Brief Papers

Asymptotic Disturbance Rejection for Hammerstein Positive Real Systems

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***Abstract*—In this paper, we present control algorithms for stabilization and asymptotic disturbance rejection for Hammerstein systems with positive real linear dynamics. To do this, we extend the nonlinear controller modification technique of Bernstein and Haddad to include matched plant disturbances. The controller is based on a novel Lyapunov function that estimates the disturbance bound. These estimates are then used to construct high-gain switching controllers that guarantee convergence of the plant output while accounting for input nonlinearities.**

***Index Terms*—Adaptive, disturbance rejection, Hammerstein, nonlinear, passivity, positive real.**

I. Introduction

WHILE many plants are nonlinear, it is often the case that the plant dynamics are inherently linear with a nonlinear input map. A linear system with an input nonlinearity is known as a Hammerstein system [4], [21], [22]. The present paper is motivated by the desire to control the response of a Hammerstein system with a matched exogenous disturbance. Our interest includes systems that have parameters, control inputs, or states that are constrained to operate in a limited region. Such constraints may arise from practical limitations such as saturation, positive-only control inputs, and constrained movement in predetermined physical gaps. For example, systems with electrostatic and electromagnetic actuators have quadratic (positive-only) control inputs, and the movement of the electrodes or the electromagnetic plates are limited by the gap between the plates.

Examples of such systems include microelectromechanical systems (MEMS) including micromachined accelerometers and gyros [5], [16], [28], electrostatically actuated micromirrors [3], [18], precision-controlled MEMS hard drive read–write heads [6], [11], [13] and electromagnetically levitated systems [24]. Most MEMS are modeled as second-order systems actuated by electromagnetic, electrostatic, and magnetic sensing and actuation techniques, which renders the controlled dynamics extremely nonlinear [20], [23]. Linear controllers designed by linearizing these systems around an operating point may be destabilizing outside of the linear operating range.

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In several applications, such as control of flexible structures, active noise control, and control of spring-mass-damper systems, the plant transfer function is known to be positive real. This property arises if the sensor and actuator are colocated and also dual, for example, force actuator and velocity sensor, torque actuator and angular rate sensor, or pressure actuator and volume velocity sensor [9]. In practice, the prospects for controlling such systems are quite good since, if the sensor and actuator dynamics are negligible, stability is unconditionally guaranteed as long as the controller is strictly positive real. For the case of positive real linear part, the stabilization problem was considered in [2], where a positive real controller was modified to account for the plant input nonlinearity. These results were extended in [8] to the case of dissipative systems. The objective of the present paper is to extend the results of [2] to include asymptotic rejection of matched but unknown disturbances. To do this, we develop a variation of the controller modification technique of [2] to include bounds on the disturbance. The controller includes states that estimate the disturbance bound. These estimates are then used by a high-gain switching controller to guarantee convergence of the plant output while accounting for the input nonlinearity. This approach is distinct from the switching controller obtained from the Lyapunov redesign technique [17, ch. 13], since our control algorithm is adaptive and does not require knowledge of the plant parameters and disturbance bound.

The contents of this paper are as follows. In Section II, we review the well-known result on the asymptotic stability of the feedback connection of a positive real (PR) plant and a strictly positive real (SPR) controller. In Section III, we consider a linear strictly proper PR plant with a matched but unknown disturbance. We augment the SPR controller of [2] with a switching term involving estimates of the disturbance bound which guarantees convergence of the plant output (Theorem 2). In Section IV, we consider the feedback interconnection of a Hammerstein plant with PR linear dynamics and a Hammerstein controller with SPR linear dynamics. Theorem 3 proposes a structure for the controller input nonlinearity to guarantee Lyapunov stability of the origin of the closed-loop system.

In Section V, we consider a Hammerstein plant with PR linear dynamics and quadratic input nonlinearity with a matched but unknown bounded disturbance. This case is of special importance since it models electrostatically and magnetically actuated systems. For this case, we construct a nonlinear controller that guarantees (Theorem 4) stability and asymptotic rejection of the unknown disturbance. In Section VI, we consider the most commonly found Hammerstein system, namely, a linear plant with control input saturation and bounded disturbance. For this case, we present a controller (Theorem 5) that guarantees stability and asymptotic disturbance rejection.

In Section VII, we provide illustrative numerical examples. In particular, we consider a quadratic input nonlinearity arising in problems of electromagnetic and electrostatic actuation [10]. First, we apply Theorem 4 to this problem and achieve asymptotic disturbance rejection. As an application of this result, in Section VII-C we consider a constant command tracking problem for an electromagnetically controlled oscillator [10]. We then apply the controller presented in Section V to this problem and achieve asymptotic tracking performance for constant and time-varying command inputs.

A. Notation

We use the following notation. For $y \in \mathbb{R}^m$, define

$$\mathrm{sign}(y) \triangleq \begin{bmatrix} \mathrm{sign}(y_1) \\ \vdots \\ \mathrm{sign}(y_m) \end{bmatrix} \tag{1}$$

where $\mathrm{sign}(y_i) = 1$ if $y_i > 0$, $\mathrm{sign}(y_i) = -1$ if $y_i < 0$, and $\mathrm{sign}(y_i) = 0$ if $y_i = 0$. For $u \in \mathbb{R}$ and $\bar{u} > 0$, define

$$\mathrm{sat}_{\bar{u}}(u) \triangleq \begin{cases} u, & |u| \le \bar{u} \\ \bar{u}\,\mathrm{sign}(u), & |u| \ge \bar{u} \end{cases} \tag{2}$$

where $\bar{u}$ is the saturation level. For $p \ge 1$, define the p-norm of $z \in \mathbb{R}^m$ by $\|z\|_p \triangleq (|z_1|^p + \cdots + |z_m|^p)^{1/p}$.

II. Stabilization of PR Plant With SPR Controller

Consider a minimal realization of the positive real plant

$$\dot{x} = Ax + Bu \tag{3}$$
$$y = Cx + Du \tag{4}$$

with control input $u \in \mathbb{R}^m$ and measurement $y \in \mathbb{R}^m$. Next, let (A_c, B_c, C_c) be minimal and strictly positive real, and consider the controller

$$\dot{x_c} = A_c x_c + B_c y \tag{5}$$
$$u = -C_c x_c \tag{6}$$

in feedback with the system (3) and (4) with $x_c \in \mathbb{R}^{n_c}$.

Although the following result is standard [7], [25], [26], we provide a proof to serve as a baseline for deriving later results.

Theorem 1: The equilibrium solution $\tilde{x} = [x^T \; x_c^T]^T = 0$ of the closed-loop system (3)–(6) is globally asymptotically stable.

Proof: Since (A, B, C, D) is positive real, the positive real lemma [1], [14], [25], [26] implies that there exist a positive-definite matrix $P \in \mathbb{R}^{n\times n}$ and matrices $L \in \mathbb{R}^{p\times n}$ and $W \in \mathbb{R}^{p\times m}$ such that

$$A^T P + PA = -L^T L \tag{7}$$
$$B^T P = C - W^T L \tag{8}$$
$$D + D^T = W^T W. \tag{9}$$

Since (A_c, B_c, C_c) is strictly positive real, there exists a positive-definite matrix $P_c \in \mathbb{R}^{n_c\times n_c}$, a matrix $L_c \in \mathbb{R}^{n_c\times p_c}$, and a real number $\varepsilon_c > 0$ such that

$$A_c^T P_c + P_c A_c = -L_c^T L_c - \varepsilon_c P_c \tag{10}$$
$$B_c^T P_c = C_c. \tag{11}$$

Next, consider the Lyapunov candidate

$$V(\tilde{x}) = x^T P x + x_c^T P_c x_c. \tag{12}$$

Then $\dot{V}(\tilde{x})$ along the closed-loop system trajectory is given by

$$\dot{V}(\tilde{x}) = -[Lx + Wu]^T [Lx + Wu] - x_c^T \left(\varepsilon_c P_c + L_c^T L_c\right) x_c \le 0. \tag{13}$$

Hence, the equilibrium solution $\tilde{x} = 0$ is Lyapunov stable. Let $\mathcal{E} \triangleq \dot{V}^{-1}(0)$. Using (13) and $u = -C_c x_c$, it follows that $\mathcal{E} = \{\tilde{x} : Lx = 0, x_c = 0\}$. Let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{E}$ and let $\tilde{x}(t) = [x^T(t) \; x_c^T(t)]^T$ be a solution of (3)–(6) in $\mathcal{M}$. Substituting $\dot{x}_c \equiv 0$ in (5) yields $B_c y \equiv 0$, which implies that $L_c A_c^{-1} B_c y \equiv 0$. Since the system (5), (6) is strictly positive real, it follows from [19] that rank $(L_c(\jmath\omega I - A_c)^{-1} B_c) = m$ for all $\omega \in \mathbb{R}$. Hence, rank $(L_c A_c^{-1} B_c) = m$, which implies $y \equiv 0$. Using (3) and (4), and noting that $Cx \equiv 0$, it follows that $CA^i x \equiv 0$ for $i = 1, \ldots, n-1$. Since (A, C) is observable, $x \equiv 0$, and therefore $\mathcal{M} = \{0\}$. By the invariant set theorem [17, Th. 3.4, p. 115], every trajectory converges to $\mathcal{M} = \{0\}$ as $t \to \infty$. □

III. Disturbance Rejection for PR Plant With High-Gain SPR Controller

Consider a minimal realization of the strictly proper plant

$$\dot{x} = Ax + B(u + d) \tag{14}$$
$$y = Cx \tag{15}$$

with control input $u \in \mathbb{R}^m$, measurement $y \in \mathbb{R}^m$, and disturbance $d \in \mathbb{R}^n$. We assume that (A, B, C) is positive real.

