, and given that the disturbance input is not known, the next step is to use adaptation to construct the desired regulators.

In order to be able to compare the existence conditions for both types of regulators, it desired to derive those conditions in terms of a common design parameter. The next section discusses the use of the Youla parametrization of stabilizing controllers in order to achieve this objective.

4. A FRAMEWORK FOR FEEDBACK SYSTEMS ANALYSIS

The Youla parametrization of the set of all stabilizing controllers for a given plant provides a unified framework for the design of the regulators needed for disturbance rejection. A common design parameter for all controllers is the Youla parameter (a transfer function or matrix). The unification in the design of the regulators is due to the fact that the regulation requirement can be translated into requirements on the Youla parameter as will be discussed in the next sections. Hence, the design requirement of different regulators can be compared based on the requirement imposed on the Youla parameter. In the following, standard results regarding the construction of the set of parametrized stabilizing controllers and the stability analysis of feedback systems are summarized, and RH_∞ denotes the set of proper asymptotically stable transfer matrices (Vidyasagar, 1985; Francis, 1987; Maciejowski, 1989).

4.1 *A Base Stabilizing Controller*

Suppose that G_{uy} has a stabilizable and detectable realization $G_{uy} \sim \left[\begin{array}{c|c} A & B_u \\ \hline C_y & D_{uy} \end{array}\right]$, and let K and L denote the state feedback and observer gains, respectively, of a feedback controller $G_{c,0}$ of the form $G_{c,0} \sim \left[\begin{array}{c|c} A + B_uK + LC_y + LD_{uy}K & -L \\ \hline K & D_{uy} \end{array}\right]$. Stabilizing controllers of this form can be obtained using standard LQG theory (Anderson and Moore, 1990). The controller $G_{c,0}$ will be called the *base stabilizing controller.*

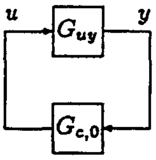

Fig. 4. The closed-loop system

4.2 *Parametrization of the set of all stabilizing controllers*

It is well known (Vidyasagar, 1985) that the plant and base stabilizing controller have a coprime fraction representations $G_{uy} = NM^{-1} = \tilde{M}^{-1}\tilde{N}$ and $G_{c,0} = UV^{-1} = \tilde{V}^{-1}\tilde{U}$, where $N, M, \tilde{N}, \tilde{M}, U, V, \tilde{U}, \tilde{V}$ are all in RH_∞. The factors in the fraction representations satisfy the double Bezout identity. Using the base controller $G_{c,0}$, the set of all stabilizing controllers can be constructed using the Youla parametrization (Youla *et al.*, 1976) which implies that for every $Q \in RH_\infty$, the controller $G_{c,Q}$ given by

$$\begin{aligned} G_{c,Q} &= (U + MQ)(V + NQ)^{-1} \\ &= (\tilde{V} + Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M}) \end{aligned} \quad (6)$$

is a stabilizing controller for the plant G_{uy}. Moreover, every stabilizing controller for the plant G_{uy} can be written in the form (6) for some $Q \in RH_\infty$.

4.3 *The Closed-Loop System*

The above section discussed the stabilization of the plant G_{uy}. The following lemma relates the stabilizability of G_{uy} to that of $\mathbf{G}$.

Lemma 1. :(Francis, 1987) Assume G_{uy} is strictly proper ($D_{uy} = 0$) and $\mathbf{G}$ has a stabilizable and detectable realization (4). Then the controller $G_{c,Q}$ stabilizes $\mathbf{G}$ if and only if $G_{c,Q}$ stabilizes G_{uy}.

Given a stabilizing controller as in (6), the closed-loop system shown in Figure 2 can be reconfigured as shown in Figure 5, where $\mathbf{T}$ is given by $\mathbf{T} = \begin{bmatrix} T_{we} & T_{se} \\ T_{wr} & 0 \end{bmatrix}$ where $T_{we} = G_{we} + G_{ue}U\tilde{M}G_{wy}$, $T_{se} = G_{ue}M$, and $T_{wr} = \tilde{M}G_{wy}$ (The realizations of T_{we}, T_{se} and T_{wr} can be found in (Maciejowski, 1989). Moreover, T_{we}, T_{se} and T_{wr} are all in RH_∞ .

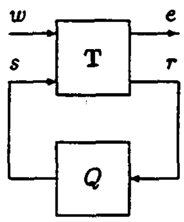

Fig. 5. Block diagram of the closed-loop system.

Letting $W(z)$ and $E(z)$ denote the $\mathcal{Z}$ transforms of the disturbance $\{w(\cdot)\}$ and the performance variable $\{e(\cdot)\}$ respectively, it follows that $E(z) = F_{T,Q}(z)W(z)$ where

$$F_{T,Q}(z) = T_{we}(z) + T_{se}(z)Q(z)T_{wr}(z) \quad (7)$$

For $Q \in RH_\infty$, it follows from Lemma 1 that $F_{T,Q}$ is asymptotically stable.

5. OFF-LINE DESIGN OF THE FIR REGULATOR

As the first step in developing the FIR-LMS approach, this section presents conditions under which asymptotic disturbance rejection in the feedback system shown in Figure 2 is realized using a controller G_c in the form of an FIR filter

$$G_c(z) = \frac{h(z)}{z^{n_h}} \tag{8}$$

where

$$h(z) = h_0 z^{n_h} + h_1 z^{n_h - 1} + \cdots + h_{n_h} \tag{9}$$

Working within the set of parametrized stabilizing controllers, a characterization of the set of Youla parameters that yield stabilizing FIR controllers is given. Additional conditions on the Youla parameter are then derived in order for the FIR controller to achieve asymptotic disturbance rejection. In the rest of the paper, only single-input single-output (SISO) systems are considered.

