Position Control Using Acceleration-Based Identification and Feedback With Unknown Measurement Bias

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A position-command-following problem for asymptotically stable linear systems is considered. To account for modeling limitations, we assume that a model is not available. Instead, acceleration data are used to construct a compliance (position-output) model, which is subsequently used to design a position servo loop. Furthermore, we assume that the acceleration measurements obtained from inertial sensors are biased. A subspace identification algorithm is used to identify the inertance (acceleration-output) model, and the biased acceleration measurements are used by the position-command-following controller, which is constructed using linear quadratic Gaussian (LQG) techniques.
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1 Introduction

Rigid-body position control using inertial sensors is difficult due to unknown sensor bias, which leads to position-estimate divergence. In particular, integration of angular velocity measurements from gyros (to obtain Euler angles) as well as double integration of accelerometer measurements from accelerometers leads to linearly or quadratically increasing position errors. In practice, drift in inertial sensors must be carefully managed over limited intervals, with supplementary measurements from noninertial sources (such as global positioning system (GPS)) used periodically for position resetting.

The difficulty associated with rigid-body position control arises from the fact that position is not observable from velocity and acceleration measurements. However, there is no fundamental impediment to the use of velocity or accelerometer measurements for estimating position when position is an observable state with such measurements. With this distinction in mind, we consider an unconventional problem in which accelerometer measurements, which may be subject to unknown, slowly drifting biases, are used for both model identification and position servo control. The approach that we take is based on the use of a backward-path controller with zero dc gain along with LQG control. The basis for this approach is developed in Ref. [1], where it is shown that rejection of unknown sensor bias is not amenable to integral control.

In the present paper, we assume that only inertial sensors are available for identification and feedback. In practice, single and double integrations of gyro and accelerometer signals with sensor bias produce position signals with ramp and parabolic noise, respectively. If estimates of the sensor biases in a servo loop are available, then the methods described in Ref. [2] can be used to achieve position-command following. Although estimates of sensor bias can be obtained offline, sensor bias generally does not remain constant over long periods of operation due to drift. In this paper, instead of integrating rate or acceleration measurements to synthesize position measurements, we use biased measurements in an observer framework within an LQG architecture along with a discrete-time version of the results of Ref. [1] to design a backward-path controller to achieve command following while rejecting sensor bias.

To account for unmodeled dynamics, we use inertial sensors in combination with system identification methods to develop a model of the compliance transfer function that can be used for position-command-following control. To obtain a compliance model of the system, we use the available measurements in conjunction with subspace identification methods [3,4]. Subspace methods provide a direct approach in constructing a state space model, although the state of the identified model lacks physical interpretation. With acceleration measurements, the identified model is an inertance, which has force input and acceleration output. To obtain a compliance model, we construct an alternative output matrix that matches the dynamics of the inertance transfer function cascaded with a double integrator. The inertial sensors are thus used offline to develop the compliance model and online as signals for feedback. This approach is applicable when only inertial sensors such as gyros and accelerometers are available, as well as when the kinematics and dynamics are not well modeled. In the present paper, we develop and illustrate an approach to this problem for systems with linear dynamics. In future work, we plan to extend this approach to kinematically and dynamically complex structures such as a 6-DOF Stewart platform using only inertial sensors.

We develop the LQG framework for acceleration-based position control in Sec. 2 and describe the identification procedure in Sec. 3. Section 4 considers controller synthesis using the identified model in the LQG framework. Next, in Sec. 5 we apply the approach to a mass-spring-damper system. The control-design methodology in this paper is discrete-time LQG theory with a backward-path controller for rejecting sensor biases as developed in Ref. [1] for continuous-time systems. A preliminary version of some of the results of this paper appeared in Ref. [5]. The goal of this paper is to demonstrate conceptually that identification-based position-following control based on biased inertial measurements is feasible. Experimental application with inertial sensors will be given in a future paper.

2 Acceleration-Based Position Control

Consider the system

$$x(k+1) = Ax(k) + Bu(k) \tag{2.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, with acceleration measurements $y_{\rm acc} \in \mathbb{R}^p$ given by

$$y_{\rm acc}(k) = C_{\rm acc}x(k) + D_{\rm acc}u(k) + v(k) \tag{2.2}$$

where $v \in \mathbb{R}^p$ is the unknown sensor bias. We assume that $(A, C_{\rm acc})$ is observable. Let the position $y_{\rm pos} \in \mathbb{R}^p$ of the system be given by

$$y_{\rm pos}(k) = C_{\rm pos}x(k) \tag{2.3}$$

so that the systems with outputs $y_{\rm pos}$ and $y_{\rm acc}$ are the compliance and inertance, respectively. Hence, the discrete-time inertance $G_{\rm inrt}(\mathbf{z})$ and discrete-time compliance $G_{\rm comp}(\mathbf{z})$ have realizations

$$G_{\rm comp}(\mathbf{z}) \sim \left[\begin{array}{c|c} A & B \\ \hline C_{\rm pos} & 0 \end{array}\right] \quad G_{\rm inrt}(\mathbf{z}) \sim \left[\begin{array}{c|c} A & B \\ \hline C_{\rm acc} & D_{\rm acc} \end{array}\right] \tag{2.4}$$

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Let $r \in \mathbb{R}^p$ be a reference position command so that, for all $k \geqslant 0$, $r(k)$ is the desired position at time k. The objective is to design a controller that uses the biased acceleration measurements y_{acc} to track the position command, that is, ensure that $y_{\mathrm{pos}}(k) - r(k) \to 0$ as $k \to \infty$. Due to the presence of sensor bias and lack of knowledge of the initial position, we cannot synthesize position measurements by integrating the acceleration measurements. Instead, we consider an LQG approach to achieve position tracking using biased acceleration measurements. We use the acceleration measurements within an observer framework to estimate the position and determine the control input based on these estimates using LQG. In order to reject the sensor bias, it is shown in Ref. [1] that a backward-path controller with zero dc gain is required. We thus include a backward-path controller G_{bp} in the control architecture.