Assumption 1: There exist a known continuous function $f : [0, \infty) \to \mathbb{R}$ and positive constants α_1, α_2 such that

$$\|d(t)\|_2 \le \alpha_1 f(t) + \alpha_2, \quad t \ge 0. \tag{16}$$

Note that Assumption 1 requires that the function f be known. However, the constants α_1 and α_2 need not be known. Next, let (A_c, B_c, C_c) be minimal and strictly positive real and consider the controller

$$\dot{x_c} = A_c x_c + B_c y \tag{17}$$
$$\dot{\hat{\alpha}}_1 = k_1 f \|y\|_1 \hat{\alpha}_1^{3/2}, \quad \hat{\alpha}_1(0) > 0 \tag{18}$$
$$\dot{\hat{\alpha}}_2 = k_2 \|y\|_1 \hat{\alpha}_2^{3/2}, \quad \hat{\alpha}_2(0) > 0 \tag{19}$$
$$u = -C_c x_c - (\hat{\alpha}_1 f + \hat{\alpha}_2)\,\mathrm{sign}(y) \tag{20}$$

where $x_c \in \mathbb{R}^{n_c}$, $\hat{\alpha}_1, \hat{\alpha}_2 \in \mathbb{R}$, and k_1, k_2 are positive constants. Note that if d is bounded, then it suffices to let $f = 0$ and omit (18).

Theorem 2: The equilibrium solution $[x, x_c, \hat{\alpha}_1, \hat{\alpha}_2] = [0, 0, \alpha_1, \alpha_2]$ of the closed-loop system (14), (15), (17)–(20) is Lyapunov stable, and $y(t), x_c(t) \to 0$ as $t \to \infty$. Moreover, $\inf_{t\ge 0} \hat{\alpha}_1(t) > 0$, $\inf_{t\ge 0} \hat{\alpha}_2(t) > 0$, and $\lim_{t\to\infty} \hat{\alpha}_1(t)$ and $\lim_{t\to\infty} \hat{\alpha}_2(t)$ exist.

Proof: Using the positive real lemma, there exist a positive-definite matrix $P \in \mathbb{R}^{n\times n}$ and a matrix $L \in \mathbb{R}^{p\times n}$ satisfying (7) and (8). Furthermore, there exist a positive-definite matrix $P_c \in \mathbb{R}^{n_c\times n_c}$, a matrix $L_c \in \mathbb{R}^{n_c\times p_c}$ and a real number $\varepsilon_c > 0$ satisfying (10) and (11). Let $\tilde{x} = [\,x^T \quad x_c^T \quad \hat{\alpha}_1 \quad \hat{\alpha}_2\,]^T$ and $\tilde{x}_0 = [\,0 \quad 0 \quad \alpha_1 \quad \alpha_2\,]^T$.

Consider the positive-definite function $V : \mathbb{R}^n \times \mathbb{R}^{n_c} \times (0,\infty) \times (0,\infty) \to \mathbb{R}$ defined by

$$V(\tilde{x}) = x^T P x + x_c^T P_c x_c + \frac{4}{k_1}\left(\sqrt{\hat{\alpha}_1} + \frac{\alpha_1}{\sqrt{\hat{\alpha}_1}}\right) + \frac{4}{k_2}\left(\sqrt{\hat{\alpha}_2} + \frac{\alpha_2}{\sqrt{\hat{\alpha}_2}}\right) - \frac{8\sqrt{\alpha_1}}{k_1} - \frac{8\sqrt{\alpha_2}}{k_2}. \quad (21)$$

Note that $V(\tilde{x})$ is minimized at $\tilde{x} = \tilde{x}_0$ with $V(\tilde{x}_0) = 0$. Suppose that either $\hat{\alpha}_1$ or $\hat{\alpha}_2$ vanishes on $(0,\infty)$ and let $t_1 = \min\{t > 0 : \hat{\alpha}_1(t) = 0 \text{ or } \hat{\alpha}_2(t) = 0\}$. Therefore, for $t \in [0, t_1)$, we have

$$\begin{aligned}\dot{V}(\tilde{x}(t),t) =& - x^T L^T L x - x_c^T\left(\varepsilon_c P_c + L_c^T L_c\right)x_c \\ &+ 2x_c^T C_c^T y + 2y^T \\ &\times[-C_c x_c - (\hat{\alpha}_1 f + \hat{\alpha}_2)\operatorname{sign}(y) + d] \\ &+ \frac{2}{k_1\hat{\alpha}_1^{3/2}}(\hat{\alpha}_1 - \alpha_1)\dot{\hat{\alpha}}_1 \\ &+ \frac{2}{k_2\hat{\alpha}_2^{3/2}}(\hat{\alpha}_2 - \alpha_2)\dot{\hat{\alpha}}_2.\end{aligned}$$

Using (18) and (19) and noting that $y^T d \le \|y\|_2\|d\|_2 \le (\alpha_1 f + \alpha_2)\|y\|_2$ and $\operatorname{sign}^T(y)y = \|y\|_1 \ge \|y\|_2$, it follows that for $t \in [0, t_1)$

$$\begin{aligned}\dot{V}(\tilde{x}(t),t) \le& - x^T L^T L x - x_c^T\left(\varepsilon_c P_c + L_c^T L_c\right)x_c \\ &+ \frac{2}{k_1\hat{\alpha}_1^{3/2}}(\hat{\alpha}_1 - \alpha_1)\left(\dot{\hat{\alpha}}_1 - k_1\hat{\alpha}_1^{3/2}\|y\|_1 f\right) \\ &+ \frac{2}{k_2\hat{\alpha}_2^{3/2}}(\hat{\alpha}_2 - \alpha_2)\left(\dot{\hat{\alpha}}_2 - \hat{\alpha}_2^{3/2}k_2\|y\|_1\right) \\ &- (\alpha_1 f + \alpha_2)(\|y\|_1 - \|y\|_2) \\ \le& - x^T L^T L x - x_c^T(\varepsilon_c P_c + L_c^T L_c)x_c \le 0. \quad (22)\end{aligned}$$

Therefore, $V(\tilde{x}(t))$ is nonincreasing and thus bounded on $t \in [0, t_1)$. This implies that for $i = 1, 2$, $\sqrt{\hat{\alpha}_i(t)}$ and $1/\sqrt{\hat{\alpha}_i(t)}$ are bounded on $[0, t_1)$. Therefore, for $i = 1, 2$, there exists $\delta_i > 0$ such that $\hat{\alpha}_i(t) > \delta_i$ for all $t \in [0, t_1)$. However, $\hat{\alpha}_1(t)$ and $\hat{\alpha}_2(t)$ are continuous, which contradicts $\hat{\alpha}_1(t_1) = 0$ or $\hat{\alpha}_2(t_1) = 0$. Therefore, both $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are nonzero on $[0,\infty)$, and thus (22) is valid for all $t \in [0,\infty)$. Hence, $V(\tilde{x}(t))$ is nonincreasing and thus bounded on $[0,\infty)$. Consequently, for $i = 1, 2$, $\hat{\alpha}_i(t)$ is bounded on $[0,\infty)$ and $\inf_{t\ge 0}\hat{\alpha}_i(t) > 0$.