5.1 *Stabilization and Structural Requirement*

A controller with transfer function G_c that is to achieve regulation should first be stabilizing. Let $G_{c,0} = UV^{-1}$ be a stabilizing controller for $G_{uy} = NM^{-1}$ such that $MV - NU = 1$. Since G_c should be stabilizing, there should exist $Q \in RH_\infty$ such that $G_c = \frac{U+QM}{V+QN}$ and $Q(z) = \frac{n_Q(z)}{d_Q(z)}$ where $n_Q(z) = q_0 z^{n_q} + q_1 z^{n_q - 1} + \cdots + q_{n_q}$ and $d_Q(z) = z^{n_q} + q_{n_q+1} z^{n_q - 1} + \cdots + q_{2n_q}$. Let $U = \frac{n_U}{d_c}$, $V = \frac{n_V}{d_c}$, $N = \frac{n_N}{d_p}$, and $M = \frac{n_M}{d_p}$, where $\deg(n_V) = \deg(d_c)$, $\deg(n_M) = \deg(d_c)$. Then G_c is expressed as

$$G_c = \frac{n_Q n_M d_c + d_Q n_U d_p}{n_Q n_N d_c + d_Q n_V d_p} \tag{10}$$

The following lemma gives conditions on the Youla parameter so that a stable controller be stabilizing (The proofs of the three lemmas given below are not included due to space limitation).

Lemma 2. : **Structure Specific Stable Stabilization.** Let d_G be a given polynomial with all roots inside the unit circle. Then a controller with transfer function $G_c = \frac{n_G}{d_G} \in RH_\infty$ is stabilizing if and only if $\exists\ Q = \frac{n_q}{d_q} \in RH_\infty$ such that

$$n_Q n_M d_c + d_Q n_U d_p = n_G (d_c d_p)^2 \tag{11}$$

$$n_Q n_N d_c + d_Q n_V d_p = d_G (d_c d_p)^2 \tag{12}$$

The above result is then specialized for the case where the stabilizing controller is of the form of an FIR filter.

Lemma 3. : **Stabilization using an FIR controller.** Assume the plant G_{uy} is strictly proper. Then a controller with transfer function $G_c(z) = \frac{h(z)}{z^{n_h}} \in RH_\infty$ is stabilizing if and only if there exists a parameter vector

$$\theta_q = [q_0 \cdots q_{2n_q}]^T \tag{13}$$

such that

(1) $n_q = n_h + \deg(d_c) + \deg(d_p)$.
(2) The polynomial d_Q has all roots inside the unit circle (i.e. the transfer function $Q = \frac{n_Q}{d_Q}$ is in RH_∞).
(3) The parameter vector θ_q satisfies

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \theta_q + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0 \tag{14}$$

where the real matrices A_1 and A_2 and the vectors B_1 and B_2 are computed from the data (4) and (5).

5.2 *Regulation Requirement*

Consider now the additional requirement of regulation. The closed loop system response is given by

$$E(z) = [T_{we}(z) + T_{se}(z)Q(z)T_{wr}(z)]W(z), \quad T_{we},\ T_{se},\ T_{wr} \in RH_\infty \tag{15}$$

In order to satisfy the regulation requirement, an additional linear constraint is imposed on the parameter vector θ_q as given by the following Lemma.

Lemma 4. : **Regulation using an FIR controller.** Consider a strictly proper plant G_{uy}. The controller G_c in (6) is an FIR filter as in (8), stabilizing, and yields asymptotic disturbance rejection if and only if there exists a parameter vector (13) satisfying conditions 1 and 2 of Lemma 3 and in addition,

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \theta_q + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = 0 \tag{16}$$

where the real matrices A_1, A_2, and A_3 and the vectors B_1, B_2, and B_3 are computed from the data (4) and (5). Moreover, A_1, A_2, B_1, and B_2 are as in Lemma 3.

Remark: For the case where $e = y$, (i.e. $G_{we} = G_{wy}$, $G_{ue} = G_{uy}$), regulation is not possible in most of the cases. In fact, the expression for $E(z)$ in this case is given by

$$E(z) = \frac{1}{1 + G_c(z)G_{ue}(z)}[G_{we}(z)W(z)], \tag{17}$$

which implies that unless G_{ue} contains some of the modes of the disturbance and G_{we} has zeros at the remaining modes of the disturbance, regulation can not be achieved, i.e., regulation is totally dependent on the plant, and not the controller.

6. THE FIR-LMS APPROACH

Assuming the regulation requirements given in the previous section are satisfied, this section presents the on-line (adaptive) construction of an FIR regulator, under the assumption the disturbance input properties are not known. A traditional approach to achieve this objective is to tune the FIR controller parameters using the Least Mean Square (LMS) algorithm. In order to describe the algorithm, let $\hat{\theta}_h$ denote an estimate of θ_h^o, the nominal parameter vector, and $\tilde{\theta}_h = \theta_h^o - \hat{\theta}_h$ the parameter estimation error. The updated parameter vector, obtained using the LMS algorithm, is given by

$$\hat{\theta}_h(k+1) = \hat{\theta}_h(k) + \mu\phi(k)e(k), \quad (18)$$

where ϕ is the regression vector, to be defined below, and e the error signal (performance variable). It is desired to have the algorithm tune the $n_h + 1$ coefficients of the FIR controller G_c in order to have the performance variable e approach zero asymptotically. The purpose of this section is to derive conditions for the asymptotic convergence of the estimated parameter vector $\hat{\theta}_h$ to the nominal parameter vector θ_h^o.

