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HYBRID CONTROL: SEPARATION IN DESIGN

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1. INTRODUCTION

Active control of sound and vibration has gained significant interest in recent years due to advances in digital signal processing hardware, the application of adaptive signal processing in the control of sound and vibration, and the development of new transduction devices for what have been frequently termed "adaptive", "smart", or "intelligent" structures. The application of adaptive control approaches, based largely upon the LMS algorithm and its derivatives, has proceeded in parallel with efforts in the controls community devoted to the design of fixed-gain, robust compensators [1]. Advantages of hybrid (adaptive feedforward and feedback) control has been discussed in recent years [2–4]; however, there has been very little effort devoted to the interpretation of the two control strategies from a common terminology base.

Within the control literature, the "standard problem" is frequently used as the basis for control system design and synthesis [5–6]. This standard problem essentially involves the development of a state variable model, obtained from analysis or experimental system identification, with a convenient structure. The block diagram describing the standard problem has been termed the two-input, two-output (TITO) model. A schematic diagram of this system is presented in Figure 1. As illustrated, there are two vector inputs, $\mathbf{w}(s)$ and $\mathbf{u}(s)$, and two vector outputs, $\mathbf{z}(s)$ and $\mathbf{y}(s)$. The two vector inputs are the disturbance and control respectively while the two vector outputs are the error and measurement respectively. The generalized plant, $\mathbf{G}(s)$, is divided into separate matrix transfer functions between each vector input and vector output. The *performance path* is defined between the disturbance input, $\mathbf{w}(s)$, and the error output, $\mathbf{z}(s)$, and this path essentially defines the cost function. The *control path* is defined between the control input, $\mathbf{u}(s)$, and the measured output, $\mathbf{y}(s)$, and is used to implement the controller, $\mathbf{K}(s)$. The *reference path* defines the relationship between the disturbance and the measured output and the *secondary path* defines the relationship between the control and error output.

The generalized plant shown in Figure 1 contains the dynamics of the system to be controlled as well as any additional frequency weighted filters that describe physical processes or penalties imposed by the designer. For example, in the structural acoustic control problem, measurements are obtained and control inputs are applied through the dynamic structure (beam, plate, shell, etc.); however, a model of the fluid–structure interaction must be constructed such that the controller is designed to minimize sound radiation as opposed to vibration. This fully coupled model describes the generalized plant, $\mathbf{G}(s)$. In general, the designer's knowledge of the application physics is conveyed in the model of the generalized plant.

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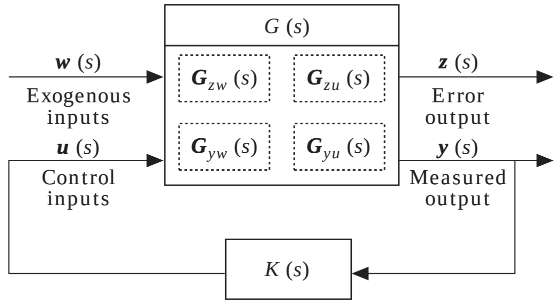

Figure 1. Block diagram of the standard problem.

Another convenience offered upon casting the system model in terms of the standard problem results when the system response is assumed linear (a basic assumption for a large class of acoustic and vibration control applications). Linear, time invariant (LTI) state variable representations can be used to represent the system dynamics, providing an efficient means of computing the system response to steady state or transient inputs. Methods of designing fixed-gain, dynamic compensators for feedback control of such systems are prevalent in the literature. The difficulty in developing such models most frequently results when additional application physics must be incorporated into the plant model to generate the error signals used in the cost functional. For example, in structural acoustic control, one can develop an expression relating the surface velocity to the radiated power. When outputs are expressed in units of power, spectral factorization must be employed to develop a state variable model of such dynamics. Thus, the application of these models required to develop the cost functional are limited to steady state predictions in control system design. However, such techniques are commonly employed in the design of feedback control as detailed by references [7–11].

The purpose and contribution of this work rests in the unified description of feedback, feedforward, and hybrid control system architectures within the general framework of the standard problem. The effects of feedforward, feedback, and hybrid control on the plant dynamics are illustrated. The standard problem is modified to demonstrate the specialized structure of the generalized plant when feedforward or hybrid control is applied. Additionally, as demonstrated in this work, the feedback and feedforward control system components are shown to be separable with respect to all disturbances which can be measured directly.