Let $\mathcal{E} \triangleq \dot{V}^{-1}(0)$. Using (22), it follows that $\mathcal{E} \subseteq \{\tilde{x} : Lx = 0,\, x_c = 0\}$. Let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{E}$ and let $\tilde{x}(t) = [x^T(t) \quad x_c^T(t) \quad \hat{\alpha}_1(t) \quad \hat{\alpha}_2(t)]^T$ be a solution of (14) and (15) and (17)–(20) in $\mathcal{M}$. Using similar argument as in the proof of Theorem 1, it can be shown that $y \equiv 0$, $\dot{\hat{\alpha}}_1 \equiv \dot{\hat{\alpha}}_2 \equiv 0$ on $\mathcal{M}$. By the invariant set theorem, every trajectory converges to $\mathcal{M}$ as $t \to \infty$. Therefore, $y(t), x_c(t) \to 0$ as $t \to \infty$. Furthermore, noting that $\hat{\alpha}_1(t), \hat{\alpha}_2(t) > 0$ for all $t \ge 0$, $\dot{\hat{\alpha}}_1(t), \dot{\hat{\alpha}}_2(t) \to 0$ as $t \to \infty$ and using the fact that $\hat{\alpha}_1(t)$ and $\hat{\alpha}_2(t)$ are bounded, it follows that $\lim_{t\to\infty}\hat{\alpha}_1(t)$ and $\lim_{t\to\infty}\hat{\alpha}_2(t)$ exist. □

Remark 1: The closed-loop system (14) and (15) and (17)–(20) is a differential equation with right-hand side discontinuity at $y = 0$. The closed-loop system can be regularized into a differential inclusion with $\operatorname{sign}(y_i)$ replaced by a set-valued-map $S(y_i)$, where $S(y_i) = 1$ if $y_i > 0$, $S(y_i) = -1$ if $y_i < 0$, and $S(y_i) \in [-1, 1]$ if $y_i = 0$. The sufficient conditions of [27] for the existence and continuity of solutions are satisfied, and the closed-loop differential inclusion has an absolutely continuous solution. Noting continuous dependence on the initial conditions and using Theorem 2.2 in [12], we conclude that the solution set of the closed-loop differential inclusion equals the solution set of the closed-loop system (14) and (15) and (17)–(20). Since the solutions are absolutely continuous, the invariant set theorem applies.

Remark 2: The presence of $\operatorname{sign}(y)$ in (20) signifies high-gain control, which is necessary to achieve complete disturbance rejection. In practice, one can approximate $\operatorname{sign}(y)$ in (20)with $\gamma\operatorname{sat}_{1/\gamma}(y)$, where $\gamma \gg 1$ is the maximum allowable gain.

As an application of Theorem 2, we consider the tracking problem in which the output of the system

$$\begin{aligned}\dot{x} =& Ax + Bu \\ y =& Cx\end{aligned}$$

is required to follow a known C^1 bounded reference signal $y_r(t)$. Assume that $\operatorname{rank}(C) = m$ and let $x_r(t) = C^T(CC^T)^{-1}y_r(t) \in \mathbb{R}^n$ so that $y_r(t) = Cx_r(t)$. Letting $\tilde{x} \triangleq x - x_r$ and $e \triangleq y - y_r$ we obtain

$$\dot{\tilde{x}} = A\tilde{x} + Bu + (Ax_r + \dot{x}_r) \quad (23)$$

$$e = C\tilde{x}. \quad (24)$$

We assume that there exists $\psi_s(t) \in \mathbb{R}^m$ such that $B\psi_s(t) = Ax_r(t) + \dot{x}_r(t)$, $t \ge 0$. Furthermore, we assume that there exist constants $\alpha_1, \alpha_2 > 0$ and a continuous function f such that (16) is satisfied with $d(t)$ replaced by $\psi_s(t)$. Consider the closed-loop system consisting of the plant (23) and (24) and the controller (17)–(20) with $y = e$. Theorem 2 applied to this closed-loop system yields asymptotic tracking performance, namely, $e(t) \to 0$ as $t \to \infty$.

Next, we consider the case in which the disturbance $d \in \mathbb{R}^n$ is constant. As above, we assume that (A_c, B_c, C_c) is strictly positive real. The following result presents a simplified controller in this case.

Proposition 1: Consider the controller

$$\dot{x_c} = A_c x_c + B_c y \quad (25)$$

$$\dot{\psi} = Ky \quad (26)$$

$$u = - C_c x_c - \psi \quad (27)$$

where $x_c \in \mathbb{R}^{n_c}$, $\psi \in \mathbb{R}^m$, and $K \in \mathbb{R}^{m\times m}$ is positive definite. Then the zero solution of the closed-loop system (14) and (15) and (25)–(27) is Lyapunov stable. Moreover, $y(t), x_c(t) \to 0$ as $t \to \infty$, and $\lim_{t\to\infty}\psi(t)$ exists.

Proof: The proof is similar to that of Theorem 2 with $V(\tilde{x})$ given by $V(\tilde{x}) = x^T P x + x_c^T P_c x_c + (\psi - d)^T K^{-1}(\psi - d)$ □

The requirement on the controller that (A_c, B_c, C_c) be SPR can be weakened to marginally strictly positive real (MSPR) [15]. It is shown in [15] that the negative feedback interconnection of a PR and MSPR system is asymptotically stable. The positive real lemma, in this case, is applicable with the exception that $L_c \in \mathbb{R}^{n_c\times p_c}$ is of the form $[0_{m\times n_1} \;\; \hat{L}_c]$, where n_1 are the number of controller poles on the imaginary axis. Theorem 1, Theorem 2, and Proposition 1 remain unchanged if (A_c, B_c, C_c) is MSPR. The proofs, however have to be modified as in [15].

In fact, the controller in Proposition 1 (with integrator state ψ) is a MSPR controller in negative feedback with a PR plant.

IV. Stabilization of Hammerstein PR Plant With Hammerstein SPR Controller

Consider the Hammerstein plant

$$\dot{x} = Ax + B\sigma(u, y) \tag{28}$$

$$y = Cx \tag{29}$$

with control input $u \in \mathbb{R}^m$, measurement $y \in \mathbb{R}^m$, and input nonlinearity $\sigma : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$. In this section, we generalize the results in [2] to input nonlinearities σ that depend on both u and y. The main contribution of [2] is that the feedback interconnection results are not based on absolute stability criteria (circle or Popov conditions), which require a gain or phase constraint on the linear portion of the loop transfer function. The idea is to modify the controller in Theorem 1 when the plant possesses an arbitrary input nonlinearity σ. We assume that the linear system (A, B, C) is minimal and positive real. In addition, we assume that σ is continuous with $\sigma(0, y) = 0$ for all $y \in \mathbb{R}^m$ and we write $\sigma(u, y) = [\, \sigma_1(u, y) \quad \cdots \quad \sigma_m(u, y) \,]^T$.

Next, let (A_c, B_c, C_c) be minimal and strictly positive real, and consider the nonlinear controller

$$\dot{x_c} = A_c x_c + B_c \beta(u, y) y \tag{30}$$

$$u = -C_c x_c \tag{31}$$

where $x_c \in \mathbb{R}^{n_c}$, in feedback with the plant (28), (29). Here, the continuous function $\beta : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m \times m}$ is constructed to satisfy

$$\beta(u, y) u = \sigma(u, y) + F(u, y) y \tag{32}$$

for all $u, y \in \mathbb{R}^m$, where $F : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m \times m}$ and $F(u, y)$ is a nonnegative-definite matrix for all $u, y \in \mathbb{R}^m$. Setting $u = 0$ in (32), it follows that $F(0, y) y = 0$ for all $y \in \mathbb{R}^m$. For example, we can choose $F(u, y) = \alpha y^T y u u^T$, where $\alpha \geq 0$.

In particular, letting $F(u, y) \equiv 0$, (32) can be satisfied by choosing

$$\beta(u, y) \triangleq \begin{bmatrix} \beta_1(u, y) & & 0 \\ & \ddots & \\ 0 & & \beta_m(u, y) \end{bmatrix} \tag{33}$$

where, for $i = 1, \ldots, m$

$$\begin{aligned} \beta_i(u, y) &= \sigma_i \frac{(u, y)}{u_i}, \quad u_s i \neq 0 \\ &= \lim_{\varepsilon \to 0} \sigma_i \frac{(u + \varepsilon e_i, y)}{\varepsilon}, \quad u_i = 0 \end{aligned} \tag{34}$$

assuming that the indicated limit exists. Here $u = [\, u_1 \quad \ldots \quad u_m \,]^T$ and e_i is the ith unit coordinate vector. Let $\tilde{x} \triangleq [\, x^T \quad x_c^T \,]^T$.

Theorem 3: The zero solution of the closed-loop system (28)–(31) is Lyapunov stable. Furthermore, if det $\beta(0, y) \neq 0$ for all $y \in \mathbb{R}^m$ or if (A, B, C) is strictly positive real, then the zero solution is globally asymptotically stable.

Proof: By assumption, there exist a positive-definite matrix $P \in \mathbb{R}^{n \times n}$ and matrices $L \in \mathbb{R}^{p \times n}$ satisfying (7)–(9) with $W = 0$. Furthermore, there exist a positive-definite matrix $P_c \in \mathbb{R}^{n_c \times n_c}$, a matrix $L_c \in \mathbb{R}^{n_c \times p_c}$ and a real number $\varepsilon_c > 0$ such that (10) and (11) are satisfied.