The analysis of the convergence properties of the algorithm (18) is performed in several steps. First, an expression for the performance variable e as a function of ϕ and $\tilde{\theta}_h$ is derived. Second, a difference equation representing the parameter estimation error dynamics (error system) is determined. The third step of the analysis is to show that the origin of the error system is an asymptotically stable equilibrium point, which implies that $\hat{\theta}_h$ converges asymptotically to θ_h^o. The proof of convergence of the parameter estimates is based on the observation that the error system can be represented as a perturbation of a system for which the origin is an exponentially stable equilibrium point. The latter property is then used to show that the origin of the error system is asymptotically stable. The details of the steps described above are given in the following.

6.1 *The performance variable*

The performance variable (error signal) e is given by

$$e = \left[G_{we} + G_{ue}G_c[1 - G_{uy}G_c]^{-1}G_{wy}\right] w, \quad (19)$$

which indicates that e is not an affine function of the controller parameter vector

$$\theta_h = [h_0 \cdots h_{n_h}]^T. \quad (20)$$

Assumption 1. : **Uniqueness of the solution to the regulation problem.** The integer n_h is such that there exists a unique parameter vector

$$\theta_h^o = [h_{o,0} \cdots h_{o,n_h}]^T \quad (21)$$

that achieves regulation.

Let G_o be the corresponding controller, and e_o the corresponding performance variable. Note that e_o is bounded above in magnitude by an exponentially decaying function. An expression for $e(k)$ as a function of $\tilde{\theta}_h(k)$ is derived below. It can be easily shown that

$$\begin{aligned} e &= e - e_o + e_o \\ &= G_{ue}[1 - G_cG_{uy}]^{-1}[G_c - G_o][1 - G_{uy}G_o]^{-1}G_{wy}w \\ &\quad + e_o. \end{aligned} \quad (22)$$

Define the signal

$$v = G_{wy}w \;=\; y - G_{uy}u \quad (23)$$

and the operators

$$\begin{aligned} L(q^{-1}) &= [1 - G_{uy}(q^{-1})G_o(q^{-1})]^{-1}, \\ \mathbf{L}(q^{-1}) &= L(q^{-1})I_{(n_h+1)\times(n_h+1)}, \\ M(k,q^{-1}) &= G_{ue}(q^{-1})[1 - G_c(k,q^{-1})G_{uy}(q^{-1})]^{-1}, \\ \mathbf{M}(q^{-1}) &= M(q^{-1})I_{(n_h+1)\times(n_h+1)}, \end{aligned}$$

where I denotes the identity matrix. It can then be shown that

$$[G_c(q^{-1}) - G_o(q^{-1})]\, L(q^{-1})G_{wy}(q^{-1})w(k) = \left[[\mathbf{L}(q^{-1})\phi(k)]^T\tilde{\theta}_h(k)\right] \quad (24)$$

where

$$\begin{aligned} \phi(k) &= [-v(k) \cdots - v(k - n_h + 1)]^T, \\ \tilde{\theta}_h(k) &= \theta_h^o - \hat{\theta}_h(k). \end{aligned} \quad (25)$$

The performance variable e in (22) is then expressed as

$$e(k) = M(k,q^{-1})\left[[\mathbf{L}(q^{-1})\phi(k)]^T\tilde{\theta}_h(k)\right] + e_o(k) \quad (26)$$

The expression given above for the error is not in a suitable form for the analysis to be conducted in the next section. A more convenient form is one where the error is the sum of two quantities, the first representing the scalar product of a "filtered version" of the regression vector and the parameter estimation error $\tilde{\theta}_h$, the second representing a bounded perturbation term. For this purpose, and following simple algebraic manipulations, the above expression for the error is rewritten as

$$e(k) = \left[\mathbf{F}(k,q^{-1})\phi(k)\right]^T \tilde{\theta}_h(k) + \delta(k) \qquad (27)$$

where $\mathbf{F}(k,q^{-1}) = \mathbf{M}(k,q^{-1})\mathbf{L}(q^{-1})$ and $\delta(k) = \Big\{(M(k,q^{-1})([\mathbf{L}(q^{-1})\phi(k)]^T\tilde{\theta}_h(k)) - \left[\mathbf{F}(k,q^{-1})\phi(k)\right]^T \tilde{\theta}_h(k))\Big\} + e_o(k)$. In the last part of this section, the boundedness of the perturbation term $\delta(k)$ is studied. As mentioned earlier, the signal $e_o(k)$ is bounded above in magnitude by an exponentially decaying function of time. It remains to study the properties of $\Big\{(M(k,q^{-1})([\mathbf{L}(q^{-1})\phi(k)]^T\tilde{\theta}_h(k)) - \left[\mathbf{F}(k,q^{-1})\phi(k)\right]^T \tilde{\theta}_h(k))\Big\}$. The following two assumptions are then invoked:

Assumption 2. : The system $G(q^{-1})$ is asymptotically stable.