Let G_{bp} have a minimal realization

$$G_{\mathrm{bp}}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\mathrm{bp}} & B_{\mathrm{bp}} \\ \hline C_{\mathrm{bp}} & D_{\mathrm{bp}} \end{array}\right] \tag{2.5}$$

with state $x_{\mathrm{bp}} \in \mathbb{R}^{n_{\mathrm{bp}}}$. To account for the backward-path controller in the LQG design, we define $\tilde{y}_{\mathrm{acc}}$ by

$$\tilde{y}_{\mathrm{acc}} = G_{\mathrm{bp}} y_{\mathrm{acc}} \tag{2.6}$$

so that

$$x_{\mathrm{bp}}(k+1) = A_{\mathrm{bp}} x_{\mathrm{bp}}(k) + B_{\mathrm{bp}} y_{\mathrm{acc}}(k) \tag{2.7}$$

$$\tilde{y}_{\mathrm{acc}}(k) = C_{\mathrm{bp}} x_{\mathrm{bp}}(k) + D_{\mathrm{bp}} y_{\mathrm{acc}}(k) \tag{2.8}$$

Next, we define the controller input y by

$$y \triangleq [\tilde{y}_{\mathrm{acc}}^T \quad r^T]^T \tag{2.9}$$

so that the LQG controller uses the output $\tilde{y}_{\mathrm{acc}}$ from the backward-path controller G_{bp} and the reference position trajectory r to produce the controller output u. Define the position-error performance variable z_{pos} by

$$z_{\mathrm{pos}} \triangleq y_{\mathrm{pos}} - r \tag{2.10}$$

where r is the position command to be followed. To include the control effort in the performance variable, we define the performance variable z by

$$z \triangleq [z_{\mathrm{pos}}^T \quad (E_u u)^T]^T \tag{2.11}$$

where the control weighting E_u has full column rank.

To facilitate LQG design, the position command r and the sensor bias v are modeled as outputs of linear filters W_r and W_b excited by white noise signals w_r and w_b, respectively. Let W_r and W_b have minimal realizations

$$W_r(\mathbf{z}) \sim \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right] \quad W_b(\mathbf{z}) \sim \left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] \tag{2.12}$$

with state $x_r \in \mathbb{R}^{n_r}$ and $x_b \in \mathbb{R}^{n_b}$, respectively. Furthermore, we define w by

$$w \triangleq [w_r^T \quad w_b^T \quad w_\epsilon^T]^T \tag{2.13}$$

where w_ϵ is a fictitious white process that facilitates LQG synthesis. It then follows from Eqs. (2.1)–(2.3) and (2.5)–(2.13) that

$$\begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{G} \begin{bmatrix} w \\ u \end{bmatrix} \tag{2.14}$$

where $\mathcal{G}$ has a realization

$$\mathcal{G} \sim \left[\begin{array}{c|cc} \mathcal{A} & \mathcal{D}_1 & \mathcal{B} \\ \hline \mathcal{E}_1 & 0 & \mathcal{E}_2 \\ \mathcal{C} & \mathcal{D}_2 & \mathcal{D} \end{array}\right] \tag{2.15}$$

with state $\tilde{x} \triangleq [x^T \; x_{\mathrm{bp}}^T \; x_r^T \; x_b^T]^T$ and

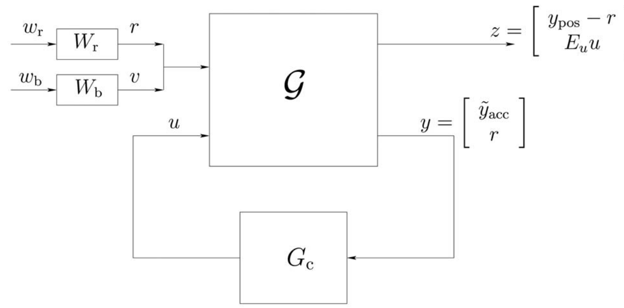

Fig. 1 Standard problem for designing a position-tracking controller G_c that uses biased acceleration measurements. To facilitate controller synthesis using LQG, the backward-path controller G_{bp} that is used to reject the sensor bias is included in the Plant $\mathcal{G}$.

$$\mathcal{A} \triangleq \begin{bmatrix} A & 0 & 0 & \\ B_{\mathrm{bp}} C_{\mathrm{acc}} & A_{\mathrm{bp}} & 0 & 0 \\ 0 & 0 & A_r & 0 \\ 0 & 0 & 0 & A_b \end{bmatrix} \quad \mathcal{B} \triangleq \begin{bmatrix} B \\ B_{\mathrm{bp}} D_{\mathrm{acc}} \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{D}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ B_r & 0 & 0 \\ 0 & B_b & 0 \end{bmatrix} \tag{2.16}$$

$$\mathcal{E}_1 \triangleq \begin{bmatrix} C_{\mathrm{pos}} & 0 & -C_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathcal{E}_2 \triangleq \begin{bmatrix} 0 \\ E_u \end{bmatrix} \tag{2.17}$$

$$\mathcal{C} \triangleq \begin{bmatrix} D_{\mathrm{bp}} C_{\mathrm{acc}} & C_{\mathrm{bp}} & 0 & D_{\mathrm{bp}} C_b \\ 0 & 0 & C_r & 0 \end{bmatrix} \quad \mathcal{D} \triangleq \begin{bmatrix} D_{\mathrm{bp}} D_{\mathrm{acc}} \\ 0 \end{bmatrix}$$

$$\mathcal{D}_2 \triangleq \begin{bmatrix} 0 & D_{\mathrm{bp}} D_b & 0 \\ D_r & 0 & \varepsilon I \end{bmatrix} \tag{2.18}$$

Next, we use the standard problem (2.15) shown in Fig. 1 and LQG (see Refs. [6,7]) to obtain a controller G_c to achieve position tracking using acceleration measurements. To solve the estimator Riccati equation, we introduce εI in Eq. (2.18) so that $\mathcal{D}_2 \mathcal{D}_2^T$ is nonsingular. The discrete-time LQG controller G_c can be obtained from the standard problem (2.15) by solving two discrete-time Riccati equations (see Ref. [8], p. 560). The resulting controller uses the reference position command r and the output $\tilde{y}_{\mathrm{acc}}$ from the backward-path controller G_{bp} to produce the control input to minimize the error between the actual position y_{pos} and the reference command. The control architecture is shown in Fig. 2. Note that the filters W_r and W_b are used only for synthesizing the LQG controller and are not implemented during position tracking.

We now use the results in Ref. [1] to choose a backward-path controller G_{bp} that ensures that the sensor bias v does not affect the position-tracking performance variable z_{pos} when used with the LQG controller G_c.

PREPOSITION 2.1. *Let the closed-loop system in Fig. 2 be internally stable and assume that $v(k)$ is constant. If $r=0$ and $G_{\mathrm{bp}}(1)=0$, then, for all $v \in \mathbb{R}^m$, $\lim_{k\to\infty} z_{\mathrm{pos}}(k) = 0$.*

Proof. Let G_c have entries

$$G_c = [G_{c,y} \quad G_{c,r}]$$

so that

$$u = G_{c,y} \tilde{y}_{\mathrm{acc}} + G_{c,r} r \tag{2.19}$$

Since $r=0$, Eq. (2.19) implies that $u = G_{c,y} \tilde{y}_{\mathrm{acc}}$ and hence it follows from Eq. (2.6) that