Consider the Lyapunov candidate

$$V(\tilde{x}) = x^T P x + x_c^T P_c x_c.$$

Then

$$\begin{aligned} \dot{V}(\tilde{x}) &= -x^T L^T L x - x_c^T (\varepsilon_c P_c + L_c^T L_c) x_c - 2 y^T F(u, y) y \\ &\leq 0. \end{aligned} \tag{35}$$

Hence, $\tilde{x} = 0$ is Lyapunov stable. Let $\mathcal{E} \triangleq \dot{V}^{-1}(0)$. Using (35), it follows that $\mathcal{E} = \{\tilde{x} : Lx = 0, x_c = 0\}$. Let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{E}$ and let $\tilde{x}(t) = [x^T(t) \; x_c^T(t)]^T$ be a solution of (28)–(31) in $\mathcal{M}$. Substituting $\dot{x}_c \equiv 0$ in (30) yields $B_c \beta(0, y) y \equiv 0$, which implies that $L_c A_c^{-1} B_c \beta(0, y) y \equiv 0$. Since (A_c, B_c, C_c) is strictly positive real, it follows that rank $(L_c A_c^{-1} B_c) = m$, which implies $\beta(0, y) y \equiv 0$.

Now assume that det $\beta(0, y) \neq 0$ for all $y \in \mathbb{R}^m$. Then $y \equiv 0$. Using (28) and (29), and noting that $Cx \equiv 0$, it follows that $CA^i x \equiv 0$ for $i = 1, \ldots, n-1$. Therefore, $\mathcal{O}x \equiv 0$, where $\mathcal{O}$ is defined in the proof of Theorem 1. Since (C, A) is observable, rank $(\mathcal{O}) = n$. Hence, $x \equiv 0$ and therefore, $\mathcal{M} = \{0\}$. By the invariant set theorem, every trajectory converges to $\mathcal{M} = \{0\}$ as $t \to \infty$.

Alternatively, assume that (A, B, C) is strictly positive real. Then

$$\begin{aligned} \dot{V}(\tilde{x}) = &-x^T (\varepsilon P x + L^T L) x \\ &- x_c^T (\varepsilon_c P_c + L_c^T L_c) x_c - 2 y^T F(u, y) y. \end{aligned}$$

Therefore, $\mathcal{E} = \mathcal{M} = \{\tilde{x} : x = 0, x_c = 0\}$ and the result follows. □

V. Disturbance Rejection for Hammerstein SPR Plant With Quadratic Input Nonlinearity

Many electromechanically and electrostatically controlled systems are modeled as second-order systems with state-dependent quadratic control inputs. In order to address the problem of controlling such systems we consider a class of Hammerstein systems with quadratic input nonlinearity and matched disturbance.

Consider the SISO plant

$$\dot{x} = Ax + Bg(y)(u^2 + d) \tag{36}$$

$$y = Cx \tag{37}$$

with scalar control input $u \in \mathbb{R}$, scalar measurement $y \in \mathbb{R}$, and bounded disturbance $d \in \mathbb{R}$ satisfying $d(t) \leq 0$ for all $t \geq 0$. Here, the piecewise continuous function $g : \mathbb{R} \to \mathbb{R}$ is bounded and satisfies $g(y) \neq 0$ for all $y \in \mathbb{R}$. Assume that (A, B, C) is minimal and strictly positive real. Next, consider the controller shown in (38)–(40) at the bottom of the next page, where $\hat{a} \in \mathbb{R}$ and $K > 0$. Assume that (A_c, B_c, C_c) is minimal and strictly positive real.

Theorem 4: The equilibrium solution $[x \; x_c \; \hat{a}] = [0 \; 0 \; a]$ of the system (36)–(40) is Lyapunov stable and $x(t), x_c(t) \to 0$

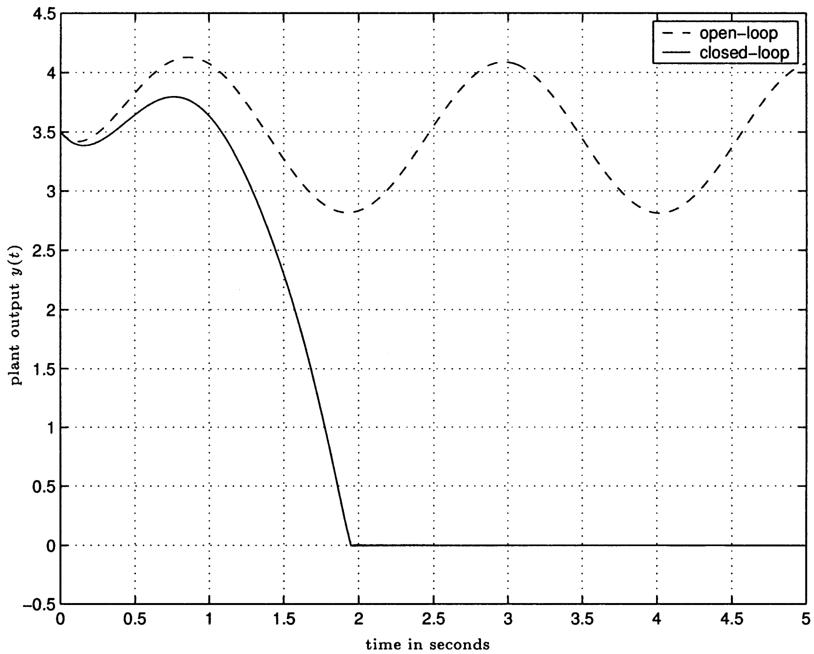

Fig. 1. Response of the system (53), (54) with the controller (17)–(20) and sinusoidal disturbance $d(t) = 3\sin(3t)$.

as $t \to \infty$. Furthermore, $\hat{a}(t)$ is bounded for all $t \geq 0$ and $\inf_{t\geq 0} \hat{a}(t) > 0$.

Proof: Let $a > 0$ satisfy $|d(t)| < a$ for all $t \geq 0$. Let $\tilde{x} = \left[x^T \; x_c^T \; \hat{a}\right]^T$ and consider the positive-definite function $V : \mathbb{R}^n \times \mathbb{R}^{n_c} \times (0,\infty) \to \mathbb{R}$ defined by

$$V(\tilde{x}) = x^T P x + x_c^T P_c x_c + \frac{1}{K}\left(\sqrt{\hat{a}} + \frac{a}{\sqrt{\hat{a}}}\right) - \frac{2\sqrt{a}}{K}.$$

It can be shown that $\dot{V}(\tilde{x}(t), t) \leq 0$ for all $t \geq 0$. The result follows using the invariant set theorem as in the proof of Theorem 2. □

Remark 3: Theorem 4 can be extended to plants with more general nonlinearities $\sigma(u)$ satisfying $\sigma(\mathbb{R}) = [0,\infty)$ or $\sigma(\mathbb{R}) = (-\infty, 0]$. Consider the plant (36), (37) with u^2 replaced by $\sigma(u)$ in feedback with the controller (38), (39), and

$$u \in \sigma^{-1}\left[s_u \frac{\frac{\hat{a}(1-\mathrm{sign}(y))}{2} + (C_c x_c)^2}{g(y)}\right]. \tag{41}$$

Then the closed-loop system yields $x(t), x_c(t) \to 0$ as $t \to \infty$.

VI. Disturbance Rejection for Hammerstein PR Plant With Input Saturation

Consider the single-input–single-output (SISO) plant

$$\dot{x} = Ax + B(\mathrm{sat}_{\bar{u}}(u) + d) \tag{42}$$

$$y = Cx \tag{43}$$

with scalar control input $u \in \mathbb{R}$ and scalar measurement $y \in \mathbb{R}$. The disturbance $d \in \mathbb{R}$ is matched and bounded so that $\sup_{t\geq 0} |d(t)| < \bar{u}$. Assume that (A, B, C) is minimal and positive real.

Next, consider the controller

$$\dot{x}_c = A_c x_c + B_c \beta(x_c, \hat{a}) y \tag{44}$$

$$\dot{\hat{a}} = K\hat{a}^{3/2}(1-\hat{a})|y|, \quad \hat{a}(0) \in (0,1) \tag{45}$$

$$u = -\bar{u}\tanh\left[\frac{C_c x_c}{\bar{u}} + \mathrm{sign}(y)\tanh^{-1}(\hat{a})\right] \tag{46}$$

where $x_c \in \mathbb{R}^{n_c}$, $\hat{a} \in \mathbb{R}$, $K > 0$ and

$$\beta(x_c, \hat{a}) = \frac{\tanh\left(\frac{C_c x_c}{\bar{u}}\right)}{\frac{C_c x_c}{\bar{u}}} \frac{1 - \hat{a}^2}{1 + \mathrm{sign}(y)\hat{a}\tanh\left(\frac{C_c x_c}{\bar{u}}\right)}. \tag{47}$$

Assume that (A_c, B_c, C_c) is minimal and positive real.

Theorem 5: The equilibrium solution $[x \; x_c \; \hat{a}] = [0 \; 0 \; a]$ of the system (42)–(46) is Lyapunov stable and $y(t), x_c(t) \to 0$ as $t \to \infty$. Furthermore, $\inf_{t\geq 0} \hat{a}(t) > 0$ and $\sup_{t\geq 0} \hat{a}(t) < 1$.