Assumption 3. : The system $M(k,q^{-1})$ is exponentially asymptotically stable, i.e. there exist $0 < \alpha < 1$ and $0 < K$ such that $|I_M(k,l)| \leq K\alpha^{(k-l)}$ where $I_M(k,l)$ is the impulse response of $M(k,q^{-1})$.

Assumption 2 is used to guarantee the boundedness of the signal v, and hence the boundedness of the regression vector ϕ. Assumption 3 is satisfied if $M(k,q^{-1})$ is exponentially stable for all $k \geq 0$ and $M(k,q^{-1})$ changes slowly with time (Sethares *et al.*, 1989). Let $\bar{\phi}(k) = \mathbf{L}(q^{-1})\phi(k)$ and $\|x\|_\infty = sup_k\|x(k)\|_\infty$. It can then be shown that

$$\Big|M(k,q^{-1})((\mathbf{L}(q^{-1})\phi(k))^T\tilde{\theta}_h(k)) - (\mathbf{M}(k,q^{-1})\mathbf{L}(q^{-1})\phi(k))^T\tilde{\theta}_h(k)\Big| \leq \mu\frac{\alpha K^2 n_h^2}{(1-\alpha)^3}\|\phi\|_\infty \|\bar{\phi}\|_\infty^2 \sup_{i\leq k}\left\|\tilde{\theta}_h(i)\right\|_\infty \qquad (28)$$

The expression for the performance variable is rewritten as

$$e(k) = \left[\mathbf{F}(k,q^{-1})\phi(k))\right]^T \tilde{\theta}_h(k) + \Delta(k,q^{-1})\tilde{\theta}_h(k) + e_o(k) \qquad (29)$$

where $\Delta(k,q^{-1})$ is a bounded operator with $\|\Delta(k,q^{-1})\|_\infty = O(\mu)$.

6.2 *The error system and convergence analysis*

The error system is obtained by subtracting θ_h^o from both sides of (18) and substituting (29) for e, resulting in the following dynamic system equation:

$$\tilde{\theta}_h(k+1) = \left[I - \mu\phi(k)[\mathbf{F}(k,q^{-1})\phi(k)]^T\right]\tilde{\theta}_h(k) - \mu\phi(k)[\Delta(k,q^{-1})\tilde{\theta}_h(k) + e_o(k)] \qquad (30)$$

It is desired to have the origin be an asymptotically stable equilibrium point for the above system. The stability analysis is conducted in two steps. In the first step, the stability of the origin for the unperturbed system

$$\tilde{\theta}_h(k+1) = \left[I - \mu\phi(k)[\mathbf{F}(k,q^{-1})\phi(k)]^T\right]\tilde{\theta}_h(k) \qquad (31)$$

is analyzed, and then the asymptotic stability of the origin for the system (30) is given. The exponential stability of the origin for the system (31) is guaranteed if there exist $\alpha > 0$ and $m > 0$ such that $\forall j, \quad \forall k > 0$,

$$\lambda_j\left[\frac{1}{m}\sum_{i=k}^{k+m-1}\phi(i)[\mathbf{F}(q^{-1},i)\phi(i)]^T\right] > \alpha. \qquad (32)$$

($\lambda_j[A]$ denotes the jth eigenvalue of A). The above persistence of excitation assumption holds if $\mathbf{F}(q^{-1},k)$ is strictly passive and ϕ is persistently spanning (i.e. (32) holds with $\mathbf{F}(q^{-1},k) = I$, the identity operator). The operator $\mathbf{F}(q^{-1},k)$ is strictly passive if $\mathbf{F}(q^{-1},k)$ is strictly positive real at each time step k and $\mathbf{F}(q^{-1},k)$ changes slowly with time (Sethares *et al.*, 1989). The usefulness of the exponential stability property of the origin stems from the fact that it allows one to add perturbations, such as those in (30), to the dynamics of the nominal unperturbed system (31) and still be able to guarantee some stability properties for the resulting perturbed system (30). In fact, assuming (32) holds, and given that ϕ is bounded, $\Delta(k,q^{-1})\tilde{\theta}_h$ is a linear bounded operator, and $e_o(k)$ decays exponentially fast to zero, then there exists a μ^* such that for $\mu \leq \mu^*$, the origin is an asymptotically stable equilibrium point for the system (30).

Theorem 1. : The origin is an asymptotically stable equilibrium point for (30) if Assumptions 1-2 are satisfied, $M(k,q^{-1})$ is exponentially asymptotically stable, ϕ is persistently exciting as given by (32) with $\mathbf{F}(k,q^{-1}) = \mathbf{M}(k,q^{-1})\mathbf{L}(k)$, the step size μ is small, $\tilde{\theta}_h(0)$ is small, and the initial conditions of $M(k,q^{-1})$ are small. The system $M(k,q^{-1})$ is exponentially asymptotically stable if

(1) The frozen time system $M(k,q^{-1})$ is exponentially stable for all k.
(2) $M(k,q^{-1})$ is slowly varying.

The persistency of excitation is guaranteed if

(SP) $M(k,q^{-1})L(q^{-1})$ is strictly passive.

(PS) The regressor $\phi(k)$ is persistently spanning.

Condition (SP) is satisfied if

(1) The frozen time system $M(k, q^{-1})L(q^{-1})$ is strictly passive for all k.
(2) $M(k, q^{-1})L(q^{-1})$ is slowly varying.

Condition (PS) is satisfied if

(1) G_{wy} has no zeros at the disturbance modes.
(2) $n_h + 1 \leq 2k_o$.