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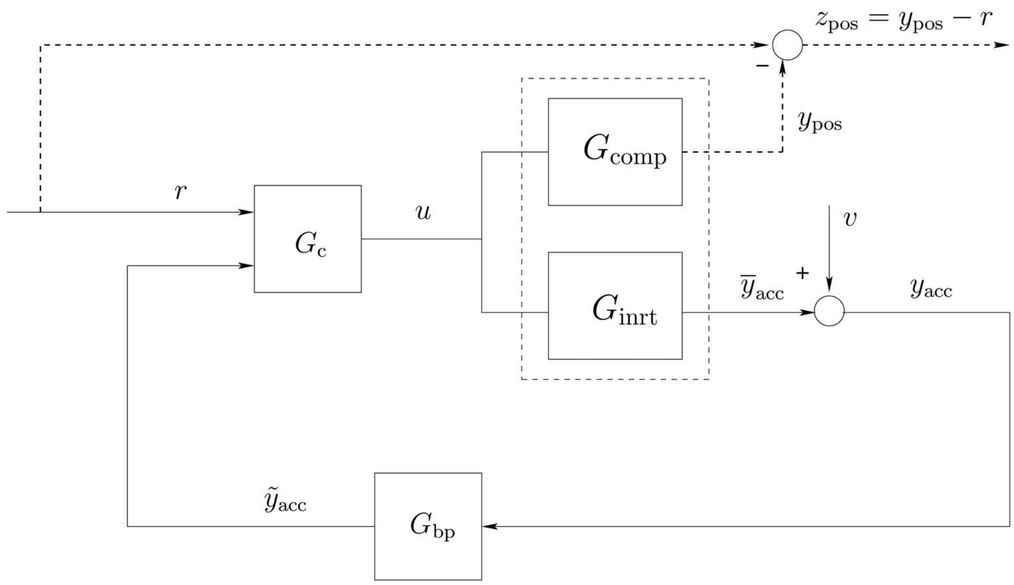

Fig. 2 Control architecture for discrete-time LQG position control using acceleration feedback and a backward-path controller $G_{\rm bp}$

$$u = G_{c,y}G_{\rm bp}(\bar{y}_{\rm acc} + v) \tag{2.20}$$

where $\bar{y}_{\rm acc}$ is the acceleration of the system given by

$$\bar{y}_{\rm acc} = G_{\rm inrt}u \tag{2.21}$$

Therefore, substituting Eq. (2.20) into Eq. (2.21) yields

$$\bar{y}_{\rm acc} = G_{y,v}v$$

where

$$G_{y,v} \triangleq (I - G_{\rm inrt}G_{c,y}G_{\rm bp})^{-1}G_{\rm inrt}G_{c,y}G_{\rm bp} \tag{2.22}$$

Substituting Eq. (2.22) into Eq. (2.20) yields

$$u = G_{c,y}G_{\rm bp}(I + G_{y,v})v \tag{2.23}$$

Since $y_{\rm pos}=G_{\rm comp}u$ and $r=0$, Eq. (2.23) implies that

$$z_{\rm pos} = G_{z,v}v \tag{2.24}$$

where

$$G_{z,v} \triangleq G_{\rm comp}G_{c,y}G_{\rm bp}(I + G_{y,v})$$

Since the closed-loop system in Fig. 2 is internally stable, there are no closed-right-half-plane pole-zero cancellations and hence $G_{z,v}(1)=0$. Since $G_{z,v}$ is asymptotically stable, the final value theorem yields

$$\lim_{k\to\infty} z_{\rm pos} = \lim_{\mathbf{z}\to 1}(\mathbf{z} - 1)G_{z,v}\frac{v}{\mathbf{z} - 1} = G_{z,v}(1)v = 0$$

Since the LQG controller ensures that the closed-loop system in Fig. 1 is internally stable, it follows from Proposition 2.1 that, as $k \to \infty$, the sensor bias has no effect on the position-tracking performance. Hence, the LQG controller along with the backward-path controller can be used for position tracking with biased acceleration measurements. Although the backward-path controller can be chosen without knowledge of the system dynamics, it follows from Eqs. (2.14)–(2.18) that LQG synthesis requires knowledge of the system dynamics, that is, knowledge of A, B, $C_{\rm acc}$, $D_{\rm acc}$, and $C_{\rm pos}$. However, if A, B, $C_{\rm acc}$, $D_{\rm acc}$, and $C_{\rm pos}$ are unknown, then we use the acceleration measurements to identify the inertance and compliance of the system and use the identified dynamics to synthesize an LQG controller. We describe the procedure in the following two sections.

3 Acceleration-Based Identification of the Compliance

We now assume that a model of the system is not available, although acceleration measurements can be used for system identification to obtain a model of the inertance. Although the sensor bias is unknown, we assume that the bias remains constant during the identification procedure. Hence, Eqs. (2.1) and (2.2) can be expressed as

$$x(k+1) = Ax(k) + \widetilde{B}\widetilde{u}(k) \tag{3.1}$$

$$y_{\rm acc}(k) = C_{\rm acc}x(k) + \widetilde{D}\widetilde{u}(k) \tag{3.2}$$

where $\widetilde{u} \in \mathbb{R}^{m+1}$ is defined by

$$\widetilde{u}(k) \triangleq [u(k)^T \quad 1]^T \tag{3.3}$$

and

$$\widetilde{B} \triangleq [B \quad 0_{n\times 1}] \qquad \widetilde{D} \triangleq [D \quad v_b] \tag{3.4}$$

For system identification, the force input u is chosen to be a white noise signal, and the outputs are the acceleration measurement $y_{\rm acc}$ given by Eq. (3.2). We use the inputs $\widetilde{u}$ and acceleration measurements $y_{\rm acc}$ in a subspace identification algorithm [3,4] to obtain discrete-time system matrices $A_{\rm id}$, $B_{\rm id}$, $C_{\rm acc,id}$, $D_{\rm acc,id}$, and an estimate $v_{\rm id}$ of the bias v, for the nth-order linear time-invariant discrete-time state space inertance model

$$\hat{x}(k+1) = A_{\rm id}\hat{x}(k) + B_{\rm id}u(k) \tag{3.5}$$

$$y_{\rm acc}(k) = C_{\rm acc,id}\hat{x}(k) + D_{\rm acc,id}u(k) + v_{\rm id} \tag{3.6}$$

The bias estimate $v_{\rm id}$ is discarded since the sensor bias is assumed to drift.