Proof: Since $\sup_{t\geq 0} |d(t)| < \bar{u}$, there exists $\alpha > 0$ such that $|d(t)| < \alpha < \bar{u}$ for all $t \geq 0$. Let $a \triangleq \alpha/\bar{u} < 1$ so that $|d(t)| < a\bar{u}$ for all $t \geq 0$. Next, noting that $\mathrm{sat}_{\bar{u}}(\bar{u}\tanh(u)) =$

$$\dot{x}_c = A_c x_c + B_c(-C_c x_c) y \tag{38}$$

$$\dot{\hat{a}} = \begin{cases} -\mathrm{sign}(g(y)) K y \hat{a}^{3/2} & \text{if sign } (g(y))y \leq 0 \\ 0 & \text{if sign } (g(y))y \geq 0 \end{cases}, \quad \hat{a}(0) > 0 \tag{39}$$

$$u = \sqrt{\left(\frac{1}{|g(y)|}\right)\left(\hat{a}\frac{(1-\mathrm{sign}(y))}{2} + (C_c x_c)^2\right)} \tag{40}$$

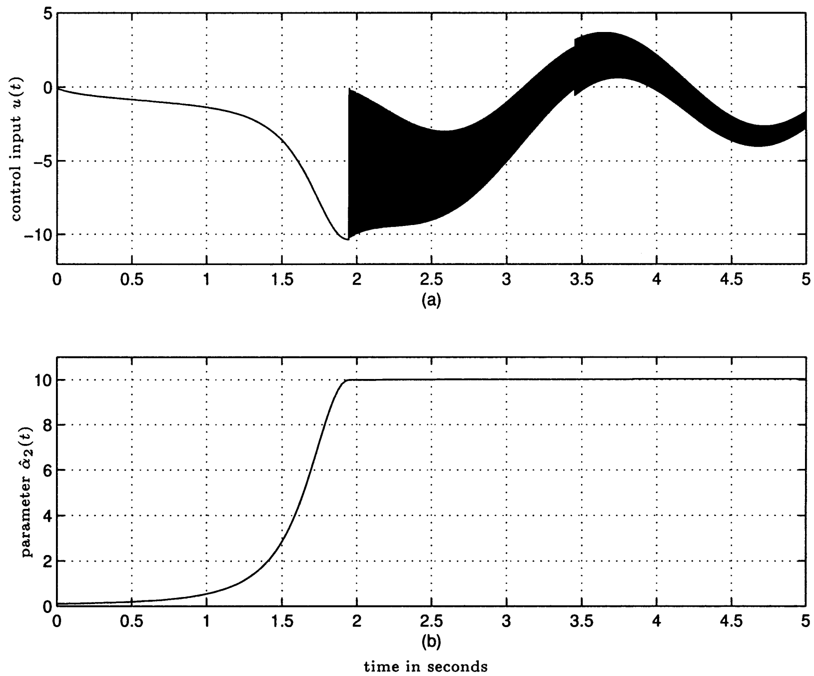

Fig. 2. (a) Control u and (b) parameter $\hat{\alpha}_2$ corresponding to the response in Fig. 1. Note the chattering of the control input $u(t)$.

$\bar{u}\tanh(u)$ for all $u \in \mathbb{R}$ and using (46), it follows that (42) can be written as

$$\dot{x} = Ax + B\bar{u}\tanh(u_c) + d \tag{48}$$

where

$$u_c \triangleq -\frac{C_c x_c}{\bar{u}} - \operatorname{sign}(y)\tanh^{-1}(\hat{a}).$$

By the positive real lemma, there exist a positive-definite matrix $P \in \mathbb{R}^{n\times n}$, and a matrix $L \in \mathbb{R}^{p\times n}$ satisfying (7), (8) with $W = 0$. Furthermore, there exist a positive-definite matrix $P_c \in \mathbb{R}^{n_c\times n_c}$, a matrix $L_c \in \mathbb{R}^{n_c\times p_c}$, and real number $\varepsilon_c > 0$ satisfying (10), (11).

Next, let $\tilde{x} \triangleq \left[x^T\ x_c^T\ \hat{a}\right]^T$ and consider the Lyapunov candidate $V : \mathbb{R}^n \times \mathbb{R}^{n_c} \times (0,1) \to \mathbb{R}$ defined by

$$V(\tilde{x}) = x^T P x + x_c^T P_c x_c + \frac{4\bar{u}}{K}\left((1-a)\tanh^{-1}(\sqrt{\hat{a}}) + \frac{a}{\hat{a}}\right) - \frac{4\bar{u}}{K}. \tag{49}$$

Note that V is radially unbounded and has its minimum at $\tilde{x}_0 = [0\ 0\ a]^T$. Let $t_1 = \min\{t : \hat{a}(t) = 0 \text{ or } \hat{a}(t) = 1\}$. Then, for all $t \in [0, t_1)$

$$\begin{aligned}
&\dot{V}(\tilde{x}(t), t)\\
&= -x^T L^T L x - x_c^T\left(\varepsilon_c P_c + L_c^T L_c\right)x_c + 2yd\\
&\quad + 2y\left[\beta(x_c,\hat{a})C_c x_c - \bar{u}\frac{\tanh\left(\frac{C_c x_c}{\bar{u}}\right) + \operatorname{sign}(y)\hat{a}}{1 + \operatorname{sign}(y)\hat{a}\tanh\left(\frac{C_c x_c}{\bar{u}}\right)}\right]\\
&\quad + \frac{2\bar{u}}{K}\frac{\hat{a} - a}{(1-\hat{a})\hat{a}^{3/2}}\dot{\hat{a}}.
\end{aligned}$$

Using (47) and noting that $yd \le a\bar{u}\|y\|_2$ and $\operatorname{sign}(y)y = |y| \ge \|y\|_2$, we obtain, for all $t \in [0, t_1)$ [see (50) at the bottom of the page]. Therefore, $V(\tilde{x}(t))$ is nonincreasing and bounded on $[0, t_1)$, which implies that both $\tanh^{-1}(\sqrt{\hat{a}})$ and $1/\hat{a}$ are bounded on $[0, t_1)$. Therefore, there exist $\delta_1, \delta_2 > 0$ such that $\delta_1 < \hat{a}(t) < \delta_2 < 1$ for all $t \in [0, t_1)$. Since $\hat{a}$ is continuous, this contradicts $\hat{a}(t_1) = 0$ and $\hat{a}(t_1) = 1$. Therefore, (50) is valid for all $t \in [0, \infty)$, and thus $V(\tilde{x}(t))$ is nonincreasing and bounded on $[0, \infty)$. Therefore, $\hat{a}(t) \in (0, 1)$ for all $t \in [0, \infty)$.

$$\begin{aligned}
\dot{V}(\tilde{x}(t), t) \le& -x^T L^T L x - x_c^T(\varepsilon_c P_c + L_c^T L_c)x_c - 2a(|y| - \|y\|_2)\\
&+ 2\bar{u}\left[y\beta(x_c,\hat{a})\frac{C_c x_c}{\bar{u}} - \frac{y\tanh\left(\frac{C_c x_c}{\bar{u}}\right) + \hat{a}|y|}{1 + \operatorname{sign}(y)\hat{a}\tanh\left(\frac{C_c x_c}{\bar{u}}\right)} + \hat{a}|y|\right]\\
&+ \frac{2\bar{u}(\hat{a} - a)}{K}\left[\frac{\dot{\hat{a}}}{(1-\hat{a})\hat{a}^{3/2}} - K|y|\right]\\
\le& -x^T L^T L x - x_c^T(\varepsilon_c P_c + L_c^T L_c)x_c\\
\le& 0.
\end{aligned} \tag{50}$$

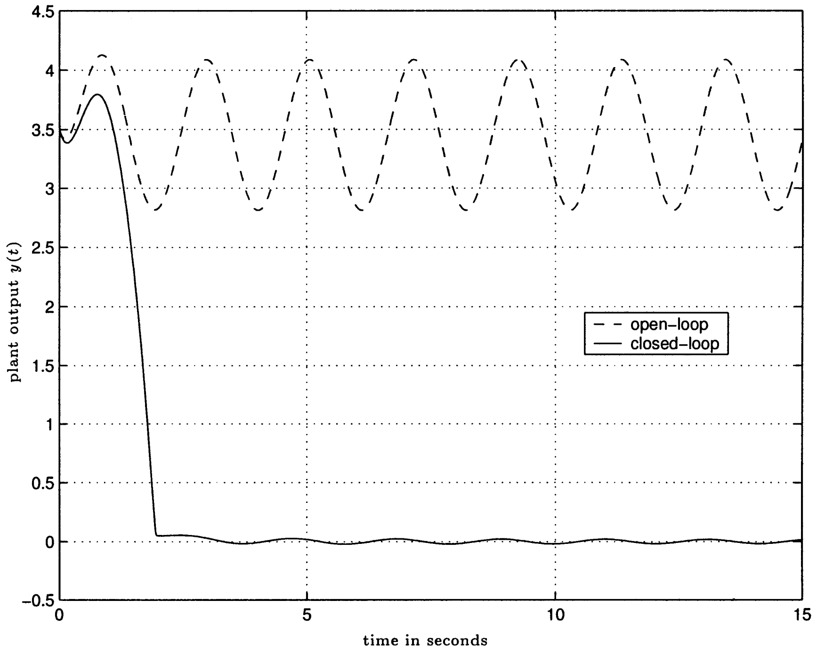

Fig. 3. Response of the system (53), (54) and the controller (17)–(20) with $\mathrm{sign}(y)$ replaced by $10\mathrm{sat}_{0.1}(y)$. Here $d(t) = 3\sin(3t)$.