2. CONTROL SYSTEM ARCHITECTURE

2.1. *Feedback control*

As detailed in the introduction, the standard problem is structured as illustrated in Figure 1, and the relationship between the inputs and outputs of the open-loop plant can be expressed as

$$\begin{bmatrix} \mathbf{z}(s) \\ \mathbf{y}(s) \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{zw}(s) & \mathbf{G}_{zu}(s) \\ \mathbf{G}_{yw}(s) & \mathbf{G}_{yu}(s) \end{bmatrix} \begin{bmatrix} \mathbf{w}(s) \\ \mathbf{u}(s) \end{bmatrix}. \tag{1}$$

As indicated in equation (1) and illustrated in Figure 1, the system matrix $\mathbf{G}(s)$ is partitioned according to $[\mathbf{z}/\mathbf{y}]$ and $[\mathbf{w}/\mathbf{u}]$. The closed-loop response at the performance output as a function of an input disturbance can be obtained through a linear fractional transformation,

$$\mathbf{z}(s) = \mathbf{T}_{zw}(s)\mathbf{w}(s), \tag{2}$$

where

$$\mathbf{T}_{zw}(s) \equiv [\mathbf{G}_{zw}(s) + \mathbf{G}_{zu}(s)\mathbf{K}(s)[\mathbf{I} - \mathbf{G}_{yu}(s)\mathbf{K}(s)]^{-1}\mathbf{G}_{yw}(s)], \tag{3}$$

$$= [\mathbf{G}_{zw}(s) + \mathbf{G}_{zu}(s)\mathbf{K}(s)\mathbf{S}(s)\mathbf{G}_{yw}(s)], \tag{4}$$

and

$$S(s) \equiv [\mathbf{I} - \mathbf{G}_{yu}(s)\mathbf{K}(s)]^{-1}, \tag{5}$$

is the sensitivity function. The closed-loop response is thus dependent upon all transfer function paths and the dynamic compensator.

As noted by Hong and Bernstein[12], a special case occurs when the control actuator is colocated with the disturbance (i.e., $\mathbf{G}_{zw}(s) = \mathbf{G}_{zu}(s)$ and $\mathbf{G}_{yw}(s) = \mathbf{G}_{yu}(s)$) or when the performance (error) variable is the same as the measured variable (i.e., $\mathbf{G}_{zw}(s) = \mathbf{G}_{yw}(s)$ and $\mathbf{G}_{zu}(s) = \mathbf{G}_{yu}(s)$). In these cases, the closed-loop transfer function, $\mathbf{T}_{zw}(s)$, reduces to the simplified expression

$$\mathbf{T}_{zw}(s) = \mathbf{S}(s)\mathbf{G}_{zw}(s). \tag{6}$$

Thus, if $\sigma_{max}(\mathbf{S}(j\omega)) < 1$ (where $\sigma_{max}(\mathbf{S}(j\omega))$ is the maximum singular value of the sensitivity function as a function of frequency), then $\sigma_{max}(\mathbf{T}_{zw}(j\omega)) < \sigma_{max}(\mathbf{G}_{zw}(j\omega))$. This special case is the multi-variable version of the classical, single input, single output servo-control problem in which open loop-shaping is used for compensator design. However, for the more general problem outlined in equation (4), a reduction in sensitivity may not lead to a direct improvement of closed loop response, and thus closed loop shaping must be applied through more general techniques.

2.2. *Feedforward control*

In feedforward control, the entries of $\mathbf{G}(s)$ can be described as illustrated in Figure 2. The disturbance input to the structure can be measured directly (i.e., $\mathbf{G}_{yw} = \mathbf{I}$), and the control signal has no influence on the measured output (i.e., $\mathbf{G}_{yu} = 0$). Although the feedforward control problem can be cast in the architecture of the standard problem, the controller cannot be synthesized through $\mathscr{H}_2$ design approaches since the plant is unobservable from the measured output, $\mathbf{y}(s)$ (i.e., the measured disturbance or reference signal as denoted in adaptive feedforward control). However, if a feedforward

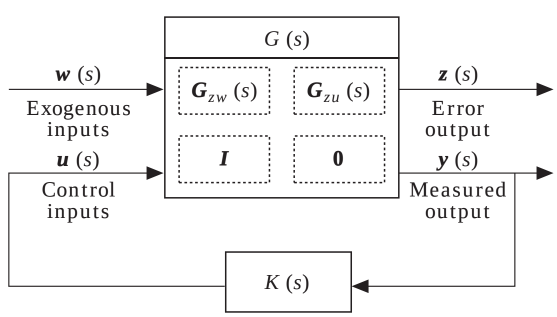

Figure 2. Block diagram of the standard problem configured for feedforward control.

compensator, $\mathbf{K}(s)$, is selected, the response between the disturbance and performance variables is given as

$$\mathbf{T}_{zw}(s) = \mathbf{G}_{zw}(s) + \mathbf{G}_{zu}(s)\mathbf{K}(s). \tag{7}$$

Thus, only the zeros of the system response can be modified since polynomial addition will occur only in the numerator. As a result, any reduction in $\sigma_{max}(\mathbf{T}_{zw}(j\omega))$ is local to the output $\mathbf{z}(s)$ since the original poles of $\mathbf{G}_{zw}(s)$ are not modified by the application of feedforward control [13].