The proofs of most of the results given above can be found in (Sethares *et al.*, 1989).

7. OFF-LINE DESIGN OF THE PS REGULATOR

As in the off-line design of the FIR regulator, it is assumed in this section that the disturbance input is completely known and proceed with the numerical design of a new class of regulators for the plant. The new class of regulators is constructed by assuming the Youla parameter Q which enters into the construction of the stabilizing controllers (6), is an FIR filter (as opposed to requiring the regulator itself to be an FIR filter). As will be discussed later, a significant advantage that results from considering such regulators is that they are very easy to design, adapt and analyze.

From section 4.3, we have

$$E(z) = [T_{we}(z) + T_{ue}(z)Q(z)T_{wy}(z)]W(z)$$

The asymptotic disturbance rejection requirement can be cast in the form of interpolation conditions.
Let $p_1, ..., p_{n_p}$ denote the poles of $W(z)$ which, according to (5), are all simple and located on the unit circle.

Lemma 5. (Ben Amara *et al.*, 1995.): Consider the closed-loop system transfer function $F_{T,Q}$ (7). Then asymptotic disturbance rejection is achieved if and only if the interpolation conditions

$$T_{we}(p_i) + T_{ue}(p_i)Q(p_i)T_{wy}(p_i) = 0, \quad i = 1, \ldots, n_p = \tag{33}$$

are satisfied.

Letting the parameter $Q(z)$ be of the form

$$Q(z) = \sum_{i=1}^{n_q} q_i z^{1-i}, \tag{34}$$

it follows that the interpolation conditions (33) are equivalent to the linear constraint

$$A\theta_q + b = 0, \tag{35}$$

where

$$\theta_q = [q_1 \cdots q_{n_q}]^T \tag{36}$$

and A is a real matrix of size $n_p \times n_q$ and b is an n_p vector. Note that, in general the number of parameters in (34) need to be greater than or equal to the number of interpolation conditions (33) in order to have at least one solution.

The adaptation process is based on adjusting the parameter vector θ_q on-line to minimize a time domain performance index. The following result relates the interpolation conditions (35) to the minimization of a time domain criterion.

Lemma 6. (Ben Amara *et al.*, 1995.): Assume the signal w is quasi-stationary and $rank\ A = rank\ [A\ b] = n_p$. Then θ_q solves the minimization problem

$$\min_{\theta_q} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} e^2(k) \tag{37}$$

if and only if θ_q satisfies the interpolation conditions (35).

First, a numerical off-line controller design procedure is presented. A recursive version of the off-line design algorithm is then used for on-line (adaptive) controller design.

The off-line numerical controller design procedure is based on the assumption that a sequence of disturbance input values is available *a priori*. A least squares optimization algorithm is used to select the controller parameters.

Let $Q(z)$ be as in (34) and define the signals

$$V_0(z) = T_{we}(z)W(z), \tag{38}$$

$$V_1(z) = T_{ue}(z)T_{wy}(z)W(z). \tag{39}$$

The disturbance response can then be expressed as

$$e(k) = v_0(k) - \phi(k)^T\theta_q, \tag{40}$$

where $\{v_i(\cdot)\} = \mathcal{Z}^{-1}(V_i(z))$, $i = 0, 1$, and

$$\phi(k) = [-v_1(k) \cdots - v_1(k - n_q + 1)]^T. \tag{41}$$

Since the sequence of disturbance input values is available a priori, the sequence $\{v_1(\cdot)\}$ can be easily computed and used to determine the sequence of vectors

$\{\phi(\cdot)\}$. The latter can then be used in a least-squares-based controller design algorithm.

Lemma 7. (Ben Amara *et al.*, 1995.): Assume the matrix A in (35) is square ($n_p = n_q$) and invertible and that $v_1(\cdot)$ is persistently exciting of order n_q. Then $\theta_q^o = -A^{-1}b$ is the unique minimizer for the mean square error and is given by

$$\theta_q^o = \left[\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \phi(k)\phi(k)^T\right]^{-1} \left[\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \phi(k)^T v_0(k)\right] \quad (42)$$

If the numerator of $T_{se}(z)T_{wr}(z)$ and the denominator of $W(z)$ are coprime, then the condition that $v_1(\cdot)$ be persistently exciting of order n_q is equivalent to that of w being persistently exciting of order n_q.

8. THE PS-RLS ADAPTIVE REGULATOR

This section presents a review of a recursive least squares (RLS) based adaptive algorithm that constructs a controller capable of asymptotically rejecting band-limited (not necessarily periodic) disturbance inputs of the form (5). The adaptive controller design approach is based on searching, *online*, within the set of parametrized stabilizing controllers for the plant, for a particular controller that achieves asymptotic disturbance rejection. The search is based on adjusting the parameters of an FIR Youla parameter Q in order to asymptotically converge to the desired regulator. A recursive version of the least squares algorithm used in the off-line controller design is presented. The weighted least squares algorithm, of which the standard least squares algorithm used in Section 7 is a particular case, is used to derive the recursive algorithm.