For LQG synthesis for position-command-following control, it is necessary to weight the position-tracking error. However, as a consequence of subspace identification, the components of $\hat{x}(k)$ do not have a physical interpretation. The state space models (2.1) and (3.5) are realizations of the same system and hence the states x and $\hat{x}$ are related by a similarity transformation

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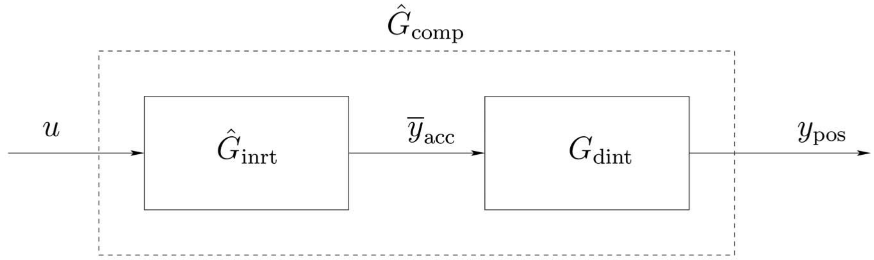

Fig. 3 Construction of the compliance $\hat{G}_{\rm comp}$ by cascading a double integrator with the identified inertance

$$x(k) = S\hat{x}(k) \tag{3.7}$$

where $S \in \mathbb{R}^{n\times n}$ is nonsingular. Hence, it follows from Eq. (2.3) that

$$y_{\rm pos}(k) = \hat{C}_{\rm pos}\hat{x}(k) \tag{3.8}$$

where

$$\hat{C}_{\rm pos} \triangleq C_{\rm pos}S \tag{3.9}$$

However, S is unknown, and thus $\hat{C}_{\rm pos}$ cannot be determined using Eq. (3.9). To overcome this difficulty, we construct an estimate of the compliance based on the identified inertance. The output of the compliance is used to form the weighted performance variable in LQG command-following synthesis.

Let $\hat{G}_{\rm inrt}$ be the identified inertance transfer function with realization

$$\hat{G}_{\rm inrt}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\rm id} & B_{\rm id} \\ \hline C_{\rm acc,id} & D_{\rm acc,id} \end{array}\right] \tag{3.10}$$

Next, consider the $p\times p$ discrete-time transfer function

$$G_{\rm dint}(\mathbf{z}) \triangleq \begin{bmatrix} \dfrac{t_s^2}{(\mathbf{z}-1)^2} & & \\ & \ddots & \\ & & \dfrac{t_s^2}{(\mathbf{z}-1)^2} \end{bmatrix} \tag{3.11}$$

where t_s is the sampling time of the discrete-time model of the plant. Note that the output of $G_{\rm dint}$ is obtained by twice integrating the input. Hence, the compliance transfer function $\hat{G}_{\rm comp}$ with position as the output is defined by (Fig. 3)

$$\hat{G}_{\rm comp}(\mathbf{z}) \triangleq G_{\rm dint}(\mathbf{z})\hat{G}_{\rm inrt}(\mathbf{z}) \tag{3.12}$$

Let $G_{\rm dint}$ have the $2p$th-order minimal realization

$$G_{\rm dint}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\rm dint} & B_{\rm dint} \\ \hline C_{\rm dint} & 0 \end{array}\right] \tag{3.13}$$

with state $x_{\rm dint} \in \mathbb{R}^{2p}$ and

$$A_{\rm int} \triangleq \begin{bmatrix} 1 & t_s & & & \\ 0 & 1 & & & \\ & & \ddots & & \\ & & & 1 & t_s \\ & & & 0 & 1 \end{bmatrix} \quad B_{\rm int} \triangleq \begin{bmatrix} t_s^2/2 & & 0 \\ t_s & & \vdots \\ 0 & \ddots & 0 \\ \vdots & & t_s^2/2 \\ 0 & & t_s \end{bmatrix}$$

$$C_{\rm int} \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & & & \ddots & \ddots & \\ 0 & \cdots & 0 & 0 & 1 & 0 \end{bmatrix}$$

It follows from Eqs. (3.10), (3.12), and (3.13) that $\hat{G}_{\rm comp}$ has a $(2p+n)$th-order realization

$$\hat{G}_{\rm comp}(\mathbf{z}) \sim \left[\begin{array}{c|c} \hat{A}_{\rm comp} & \hat{B}_{\rm comp} \\ \hline \hat{C}_{\rm comp} & 0 \end{array}\right] \tag{3.14}$$

where

$$\hat{A}_{\rm comp} \triangleq \begin{bmatrix} A_{\rm id} & 0_{4\times 4} \\ B_{\rm dint}C_{\rm acc,id} & A_{\rm dint} \end{bmatrix} \quad \hat{B}_{\rm comp} \triangleq \begin{bmatrix} B_{\rm id} \\ B_{\rm dint}D_{\rm acc,id} \end{bmatrix}$$
$$\hat{C}_{\rm comp} \triangleq [0 \ C_{\rm dint}] \tag{3.15}$$

Therefore, the state $\hat{x}_{\rm comp} \triangleq [\hat{x}^T \quad x_{\rm dint}^T]^T$ satisfies

$$\hat{x}_{\rm comp}(k+1) = \hat{A}_{\rm comp}\hat{x}_{\rm comp}(k) + \hat{B}_{\rm comp}u(k) \tag{3.16}$$

$$y_{\rm pos}(k) = \hat{C}_{\rm comp}\hat{x}_{\rm comp}(k) \tag{3.17}$$

Furthermore, it follows from Eq. (3.6) that

$$y_{\rm acc}(k) = [C_{\rm acc,id} \quad 0_{2\times 4}]\hat{x}_{\rm comp}(k) + D_{\rm acc,id}u(k) \tag{3.18}$$

Note that all of the matrices in Eqs. (3.16)–(3.18) are known. However, the states $x_{\rm dint}$ of the double integrator are not observable through the acceleration measurement $y_{\rm acc}$, that is, $(\hat{A}_{\rm comp}, [C_{\rm acc,id} \quad 0_{2\times 4}])$ is not observable. Since the eigenvalues of $A_{\rm dint}$ are not observable, the realization $\hat{G}_{\rm comp}$ in Eq. (3.14) is not suitable for LQG synthesis.

Instead, we determine an output matrix $\hat{C}_{\rm pos,id}$ so that the identified compliance $\hat{G}_{\rm comp}$ has the minimal realization

$$\hat{G}_{\rm comp}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\rm id} & B_{\rm id} \\ \hline \hat{C}_{\rm pos,id} & 0 \end{array}\right] \tag{3.19}$$

and the position $y_{\rm pos}$ is given by

$$y_{\rm pos}(k) = \hat{C}_{\rm pos,id}\hat{x}(k) \tag{3.20}$$

In particular, $\hat{C}_{\rm pos,id}$ is identified by comparing the Markov parameters of $\hat{G}_{\rm comp}$ in Eqs. (3.14) and (3.19). It follows from Eqs. (3.14) and (3.19) that, for all $i \geqslant 1$,

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$$\hat{C}_{\rm pos,id}A_{\rm id}^{i-1}B_{\rm id} = \hat{C}_{\rm comp}\hat{A}_{\rm comp}^{i-1}\hat{B}_{\rm comp} \tag{3.21}$$

and hence

$$F = \hat{C}_{\rm pos,id}G \tag{3.22}$$

where

$$F \triangleq [\hat{C}_{\rm comp}\hat{B}_{\rm comp} \ \cdots \ \hat{C}_{\rm comp}\hat{A}_{\rm comp}^{n}\hat{B}_{\rm comp}] \quad G \triangleq [B_{\rm id} \ \cdots \ A_{\rm id}^{n}B_{\rm id}] \tag{3.23}$$

The least squares fit is given by

$$\hat{C}_{\rm pos,id} = (G^{\dagger}F)^T \tag{3.24}$$

Next, we use the compliance model in Eq. (3.19) with $\hat{C}_{\rm pos,id}$ given by Eq. (3.24) for LQG synthesis of the position-tracking controller.