2.3. *Hybrid control*

Hybrid control, in this work, is defined as the combination of feedback and feedforward control. The hybrid control problem is cast in terms of the standard problem in Figure 3. The disturbance has been partitioned into a random input, $\mathbf{w}_r(s)$, and a measured input, $\mathbf{w}_m(s)$. The measured output has also been partitioned into an output measured directly from the plant response, $\mathbf{y}_P(s)$, due to the inputs $\mathbf{u}(s)$ and $\mathbf{w}(s)$ and a direct measure of the measured input, $\mathbf{y}_{w_m}(s)$. The generalized plant, $\mathbf{G}(s)$, has been partitioned appropriately for the input and output variables. Notice also that the dynamic compensator has been partitioned to reflect the portion associated with feedback control, $\mathbf{K}_{uy_P}(s)$ and the portion associated with feedforward control, $\mathbf{K}_{uw_m}(s)$. Through simple matrix algebra, one can show that the controlled response between the disturbance and performance can be described as

$$\begin{aligned}\mathbf{z}(s) = {} & [\mathbf{G}_{zw}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)[\mathbf{I} - \mathbf{G}_{y_Pw}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{y_Pu}(s)]\mathbf{w}(s) \\ & + [\mathbf{G}_{zu}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)[\mathbf{I} - \mathbf{G}_{y_Pu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{y_Pu}(s)]\mathbf{K}_{uw_m}(s)w_m(s).\end{aligned} \tag{8}$$

Equation (8) can be partitioned so as to separate the disturbance due to the random input, $\mathbf{w}_r(s)$ and that due to the measured input, $\mathbf{w}_m(s)$:

$$\begin{aligned}\mathbf{z}(s) = {} & [\mathbf{G}_{zw_r}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)[\mathbf{I} - \mathbf{G}_{y_Pu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{y_Pw_r}(s)]\mathbf{w}_r(s) \\ & + ([\mathbf{G}_{zw_m}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)[\mathbf{I} - \mathbf{G}_{y_Pu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{y_Pw_m}(s)] \\ & + [\mathbf{G}_{zu}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)[\mathbf{I} - \mathbf{G}_{y_Pu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{y_Pu}(s)]\mathbf{K}_{uw_m}(s))\mathbf{w}_m(s).\end{aligned} \tag{9}$$

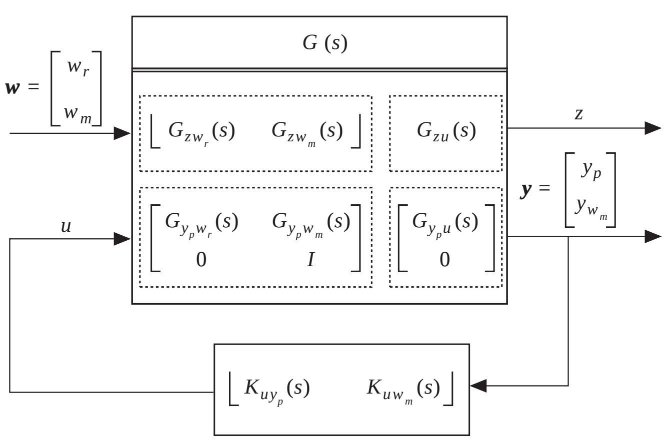

Figure 3. Block diagram of the standard problem configured for hybrid control.

In adapative feedforward control applications, the performance output variables, and the measured output variables are frequently different; however, if they are the same (i.e., when $\mathbf{z}(s) = \mathbf{y}_P(s)$) equation (9) can be simplified to

$$\mathbf{z}(s) = ([\mathbf{I} - \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{zw_m}(s) + [\mathbf{I} - \mathbf{G}_{zu}(s)\mathbf{K}_{uy_P}(s)]^{-1}\mathbf{G}_{zu}(s)\mathbf{K}_{uw_m}(s))\mathbf{w}_m(s) + [\mathbf{G}_{zw_r}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uz}(s)[\mathbf{I} - \mathbf{G}_{zu}(s)\mathbf{K}_{uz}(s)]^{-1}\mathbf{G}_{zw_r}(s)]\mathbf{w}_r(s). \quad (10)$$