Define the signal $r(k) = \tilde{M}(q^{-1})y(k) - \tilde{N}(q^{-1})u(k)$ and consider the performance variable

$$\begin{aligned} e(k) &= [T_{we}(q^{-1}) + T_{se}(q^{-1})QT_{wr}(q^{-1})]w(k) \\ &= T_{we}(q^{-1})w(k) + T_{se}(q^{-1})Qr(k) \\ &= T_{we}(q^{-1})w(k) + QT_{se}(q^{-1})r(k) \\ &\quad + [T_{se}(q^{-1})Q - QT_{se}(q^{-1})]r(k). \end{aligned}$$

Let $e_2(k) = [T_{se}(q^{-1})Q - QT_{se}(q^{-1})]r(k)$ and define a pseudo performance variable $\bar{e}(k) = e(k) - e_2(k)$. Assuming Q is of the form (34), then $\bar{e}(k)$ is given by

$$\begin{aligned} \bar{e}(k) &= T_{we}(q^{-1})w(k) + QT_{se}(q^{-1})r(k) \\ &= v_0(k) + \sum_{i=0}^{n_q-1} q_i(k)v_1(k-i) \\ &= v_0(k) - \phi^T(k)\theta_q(k) \end{aligned} \quad (43)$$

where $v_0(k) = T_{we}(q^{-1})w(k)$, $v_1(k) = T_{se}(q^{-1})r(k)$, and ϕ is given by (41). It can be seen from the above that the construction of the vector $\phi(\cdot)$ requires knowledge of only the control signal $u(\cdot)$ and the measurement $y(\cdot)$.

The recursive least squares algorithm with time varying forgetting factor is used to adjust the parameter vector θ_q online in order to achieve an asymptotic controller capable of rejecting the disturbance input. The parameter adjustment algorithm is given by

$$\hat{\theta}_q(k+1) = \hat{\theta}_q(k) + L(k+1)\bar{e}(k+1), \quad (44)$$

$$\mathbf{P}(k+1) = \frac{1}{\lambda(k+1)}[\mathbf{P}(k) - L(k+1)\phi(k+1)^T\mathbf{P}(k)], \quad (45)$$

$$L(k+1) = \frac{\mathbf{P}(k)\phi(k+1)}{1+\phi(k+1)^T\mathbf{P}(k)\phi(k+1)}, \quad (46)$$

with $\hat{\theta}_q(0) = \hat{\theta}_0$, $\mathbf{P}(0) = \mathbf{P}_0 > 0$, and where $\lambda(k)$ is the time-varying forgetting factor satisfying $0 < \lambda_{\min} \leq \lambda(k) \leq \lambda_{\max} < 1$.

Let θ_q^o denote the parameter vector satisfying the interpolation conditions. The parameter error at time k can be defined as

$$\tilde{\theta}_q(k) = \theta_q^o - \hat{\theta}_q(k) \quad (47)$$

The parameter estimation error (47) asymptotically converges to zero provided the interpolation equations (33) admit a unique solution and that the signal $v_1(\cdot)$ in the regression vector $\phi(\cdot)$ is such that

$$\lim_{N\to\infty} \lambda_{\min}[\sum_{i=1}^{N} \phi(i)\phi^T(i)] > \alpha > 0, \quad (48)$$

hence, resulting in an adaptive implementation of the Internal Model Principle when the performance e and the measurement y are the same ($\lambda_{\min}[A]$ denotes the smallest eigenvalue of A).

Remark: The adaptive parameter adjustment algorithm where the RLS algorithm with a forgetting factor is used can also deal with situations where the coefficients c_i, frequencies ω_i, and phases ϕ_i, $i = 0, \cdots, r$, in (5) are unknown and possibly piece-wise constant. In fact, for $0 < \lambda(k) < 1$, the gain $L(\cdot)$ does not decay to zero which allows the algorithm to remain alert to changes in the nominal parameter vector θ_q^o.

9. SIMULATION EXAMPLES

In this section, the FIR-LMS and PS-RLS approaches are applied to the noise cancellation problem in an acoustic duct. The acoustic duct model is obtained by considering only five modes of vibration. With a sampling period $T_s = .5$ msec, the discrete time state space representation ($\mathbf{A} = [\mathbf{A}_1\ \mathbf{A}_2]$, $\mathbf{B} = [\mathbf{B}_1\ \mathbf{B}_2]^T$, $\mathbf{C} = [\mathbf{C}_1\ \mathbf{C}_2]$) of the plant is given by

$$\mathbf{A}_1 = \begin{bmatrix} 0.7376 & -0.6205 & 0.0441 & -0.0573 & 0.0323 \\ 0.5468 & 0.7398 & 0.0888 & -0.1953 & 0.0439 \\ 0.0357 & 0.0583 & 0.8483 & 0.4889 & 0.0124 \\ 0.0977 & 0.1335 & -0.4770 & 0.7059 & 0.1823 \\ 0.0050 & 0.0298 & 0.0064 & -0.1582 & 0.7277 \\ 0.0135 & 0.0448 & -0.0295 & -0.0627 & -0.5768 \\ -0.1238 & -0.0307 & 0.0736 & 0.0424 & 0.2110 \\ 0.0044 & 0.0420 & -0.0395 & 0.0244 & -0.1478 \\ 0.0009 & 0.0103 & 0.0219 & -0.1030 & 0.0387 \\ 0.0110 & 0.0098 & -0.0106 & 0.0015 & -0.0284 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} -0.0375 & 0.0567 & 0.0037 & 0.0130 & -0.0036 \\ -0.0944 & 0.0793 & -0.0047 & 0.0114 & -0.0107 \\ 0.0564 & -0.0113 & -0.0235 & 0.0188 & 0.0110 \\ -0.0944 & 0.2040 & -0.0608 & 0.0730 & 0.0137 \\ 0.6488 & -0.0248 & -0.0141 & 0.0196 & 0.0252 \\ 0.6104 & 0.4816 & -0.0453 & 0.0688 & 0.0095 \\ -0.1357 & 0.6092 & 0.5478 & -0.1327 & -0.0607 \\ 0.1294 & -0.4521 & 0.7550 & 0.1937 & -0.0058 \\ -0.1390 & 0.1253 & 0.0388 & 0.5817 & 0.7302 \\ 0.0187 & -0.0411 & 0.0641 & -0.7330 & 0.6314 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0.0229 & -0.5343 & 0.3014 & 0.4218 & 0.1799 \\ -0.5228 & -0.1281 & 0.0027 & -0.1584 & -0.0470 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} 0.1835 & -0.3094 & 0.1040 & 0.1436 & 0.0469 \\ -0.1136 & -0.3564 & -0.1864 & 0.0234 & -0.0025 \end{bmatrix},$$