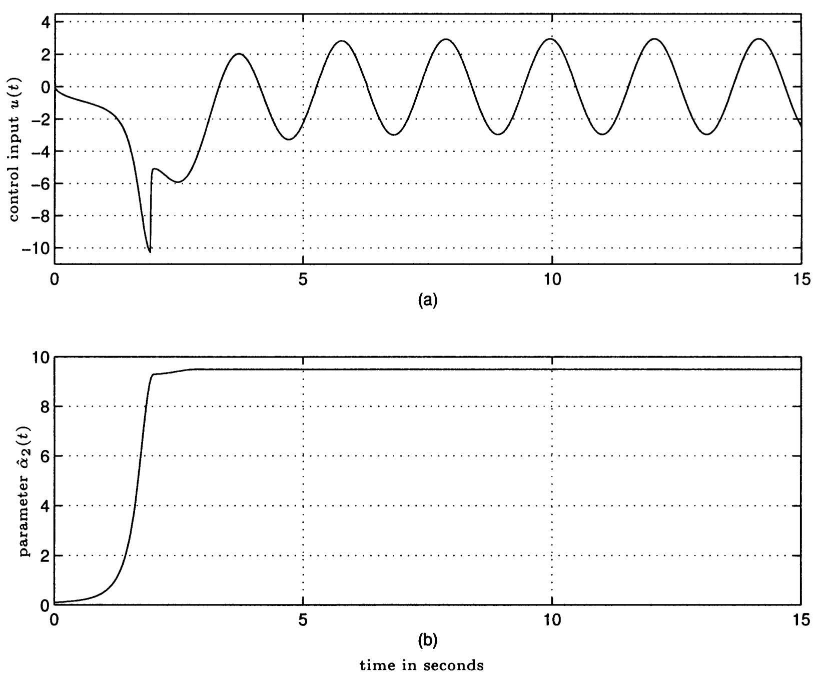

Fig. 4. (a) control u and (b) parameter $\hat{\alpha}_2$ corresponding to the response in Fig. 3.

Let $\mathcal{E} \triangleq \dot{V}^{-1}(0)$. Then it follows that $\mathcal{E} = \{\tilde{x} : Lx = 0,\, x_c = 0\}$. Let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{E}$. Let $\tilde{x}(t) = [x^T(t) \quad x_c^T(t) \quad \hat{a}(t)^T]^T$ be a solution of (42)–(46) on $\mathcal{M}$. Substituting $\dot{x}_c \equiv 0$ in (44) yields $B_c\beta(u_c, \hat{a})y \equiv 0$, which implies that $L_cA_c^{-1}B_c\beta(u_c, \hat{a})y \equiv 0$. Noting that $\hat{a}(t) \in (0,1)$ for all $t \geq 0$ it follows that $1 - \hat{a}^2 > 0$ and $1 - \mathrm{sign}(y)\hat{a}\tanh(u_c) > 0$ for all $t \geq 0$. Therefore, $\beta(u_c, \hat{a}) > 0$ for all $t \geq 0$. Since (A_c, B_c, C_c) is strictly positive real, it follows that rank $(L_cA_c^{-1}B_c) = m$, which implies $y \equiv 0$ on $\mathcal{M}$. Using (45), it follows that $\hat{a} \equiv 0$. By the invariant set theorem, every trajectory converges to $\mathcal{M}$ as $t \to \infty$. Therefore, $y(t), x_c(t) \to 0$ as $t \to \infty$. Furthermore, since $\dot{\hat{a}}(t) \to 0$, and $\hat{a}(t)$ is bounded, it follows that $\lim_{t\to\infty} \hat{a}(t)$ exists. □

VII. Numerical Examples

For the following numerical examples, let

$$A = \begin{bmatrix} -7 & -10 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [1 \quad 4 \quad 3] \tag{51}$$

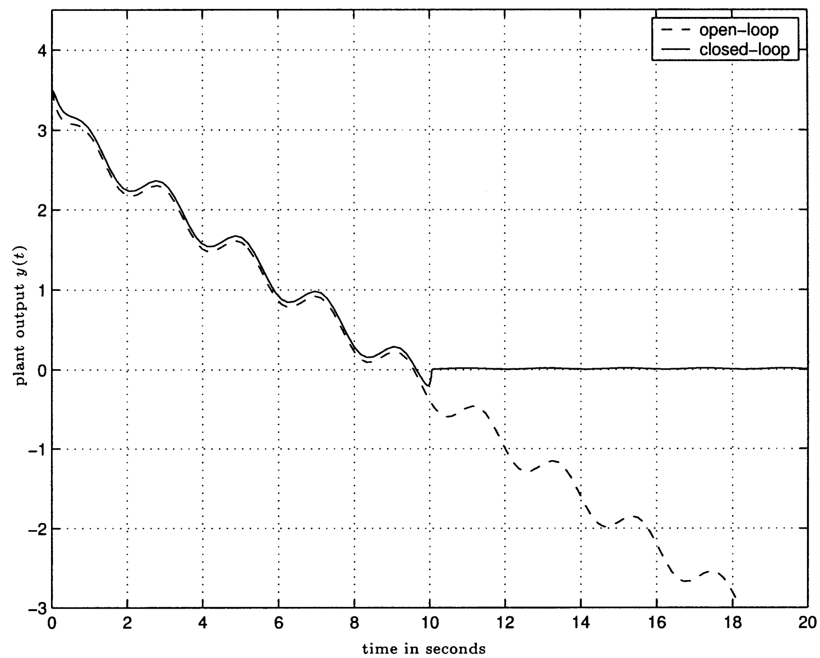

Fig. 5. Hammerstein plant with quadratic input nonlinearity. The closed-loop response of (36)–(40) is shown for the disturbance $d(t) = -1.0 - \sin(3t)$.

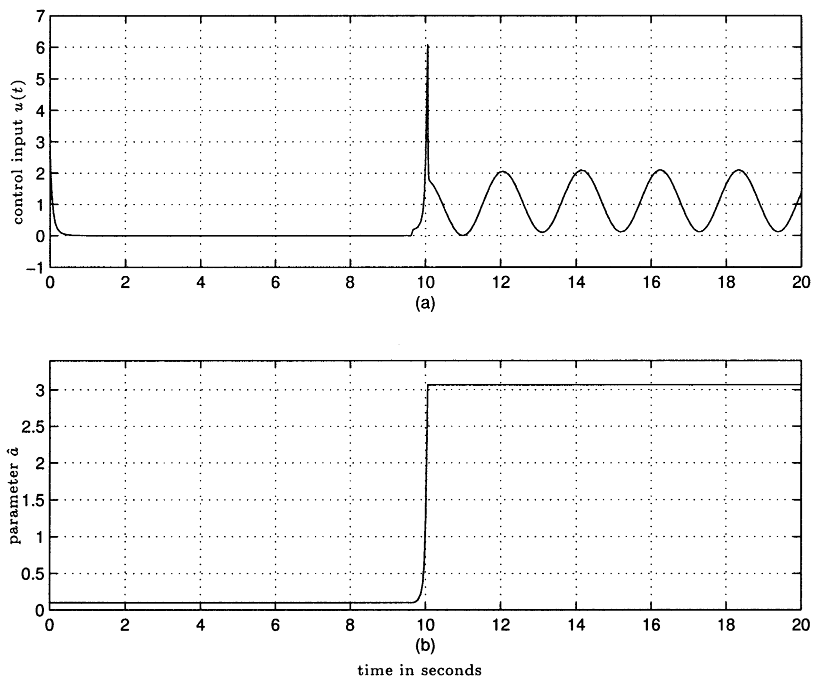

Fig. 6. (a) Control u and (b) parameter $\hat{a}$ for the response in Fig. 5.

and

$$A_c = \begin{bmatrix} -12.5 & -40.5 & -36 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C_c = [1 \quad 6.5 \quad 9]. \tag{52}$$

Here, (A, B, C) is PR and (A_c, B_c, C_c) is SPR. The matrix A has an eigenvalue at zero.

A. Linear Plant With Disturbance

To illustrate Theorem 2, consider the plant

$$\dot{x} = Ax + B(u + d) \tag{53}$$

$$y = Cx \tag{54}$$

where $d(t) = 3.0\sin(3t)$. Since d is bounded (but otherwise unknown), (16) is satisfied with $\alpha_1 = 0$ and, thus, $\hat{\alpha}_1$ can be ignored in controller (17)–(20). Figs. 1 and 2 show that the controller converges sufficiently fast to reject $d(t)$ and that $\hat{\alpha}_2(t)$ converges.