Through manipulation one can show that

$$\mathbf{z}(s) = \mathbf{S}(s)[\mathbf{G}_{zw_m}(s) + \mathbf{G}_{zu}(s)\mathbf{K}_{uw_m}(s)]\mathbf{w}_m(s) + \mathbf{S}(s)\mathbf{G}_{zw_r}(s)\mathbf{w}_r(s). \quad (11)$$

Thus, if $\sigma_{max}(S(\mathrm{j}\omega)) < 1$, then a reduction in the response due to the random input disturbance, $\mathbf{w}_r(j\omega)$, will result. However, the response due to the measured disturbance input, $\mathbf{w}_m(s)$, is controlled through a combination of zero placement (feedforward control) and the magnitude reduction in the sensitivity (feedback control). For control of harmonics, the zeros are readily modified to ‘‘cancel’’ the output due to the disturbance.

3. LQG CONTROL WITH A HYBRID MEASUREMENT STRUCTURE

In this section one applies LQG theory to the hybrid system shown in Figure 3. One demonstrates that the LQG controller can be decomposed into two components, namely, a feedback controller and a feedforward controller. In addition, the LQG controller is shown to satisfy a *separation* property in the sense that the feedback component of the controller coincides precisely with the LQG controller that is obtained if the tonal disturbance is absent.

Consider the following linear time-invariant (LTI) state equation:

$$\dot{x}(t) = Ax(t) + Bu(t) + D_{1r}w_r(t) + D_{1m}w_m(t), \quad (12)$$

where $w_r(t)$ is a random disturbance and $w_m(t)$ is a disturbance that can be measured directly. One can construct two outputs:

$$y_P(t) = C_p x(t) + D_{2P}w_P(t), \quad (13)$$

and

$$y_{w_m}(t) = w_m(t) + D_{ms}w_{ms}(t), \quad (14)$$

where $y_P(t)$ is the output filtered by the plant to be controlled, and $y_{w_m}(t)$ is the output corresponding to the disturbance which can be measured directly. Sensor noise is added to each output and is represented by $D_{2P}w_P(t)$ and $D_{ms}w_{ms}(t)$ respectively. The disturbance and sensor noise signals w_r, w_m, w_P, and w_{ms} are mutually uncorrelated white noise inputs with unit intensity. The performance or error output is represented as

$$z(t) = E_1x(t) + E_2u(t), \quad (15)$$

in terms of the system states, $x(t)$, and control input, $u(t)$.

In the usual notation, the system equations can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + D_1w, \qquad y(t) = Cx(t) + D_2w(t), \quad (16,17)$$

where

$$D_1 = [D_{1r}\ \ D_{1m}\ \ 0\ \ 0], \qquad D_2 = \begin{bmatrix} 0 & 0 & D_{2P} & 0 \\ 0 & I & 0 & D_{ms} \end{bmatrix}, \qquad C = \begin{bmatrix} C_P \\ 0 \end{bmatrix}, \quad (18\text{–}20)$$

and

$$w(t) = \begin{bmatrix} w_r(t) \\ w_m(t) \\ w_P(t) \\ w_{ms}(t) \end{bmatrix}. \tag{21}$$

The resulting Riccati equations for the hybrid LQG problem can be expressed as

$$0 = (A - BR_2^{-1}R_{12}^{\mathrm{T}})^{\mathrm{T}}P + P(A - BR_2^{-1}R_{12}^{\mathrm{T}}) - PBR_2^{-1}B^{\mathrm{T}}P + R_1 - R_{12}R_2^{-1}R_{12}^{\mathrm{T}}, \tag{22}$$

$$0 = (A - V_{12}V_2^{-1}C)Q + Q(A - V_{12}V_2^{-1}C)^{\mathrm{T}} - QC^{\mathrm{T}}V_2^{-1}CQ + V_1 - V_{12}V_2^{-1}V_{12}^{\mathrm{T}}, \tag{23}$$

where

$$R_1 = E_1^{\mathrm{T}}E_1, \qquad R_{12} = E_1^{\mathrm{T}}E_2 = 0, \qquad R_2 = E_2^{\mathrm{T}}E_2,$$

$$V_1 = D_1D_1^{\mathrm{T}}, \qquad V_{12} = D_1D_2^{\mathrm{T}}, \qquad \mathrm{V}_2 = \mathrm{D}_2\mathrm{D}_2^{\mathrm{T}}.$$