$$\mathbf{C}_1 = \begin{bmatrix} 0.0148 & 0.0271 & -0.0391 & -0.0225 & 0.0896 \\ 0.7105 & 0.2944 & -0.2970 & 0.4395 & -0.2010 \end{bmatrix},$$

$$\mathbf{C}_2 = \begin{bmatrix} -0.0273 & -0.0383 & -0.0617 & 0.0752 & -0.0711 \\ 0.3056 & -0.3081 & 0.0971 & -0.1032 & 0.0187 \end{bmatrix}.$$

The disturbance $w(.)$ is a single sinusoid given by

$$w(k) = c\sin(\omega k T_s + \phi), \tag{49}$$

where c, ω, and ϕ are the amplitude, frequency, and phase of the continuous time sinusoid. The triple (c, ω, ϕ) is (1, 1256 rad/sec, 3 rad) for $0 \leq k < T$ and then changes to (.5, 1194 rad/sec, 1.5 rad) for $T \leq k$ where $T = 40$ *sec* for the FIR-LMS case and $T = .5$ *sec* for PS-RLS case.

In the case of the PS-RLS approach, the base stabilizing LQG controller $G_{c,0}$ was designed using the LQG theory. The forgetting factor in the RLS algorithm is chosen to be $\lambda = .95$. The initial conditions of the algorithm are $\hat{\theta}_q(0) = [0,\ 0]^T$ and $P(0) = 100I$ where I is 2×2 identity matrix.

The performances of the closed-loop control systems are shown in Figures 6 - 7. It can be seen that the two adaptive control systems were both capable of rejecting the disturbance input even when the frequency of the disturbance input changes. However, it took the FIR-LMS algorithm a much longer time to converge compared to the time it took the PS-RLS approach to converge. In all cases, and as required by the interpolation conditions, the frequency response of the closed-loop system transfer function $F_{T,Q}$ after parameter convergence indicates that the latter has zeros at the modes of the disturbance (not shown here due to space limitation).

10. COMPARISON OF THE ADAPTIVE DISTURBANCE REJECTION ALGORITHMS

This section compares the properties of the FIR-LMS and PS-RLS approaches to the adaptive disturbance rejection problem. First, a comparison of the off-line designs of the FIR controller and PS controller is given. The framework for the analysis of feedback system, presented in section 4, allows the comparison to be performed by considering the properties of the Youla parameter Q, which is the common design parameter in both controllers. A comparison of the convergence properties of the two adaptive algorithms is then given. The latter properties are dependent on the particular parametrization of each controller.

10.1 *Comparison of the off-line designs*

The off-line design approach assumes knowledge of the disturbance input properties. The comparison is given in terms of the properties of the Youla parameter used in constructing the FIR or PS regulators.

In the case of the FIR regulator, the Youla parameter Q was chosen to be a fully parametrized transfer function, that is, a transfer function with parametrized numerator and denominator. The design of the FIR regulator involved satisfying two sets of constraints on the parameters of Q. The first set of constraints result from the structural and stabilization requirements on the controller. These constraints are used to set conditions on the Youla parameter so that the resulting controller is of the form of an FIR filter and stabilizing. The second set of constraints represent interpolation conditions that have to be satisfied in order for regulation to be achieved. All the constraints are in the form of linear equations in the unknown parameter vector θ_q (13). In the PS controller case, the Youla parameter was chosen to be an FIR filter. The controller is designed by having the parameter vector θ_q (36) satisfy only the set of linear constraints corresponding to interpolation conditions for regulation. Stability is guaranteed since $Q \in RH_\infty$. For a Q of order n_q, the number of free parameters to be determined is $2n_q + 1$ parameters in the case of the FIR controller, and n_q parameters in the case of the PS controller .

Based on the above, it can be seen that, for a given n_q, the set of stabilizing controllers considered in the search for an FIR controller that achieves regulation is larger than the set of stabilizing controllers considered in the search for a PS controller that achieves regulation. However, the number of constraints imposed on Q in the design of the FIR regulator is much larger than the number of constraints imposed on Q in the design of the PS regulator, which may lead to a smaller set of FIR regulators, if any at all, compared to the set of PS controllers that achieve regulation.

10.2 *Comparison of the adaptive algorithms*

In the adaptive controller design case, the disturbance input properties, that is the amplitudes, frequencies, and phases, are not known.