4 Acceleration-Based Position Control Using the Identified Model

In this section, we obtain a position-tracking controller by applying discrete-time LQG synthesis using the identified compliance and inertance models. We consider Eqs. (2.14)–(2.18) with A, B, $C_{\rm acc}$, $D_{\rm acc}$, and $C_{\rm pos}$ replaced by $A_{\rm id}$, $B_{\rm id}$, $C_{\rm acc,id}$, $D_{\rm acc,id}$, and $\hat{C}_{\rm pos,id}$, respectively. The standard problem for LQG synthesis is given by Eq. (2.15) with $\tilde{x}$ defined by $\tilde{x} \triangleq [\hat{x}^T \quad x_{\rm bp}^T \quad x_r^T \quad x_b^T]^T$. The implementation of the controller is shown in Fig. 2.

Let the LQG controller G_c have the minimal realization

$$G_c \sim \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array}\right] \tag{4.1}$$

with state $x_c \in \mathbb{R}^{n_c}$. Note that the order of the controller G_c is the same as the dimension of $\tilde{x}$, that is, $n_c = n + n_{\rm bp} + n_r + n_b$. To analyze the closed-loop dynamics, define $x_{\rm cl}$ by

$$x_{\rm cl} \triangleq [x^T \quad x_c^T \quad x_{\rm bp}^T]^T \tag{4.2}$$

where x is given by Eq. (2.1) and $x_{\rm bp}$ is the state of the backward-path controller $G_{\rm bp}$. Note that $z_{\rm pos}$ is the error between the position command and the positions of the two masses. The closed-loop system dynamics are then given by

$$x_{\rm cl}(k+1) = A_{\rm cl}x_{\rm cl}(k) + B_{\rm cl}r(k) + D_{1,\rm cl}v(k) \tag{4.3}$$

$$z_{\rm pos}(k) = C_{\rm cl}x_{\rm cl}(k) + D_{\rm cl}r(k) + D_{2,\rm cl}v(k) \tag{4.4}$$

where

$$A_{\rm cl} \triangleq \begin{bmatrix} A & BC_c & 0 \\ B_{c,y}D_{\rm bp}C_{\rm acc} & A_c + B_{c,y}D_{\rm bp}D_{\rm acc}C_c & B_{c,y}C_{\rm bp} \\ B_{\rm bp}C_{\rm acc} & B_{\rm bp}D_{\rm acc}C_c & A_{\rm bp} \end{bmatrix}$$
$$B_{\rm cl} \triangleq \begin{bmatrix} 0 \\ B_{c,r} \\ 0 \end{bmatrix} \quad D_{1,\rm cl} \triangleq \begin{bmatrix} 0 \\ B_{c,y} \\ B_{\rm bp} \end{bmatrix} \tag{4.5}$$

$$C_{\rm cl} \triangleq [C_{\rm pos} \quad 0 \quad 0] \quad D_{\rm cl} = -I \quad D_{2,\rm cl} = 0 \tag{4.6}$$

and B_c has entries $B_c = [B_{c,y} \quad B_{c,r}]$, with $B_{c,y} \in \mathbb{R}^{n_c \times 2}$ and $B_{c,r} \in \mathbb{R}^{n_c \times 2}$.

Let $G_{{\rm sens},r}$ be the sensitivity transfer function with the position command r as input and the error $z_{\rm pos}$ as the output. It follows from Eqs. (4.3)–(4.6) that $G_{{\rm sens},r}$ is realized by

$$G_{{\rm sens},r}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\rm cl} & B_{\rm cl} \\ \hline C_{\rm cl} & D_{\rm cl} \end{array}\right] \tag{4.7}$$

Similarly, the sensitivity transfer function $G_{{\rm sens},v}$ with the sensor bias v as input and the actual position $y_{\rm pos}$ as the output is realized by

$$G_{{\rm sens},v}(\mathbf{z}) \sim \left[\begin{array}{c|c} A_{\rm cl} & D_{1,\rm cl} \\ \hline C_{\rm cl} & D_{2,\rm cl} \end{array}\right] \tag{4.8}$$

Note that the dynamics of the plant are unknown and hence the sensitivity functions $G_{{\rm sens},r}$ and $G_{{\rm sens},v}$ in Eqs. (4.7) and (4.8), respectively, cannot be constructed in practice. However, these sensitivity functions can be constructed for simulation examples and can be used to evaluate the performance of the position-tracking controller designed using the procedure presented in this paper. Next, we design a position-tracking controller for a linear mass-spring-damper system by using biased acceleration measurements of the masses for identification and feedback.

5 Two-Mass System

Consider the two-mass system shown in Fig. 4 with force inputs u_1, u_2 and two acceleration sensors (accelerometers) measuring $\ddot{x}_1$ and $\ddot{x}_2$. The equations of motion are

$$m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = -u_1 \tag{5.1}$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_2\dot{x}_1 - k_2x_1 = u_1 + u_2 \tag{5.2}$$

The state space representation of Eqs. (5.1) and (5.2) is

$$\dot{x} = A_{\rm ct}x + B_{\rm ct}u \tag{5.3}$$

where $x \in \mathbb{R}^4$ and $u \in \mathbb{R}^2$ are defined by

$$x \triangleq [x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2]^T \quad u \triangleq [u_1 \quad u_2]^T \tag{5.4}$$

and $A_{\rm ct} \in \mathbb{R}^{4\times4}$ and $B_{\rm ct} \in \mathbb{R}^{4\times2}$ are defined by

$$A_{\rm ct} \triangleq \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix} \quad B_{\rm ct} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{1}{m_1} & 0 \\ \frac{1}{m_2} & \frac{1}{m_2} \end{bmatrix} \tag{5.5}$$

Let the acceleration measurement $y_{\rm acc}$ of $\ddot{x}_1$ and $\ddot{x}_2$ be given by

$$y_{\rm acc} = C_{\rm acc}x + D_{\rm acc}u + v \tag{5.6}$$

where

$$C_{\rm acc} \triangleq \begin{bmatrix} -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix}$$
$$D_{\rm acc} \triangleq \begin{bmatrix} -\frac{1}{m_1} & 0 \\ \frac{1}{m_2} & \frac{1}{m_2} \end{bmatrix} \tag{5.7}$$

and $v \in \mathbb{R}^2$ is the unknown sensor bias. Let the positions $y_{\rm pos}$ of the two masses be given by

$$y_{\rm pos} = C_{\rm pos}x \tag{5.8}$$

where

$$C_{\rm pos} \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{5.9}$$

The systems with outputs $y_{\rm pos}$ and $y_{\rm acc}$ are the compliance and inertance, respectively.