Note that there is no cross-weighting assumed in the performance measure. The state variable description for the compensator can be expressed as

$$A_c = A + BC_c - B_cC, \qquad B_c = (QC^{\mathrm{T}} + V_{12})V_2^{-1}, \qquad C_c = -R_2^{-1}(B^{\mathrm{T}}P + R_{12}^{\mathrm{T}}). \tag{24–26}$$

Next one notes that V_1, V_{12}, and V_2 are given by

$$V_1 = D_{1r}D_{1r}^{\mathrm{T}} + D_{1m}D_{1m}^{\mathrm{T}} = V_{1r} + V_{1m}, \qquad V_{12} = [0 \;\; D_{1m}], \tag{27, 28}$$

$$V_2 = \begin{bmatrix} D_{2P}D_{2P}^{\mathrm{T}} & 0 \\ 0 & I + D_{ms}D_{ms}^{\mathrm{T}} \end{bmatrix}, \tag{29}$$

where

$$V_{1r} = D_{1r}D_{1r}^{\mathrm{T}} \qquad V_{1m} = D_{1m}D_{1m}^{\mathrm{T}}.$$

Now, assuming that D_{ms} is zero,

$$V_{12}V_2^{-1}C = [0 \;\; D_{1m}]\begin{bmatrix} V_{2P}^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} C_P \\ 0 \end{bmatrix} = 0, \tag{30}$$

and

$$V_{12}V_2^{-1}V_{12}^{\mathrm{T}} = [0 \;\; D_{1m}]\begin{bmatrix} V_{2P}^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} 0 \\ D_{1m}^{\mathrm{T}} \end{bmatrix} = V_{1m}. \tag{31}$$

Substracting equation (31) from equation (27), one obtains

$$V_1 - V_{12}V_2^{-1}V_{12}^{\mathrm{T}} = V_{1r}. \tag{32}$$

Additionally,

$$CV_2^{-1}C = [C_P^{\mathrm{T}} \;\; 0]\begin{bmatrix} V_{2P}^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} C_P \\ 0 \end{bmatrix} = C_P^{\mathrm{T}}V_{2P}C_P. \tag{33}$$

Thus, equation (23) can be expressed as

$$0 = AQ + QA^{\mathrm{T}} - QC_P^{\mathrm{T}} V_{2P}^{-1} C_P Q + V_{1r}. \tag{34}$$

However, this is exactly the Riccati equation that would result if one *ignores* w_m throughout the development and simply uses y_P.

Making appropriate substitutions in equation (25), one can show that

$$B_c = [QC_P^{\mathrm{T}} V_{2p}^{-1} \quad D_{1m}]. \tag{35}$$

Thus, $QC_P^{\mathrm{T}} V_{2p}^{-1}$ is the gain for y_p, and D_{1m} is the gain for y_m. The final compensator can be written as

$$\dot{x}_c(t) = A_c x_c(t) + B_c y(t) = A_c x_x(t) + QC_P^{\mathrm{T}} V_{2P}^{-1} y_p(t) + D_{1m} y_m(t) \tag{36}$$

and

$$u(t) = C_c x_c(t). \tag{37}$$

Note that if $w_m = 0$ (i.e., the measured input disturbance is no longer applied to the system), then $y_{w_m} = 0$. For this case,

$$\dot{x}_c(t) = A_c x_c(t) + QC_P^{\mathrm{T}} V_{2P}^{-1} y_P(t). \tag{38}$$

This is identical to the LQG controller that would result if w_m were set to zero from the start and serves to demonstrate the independence of the feedback and feedforward design for stochastic and measured disturbance inputs respectively. Thus, if a portion of the disturbance can be measured and controlled with feedforward, or adaptive feedforward control, this portion can be ignored during the design process required to develop a compensator for control of stochastic disturbance inputs.

4. CONCLUSIONS

The hybrid (feedback and feedforward) control problem was cast in the format of the standard problem described in the controls literature. Since both feedback and feedforward control approaches are frequently applied to noise and vibration problems, a method of considering the design objectives in a concurrent format was presented.

The hybrid measurement structure and problem was formulated to demonstrate that the LQG compensator design for control of stochastic inputs is separable with respect to inputs which can be measured directly. Thus, one can design a feedback controller in the absence of measureable input disturbances if the objective is to combine both feedback and feedforward or adaptive feedforward control for stochastic and measureable inputs respectively. Once the feedback controller is designed for the stochastic inputs, the feedforward controller required for the measurable input disturbances can be designed in a subsequent formulation of the control problem. Realizing that the design of such compensators is separable serves to simplify the hybrid design process.

Future work will be devoted to the application of such hybrid control system design to problems in vibration and acoustics. Additionally, the hybrid design approach will be explored to determine its affect on order reduction for dynamic compensation.

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