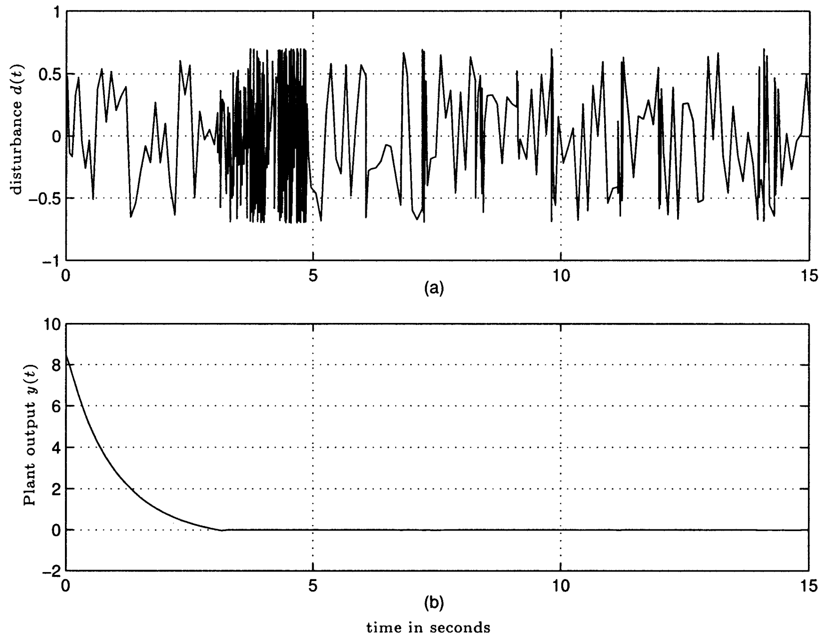

Fig. 7. Hammerstein PR system with input saturation and disturbance. (a) Random bounded disturbance $d(t)$ and (b) response of the closed-loop system (55), (56) and (44)–(46).

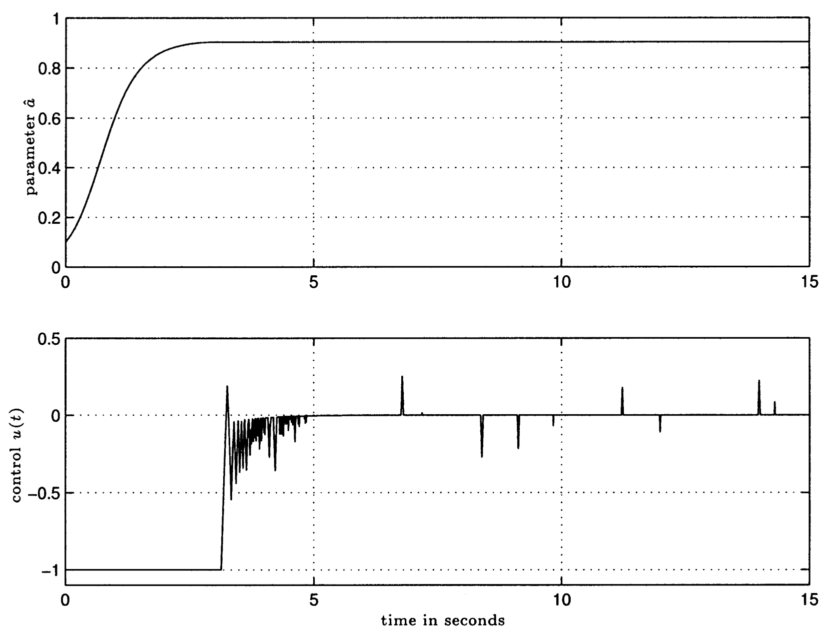

Fig. 8. Control parameter $\hat{a}(t)$ and the control input $\tanh(u(t))$ corresponding to the response in Fig. 7.

However, the control input chatters (see Fig. 2) for small values of the output $y(t)$ due to the presence of the term $\mathrm{sign}(y)$ in (20). As in Remark 2 we approximate $\mathrm{sign}(y)$ by $10\mathrm{sat}_{0.1}(y)$, hence effectively limiting the gain of that term to ten. Figs. 3 and 4 show that the controller is able to attenuate the sinusoidal disturbance although asymptotic disturbance is no longer guaranteed.

B. Hammerstein PR Plant With Disturbance

To illustrate Theorem 4, consider the Hammerstein PR plant with a quadratic nonlinearity given by (36), (37) with $g(y) = 1$ and nonlinear controller (38)–(40) with $K = 10$. We consider the bounded disturbance $d(t) = -1.0 - \sin(3t)$. It can be seen from Figs. 5 and 6 that the controller rejects the disturbance, and

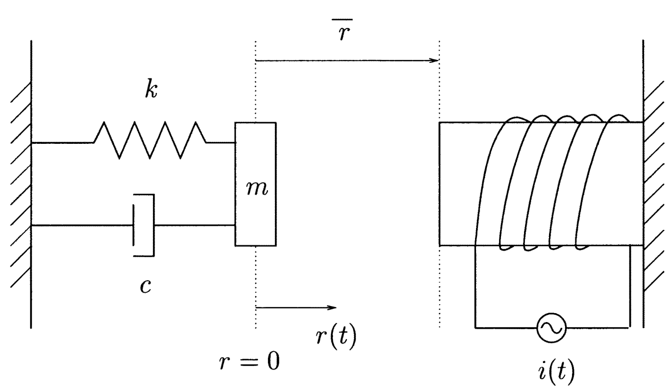

Fig. 9. Schematic of the electromagnetically controlled oscillator.

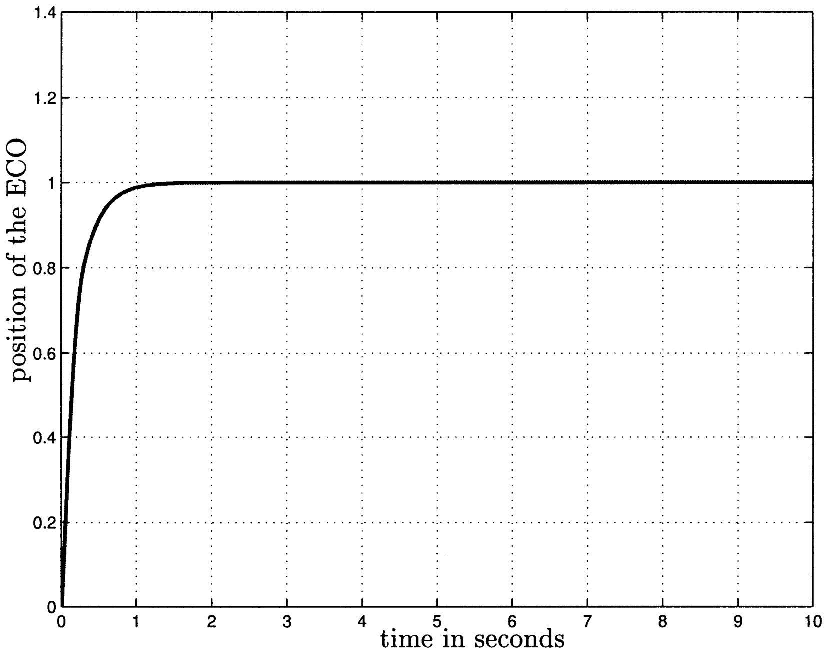

Fig. 10. Mass position of an electromagnetically controlled oscillator for the constant position command $r_{\rm eq} = 1.0$.

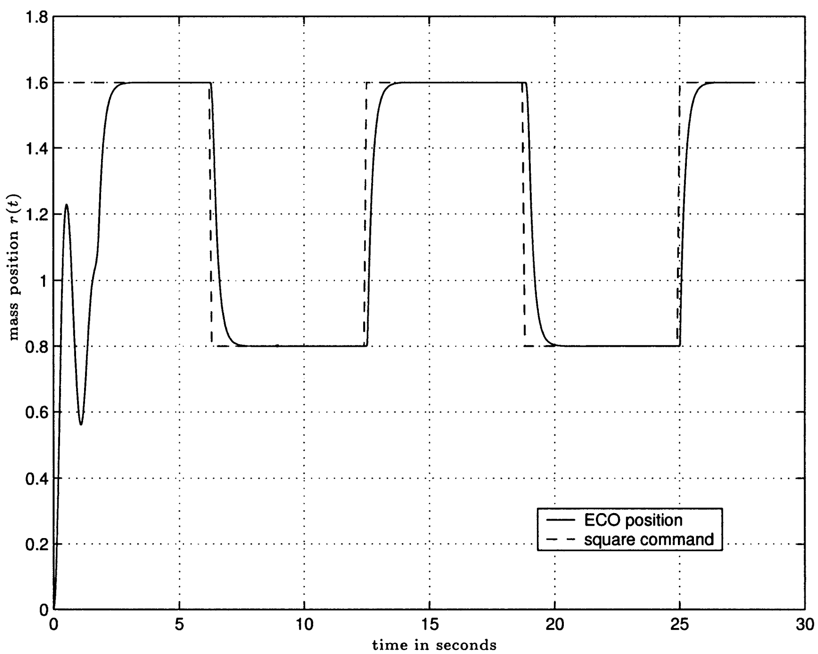

Fig. 11. Mass position of the electromagnetically controlled oscillator for the square wave position command $r_{\rm eq}(t)$ with period 12.5 s.

it is observed that the controller state $\hat{a}$ converges [Fig. 6(a)]. The open-loop response (dashed-line in Fig. 5) of the system diverges due to the eigenvalue at the origin.