The equivalent zero-order-hold discrete-time state space representation of Eqs. (5.1), (5.6), and (5.8) with sampling time t_s is

$$x(k+1) = Ax(k) + Bu(k) \tag{5.10}$$

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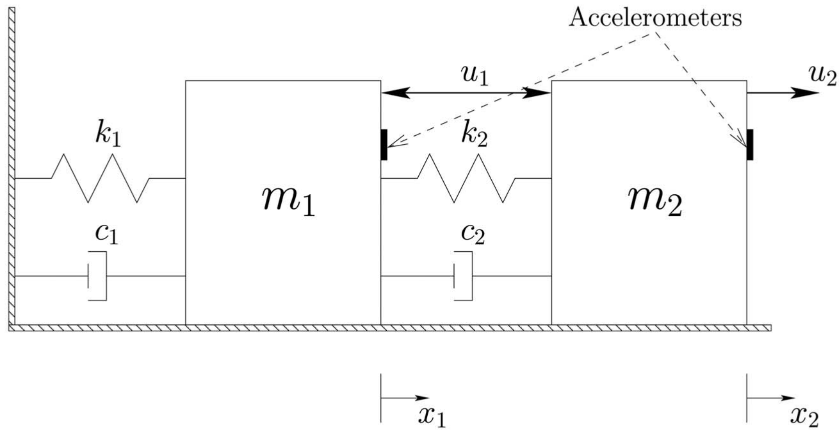

Fig. 4 Two-mass system

$$y_{\text{acc}}(k) = C_{\text{acc}}x(k) + D_{\text{acc}}u(k) + v(k) \tag{5.11}$$

$$y_{\text{pos}}(k) = C_{\text{pos}}x(k) \tag{5.12}$$

where

$$A \triangleq e^{A_{\text{ct}}t_s} \quad B \triangleq \int_0^{t_s} e^{A_{\text{ct}}t_s}B_{\text{ct}}ds \tag{5.13}$$

To illustrate position-following control with acceleration-based identification and acceleration feedback, we excite the two-mass system with white noise inputs u_1 and u_2, and corrupt the acceleration measurements with a bias but no other noise. Next, we identify the inertance and compliance transfer functions using the procedure described in Sec. 4. To compare the true system with the identified model, we plot the position $y_{\text{pos},1}$ of m_1 when u_1 is an impulse and $u_2=0$, and when $u_1=0$ and u_2 is an impulse in Figs. 5 and 6, respectively. The errors between the position measurements and the outputs of the identified model are small, and thus the identified inertance and compliance models are good approximations of the inertance and compliance.

The control objective is to have the positions of m_1 and m_2 follow commands that are sinusoidal with a spectral bandwidth between 0.1 Hz and 1 Hz. In accordance with this specification, the transfer function W_r defined in Eq. (2.12) is chosen to be

$$W_r(\mathbf{z}) = \frac{(\mathbf{z}-1)}{(\mathbf{z}-0.995)(\mathbf{z}-0.9995)}I_2 \tag{5.14}$$

so that W_r has high gain in the required bandwidth. The magnitude of the diagonal entry of W_r is shown in Fig. 7. The LQG controller is designed using the identified model using the procedure described in Sec. 5. The position command for m_1 is a sinusoid of amplitude 0.5 m and frequency 0.25 Hz, while the position command for m_2 is a sinusoid of amplitude 1.0 m and frequency 0.125 Hz. Furthermore, we assume that the acceleration measurements of m_1 and m_2 have constant biases of 5 m/s^2 and

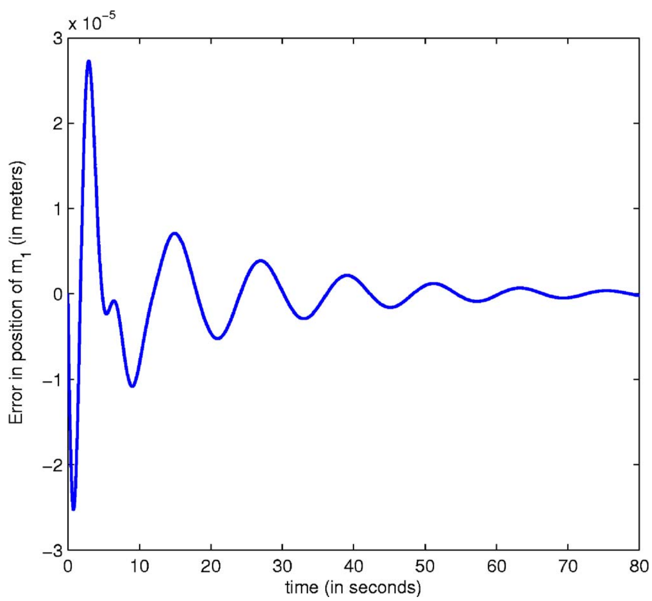

Fig. 5 Error between the actual position of m_1 and the output $y_{\text{pos},1}$ of the identified compliance model when u_1 is an impulse and $u_2=0$. For position-tracking controller synthesis, the identified compliance is used.

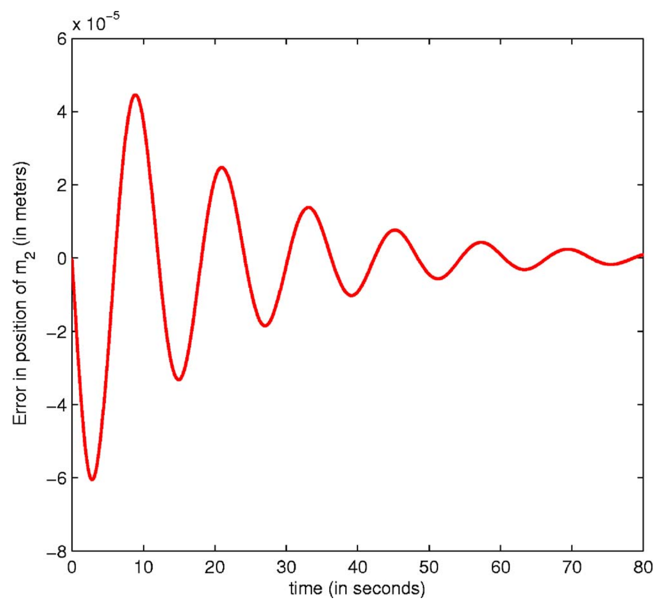

Fig. 6 Error between the position of m_2 and the output $y_{\text{pos},2}$ from the identified compliance model when $u_1=0$ and u_2 is an impulse