The FIR controller parametrization, with only the numerator coefficients to be adjusted, makes it easy to directly adjust the unknown controller parameters on-line as given by (18). However, the expression for the performance variable e is not affine in the parameter vector θ_h, and therefore, the analysis of the FIR-LMS adaptive control system properties is not easy to conduct. The sufficient conditions for convergence given by Theorem 1 place some stringent requirements on the operator $M(k, q^{-1})L(q^{-1})$. Pointwise in time exponential stability of $M(k, q^{-1})$ and strict passivity of $M(k, q^{-1})L(q^{-1})$ were among the requirements for asymptotic convergence. There is no mechanism in the algorithm (18) to guarantee that such requirements are met. It should be noted that the strict passivity requirement was invoked as part of a sufficient condition to have (32) satisfied. However, (32) may still be satisfied in the frequency range of interest without having the pointwise in time strict passivity of $M(k, q^{-1})L(q^{-1})$ be satisfied. Based on the observations given above, it is also clear that the bounded input bounded output (BIBO) stability of the adaptive closed loop system can not be guaranteed *a priori*. Regarding the speed of convergence, and as illustrated in the simulation example, the convergence of the estimated parameters to their nominal values is very slow.

The main advantages the PS-RLS approach has over the FIR-LMS approach can be stated as follows.

1- Given that the closed loop system transfer function is affine in Q and that Q is a linear combination of stable transfer functions, the resulting expression for the performance variable e is affine in θ_q which helped in conducting the performance analysis of the closed loop system as given in (Ben Amara *et al.*, 1995.) and summarized in section 8.

2- The only requirement for asymptotic convergence is the persistence of excitation assumption (48) which represents a very mild requirement. This convergence property holds for any $\hat{\theta}_q(0)$ whereas those given for the FIR-LMS approach hold only for $\hat{\theta}_h(0)$ in the neighborhood of $\hat{\theta}_h^0$.

3- The speed of convergence in the simulation of the estimated parameters is much higher than that of the FIR-LMS approach.

4- The BIBO stability of the adaptive closed loop system is guaranteed since the performance variable e is driven to zero asymptotically.

The main drawback of the PS-RLS approach is that it is computationally more expensive than the FIR-LMS approach as is obvious from (18) and (44)-(46).

11. SUMMARY AND CONCLUSIONS

This paper presented a comparison of a conventional approach to the adaptive disturbance rejection, namely the FIR-LMS approach, to the newly proposed PS-RLS approach. First, conditions for the existence of an FIR controller that achieves regulation are given. The FIR-LMS approach is then presented. The convergence properties of the approach are discussed. The sufficient conditions for convergence are stringent and not possible to guarantee. The PS-RLS approach is then summarized and its convergence properties given. Simulation results, using the acoustic duct example, are used to illustrate the performances of the two approaches. The comparison results indicate that the PS-RLS approach is more amenable to analysis studies and has better stability and convergence properties than the FIR-LMS approach. On the other hand, the latter has less computational requirements and is easier to implement. The two

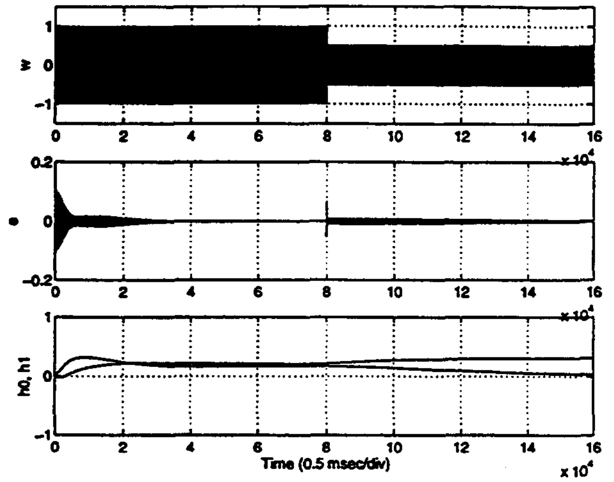

Fig. 6. Response of the adaptive control system for duct using the FIR-LMS algorithm with $\mu = .001$. Top: Disturbance input w(k). Middle: Response of the adaptive control system to the disturbance input w(k). Bottom : Coefficients of the FIR controller.

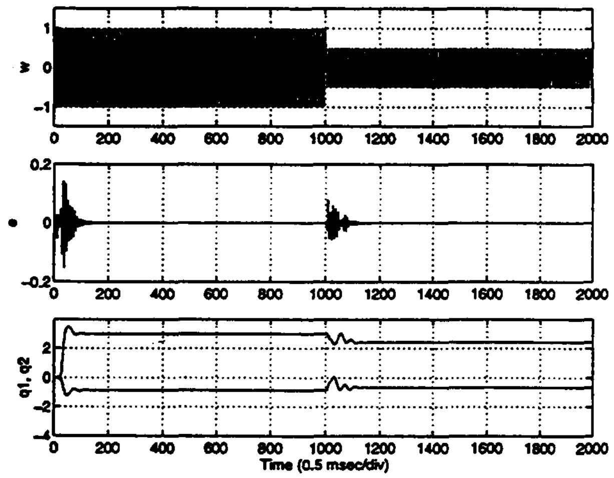

Fig. 7. Response of the adaptive control system using the PS-RLS algorithm. Top: Disturbance input w(k). Middle: Response of the adaptive control system to the disturbance input w(k). Bottom : Parameters of the controller parametrizing mapping Q.

approaches are currently in the process of being experimentally implemented and evaluated on the acoustic duct shown in Figure 2.

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