To illustrate Theorem 5, we consider the Hammerstein PR plant with saturated control input

$$\dot{x} = Ax + B\left(\text{sat}_1(u) + d\right) \quad (55)$$

$$y = Cx \quad (56)$$

where d is a random bounded disturbance, and the nonlinear controller (44)–(46) with $K = 1$ and $\text{sign}(y)$ approximated by $10\text{sat}_{0.1}(y)$. It can be seen from Figs. 7 and 8 that the controller recovers from an initial saturation and rejects the random disturbance.

C. Electromagnetically Controlled Oscillator

Consider the electromagnetically controlled oscillator shown in Fig. 9. The dynamics of the oscillator [10] are given by

$$\ddot{r} + c\dot{r} + kr = \frac{i^2}{(\bar{r} - r)^2} \quad (57)$$

where i is the input current to the electromagnet, $c > 0$ is the damping constant, and $k > 0$ is the spring constant. Here $r = 0$ corresponds to the position of the mass when the spring is relaxed, and $\bar{r}$ is the location of the electromagnet. Several electrostatically or electromagnetically actuated systems such as MEMS can be modeled by (57). It can be shown that the linearized system around an operating point $r = r_{\text{eq}}$ is unstable if $r_{\text{eq}} > (1/3)\bar{r}$. With $\xi = r - r_{\text{eq}}$, we have

$$\begin{bmatrix} \dot{\xi} \\ \ddot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -c \end{bmatrix} \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{(\bar{r} - r_{\text{eq}} - \xi)^2} \end{bmatrix} i^2 + \begin{bmatrix} 0 \\ -k r_{\text{eq}} \end{bmatrix} \qquad (58)$$

where $\bar{r} = 2$, $k = 16$ and $c = 4$. Assume that the state $x = [\xi \;\; \dot{\xi}]^T$ is available for feedback and let $y = Cx$ with $C = [4 \;\; 1]$ so that the system $\left(\begin{bmatrix} 0 & 1 \\ -k & -c \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C \right)$ is SPR. Note that with $i = u(\bar{r} - r_{\text{eq}} - \xi)$, (58) has the same form as (36) with $d = -k r_{\text{eq}}$. Next, we choose the controller (38)–(40) in Theorem 4 with $K = 1$ and $\text{sign}(y)$ approximated by $10\text{sat}_{0.1}(y)$. For a constant command input $r_{\text{eq}} = 1.0$, it can be seen in Fig. 10 that the controller stabilizes the plant and follows the command input. For a square wave command switching between 0.8 and 1.6 with a period of 12.5 s, Fig. 11 shows that the controller attenuates the tracking error ξ. The tracking performance improves with better approximation (higher γ in $\gamma\text{sat}_{1/\gamma}(y)$) of the $\text{sign}(y)$ function. Note that both the command inputs are beyond the one-third gap and need to be stabilized.

REFERENCES

[1] B. D. O. Anderson, “A system theory criterion for positive real matrices,” *SIAM J. Contr.*, vol. 6, pp. 171–182, 1967.

[2] D. S. Bernstein and W. Haddad, “Nonlinear controllers for positive real systems with arbitrary input nonlinearities,” *IEEE Trans. Automat. Contr.*, vol. 39, pp. 1513–1517, July 1994.

[3] T. G. Bifano, J. Perreault, R. K. Mali, and M. N. Horenstein, “Microelectromechanical deformable mirrors,” *IEEE J. Select. Topics Quantum Electron.*, vol. 5, pp. 83–89, Jan. 1999.

[4] S. A. Billings and S. Y. Fakhouri, “Nonlinear system identification using hammerstein models,” *Int. J. Syst. Sci.*, vol. 10, no. 5, pp. 567–579, 1979.

[5] W. A. Clark, “Micromachined Vibratory Rate Gyroscopes,” Ph.D. dissertation, Univ. California, Berkeley, 1997.

[6] L. S. Fan, H. H. Ottesen, T. C. Reily, and R. W. Wood, “Magnetic recording-head positioning at very high track densities using microactuator-based, two stage servo system,” *IEEE Trans. Ind. Electron.*, vol. 42, pp. 222–233, June 1995.

[7] W. M. Haddad and D. S. Bernstein, “Explicit construction of quadratic lyapunov functions for the small gain, positivity, circle, and Popov theorems and their application to robust stability. Part I: Continuous time theory,” *Int. J. Robust Nonlinear Contr.*, vol. 3, pp. 313–319, 1993.

[8] W. M. Haddad and V. Chellaboina, “Nonlinear controllers for hammerstein systems with passive nonlinear dynamics,” *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1630–1634, Oct. 2001.

[9] J. Hong and D. S. Bernstein, “Bode integral constraints, colocation, and spillover in active noise and vibration control,” *IEEE Trans. Contr. Syst. Technol.*, vol. 6, pp. 111–120, Jan. 1998.

[10] ——, “Experimental application of direct adaptive control laws for adaptive stabilization and command following,” in *Proc. Conf. Decision Contr.*, Phoenix, AZ, Dec. 1999, pp. 779–783.

[11] D. A. Horsley, A. Singh, R. Horowitz, and A. P. Pisano, “Angular micropositioner for disk drives,” in *Proc. 10th Int. Conf. Microelectromech. Syst.*, Nagoya, Japan, 1997, pp. 454–459.

[12] S. Hu, “Differential equations with discontinuous right-hand sides,” *J. Math. Anal. Applicat.*, vol. 154, pp. 377–390, 1991.

[13] T. Imanura, T. Koshikawa, and M. Katayama, “Transverse mode electrostatic microactuator for MEMS-based HDD slider,” in *Proc. 9th Int. Conf. MEMS*, San Diego, CA, 1996, pp. 216–221.

[14] P. Ioannou and G. Tao, “Frequency domain conditions for strictly positive real transfer functions,” *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 53–54, Jan. 1987.

[15] S. M. Joshi and S. Gupta, “On a class of marginally stable positive real systems,” *IEEE Trans. Automat. Contr.*, vol. 41, pp. 152–155, Jan. 1996.

[16] T. N. Juneau, “Micromachined Dual Input Axis Rate Gyroscope,” Ph.D. dissertation, Univ. California, Berkeley, 1991.

[17] H. K. Khalil, *Nonlinear Syst.*, 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 1996.

[18] L. Y. Lin, E. L. Goldstein, and R. W. Tkach, “Free-space micromachined optical switches for optical networking,” *IEEE J. Select. Topics Quantum Electron.*, vol. 5, pp. 4–9, Jan. 1999.

[19] R. Lozano-Leal and S. M. Joshi, “Strictly positive real transfer functions revisited,” *IEEE Trans. Automat. Contr.*, vol. 35, pp. 1243–1245, Nov. 1990.

[20] S. E. Lyshevski and M. A. Lyshevski, “Analysis, dynamics and control of micro-electromechanical systems,” in *Proc. American Control Conf.*, Chicago, IL, June 2000, pp. 3091–3095.

[21] K. S. Narendra and P. G. Gallman, “An iterative method for the identification of nonlinear systems using a Hammerstein model,” *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 546–550, May 1966.

[22] D. Nesic and I. Mareels, “Dead-beat control of simple hammerstein models,” *IEEE Trans. Automat. Contr.*, vol. 43, pp. 1184–1188, Aug. 1998.

[23] J. I. Seeger and S. B. Crary, “Stabilization of electrostatically actuated mechanical devices,” in *Proc. Int. Conf. Solid-State Sensors Actuators*, Chicago, IL, June 1997, pp. 1133–1136.

[24] P. K. Sinha, *Electromagnetic Suspension: Dynamics and Control*. Exeter, U.K.: Peter Peregrinus, 1987.

[25] G. Tao and P. Ioannou, “Strictly positive real matrices and the Lefschez-Kalman-Yakubovich lemma,” *IEEE Trans. Automat. Contr.*, vol. 33, pp. 1183–1185, Nov. 1988.

[26] J. H. Taylor, “Strictly positive real functions and Lefschetz-Kalman-Yakubovich lemma,” *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 310–311, Mar. 1974.

[27] A. Teel and L. Praly, “Results on converse lyapunov functions from class-KL estimates,” in *Proc. Conf. Decision Control*, Phoenix, AZ, Dec. 1999, pp. 2545–2550.

[28] N. Yazdi and K. Najafi, “All-silicon single wafer micro-g accelerometer with combined surface and bulk micromachining process,” *J. Microelectromech. Syst.*, vol. 9, pp. 544–550, Dec. 2000.