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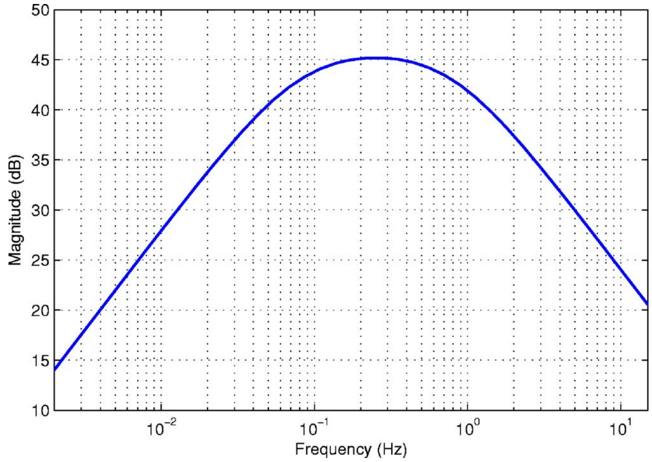

Fig. 7 Magnitude of the diagonal entries of $W_r(\mathbf{z})$

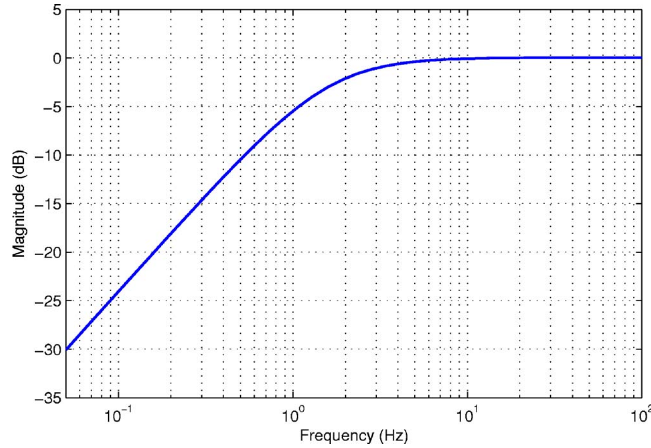

Fig. 8 Magnitude of the diagonal entries of $G_{bp}(\mathbf{z})$

$7\ \mathrm{m/s^2}$, respectively, during position-command following. The backward-path controller G_{bp} is chosen to be

$$G_{bp}(\mathbf{z}) = \frac{\mathbf{z}-1}{\mathbf{z}-0.99} I_2 \tag{5.15}$$

so that G_{bp} is asymptotically stable and $G_{bp}(1)=0$. Note that the backward-path controller is proper and thus does not require computation of any signal derivatives, and hence can be implemented in practice. The magnitude plot of the diagonal entries of the backward-path controller is shown in Fig. 8.

Finally, we design the LQG controller using the procedure described in Secs. 3 and 5. The position commands and the actual positions of the two masses with the discrete-time LQG controller and the backward-path controller are shown in Fig. 9. Note that in a real-world application, the positions of the two masses are not available. However, in the two-mass system simulation, although

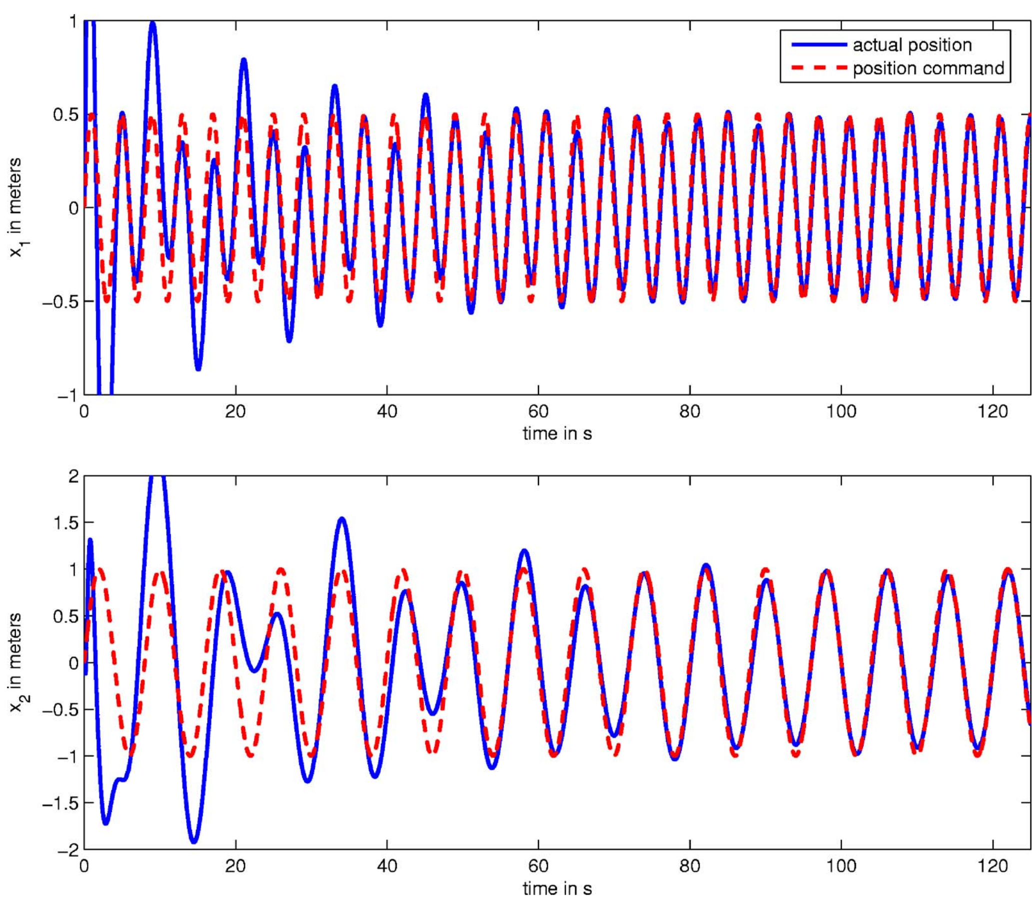

Fig. 9 Position-command following for the two-mass system using an LQG controller and a backward-path controller. The LQG controller G_c produces the control input u to track the position command r, while the backward-path controller with zero dc gain, that is, a zero at z=1, rejects the sensor bias.

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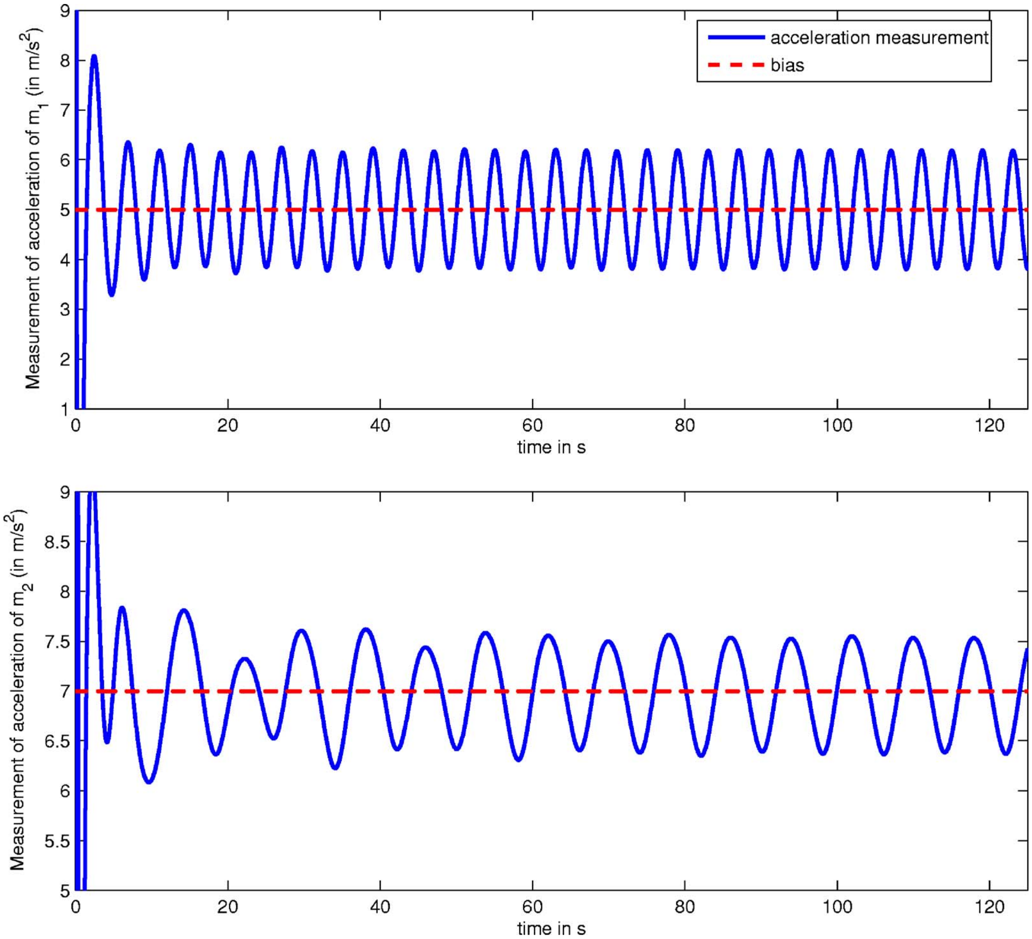

Fig. 10 Acceleration measurements of the two masses. The sensor biases in the accelerometers are shown as dashed lines.

we do not use the position output from the model for tracking, we plot the position output to illustrate the performance of the implemented controller. The biased acceleration measurements during position tracking are shown in Fig. 10. In spite of the presence of the bias, the positions of the two masses accurately follow the reference command. The magnitude of the diagonal entries of the sensitivity transfer function $G_{\mathrm{sens},r}$ given by Eq. (4.7) is shown in Fig. 11. It can be seen that the sensitivity is low in the desired frequency range between 0.1 Hz and 1 Hz. Furthermore, the input-output characteristic of the closed-loop system is highly decoupled in the sense that the position command for one mass has minimal effect on the position of the other mass. The magnitudes

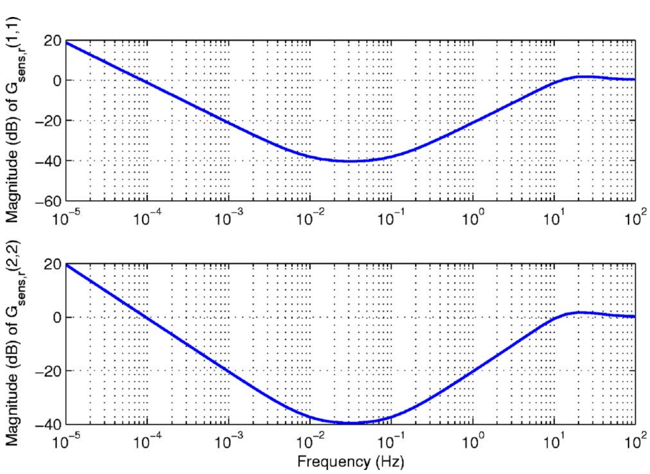

Fig. 11 Magnitudes of the diagonal entries of $G_{\mathrm{sens},r}$, the sensitivity transfer function between the reference position command r and the position-tracking error z_{pos}. The magnitude of the sensitivity function is low in the required bandwidth between 0.1 Hz and 1 Hz.

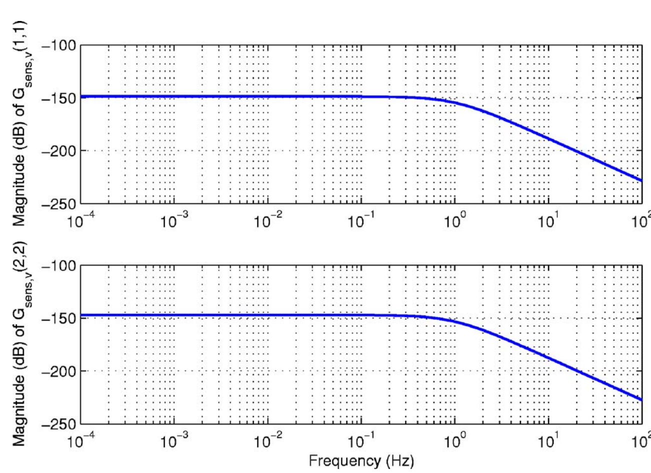

Fig. 12 Magnitudes of the diagonal entries of $G_{\mathrm{sens},v}$, the sensitivity function between the bias v and position-tracking error z_{pos}. The inclusion of a backward-path controller with zero dc gain ensures that as $k \to \infty$ the position-tracking performance is not affected by the sensor bias v.

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of the diagonal entries of the sensitivity function $G_{\text{sens},v}$ in Eq. (4.8) are plotted in Fig. 12. Note that $G_{\text{bp}}(1)=0$, and hence Proposition 2.1 guarantees that $z_{\text{pos}}(k)\to 0$ as $k\to\infty$ when $r=0$. However, in this example, the reference position command r is nonzero, and therefore $z_{\text{pos}}(k)$ may not converge to 0 as $k\to\infty$. However, since the sensitivity between z_{pos} and r is small between 0.1 Hz and 1 Hz, the steady-state position-tracking performance is satisfactory.

6 Conclusion

In this paper, we developed a position-command-following controller for linear systems using acceleration measurements that are biased for both system identification and feedback. The method outlined here is applicable to systems that have stable dynamics and when the measurement biases are unknown. Since a system identification procedure is used to obtain the inertance and compliance models, no modeling information is required. This method is easy to implement because displacement measurements, which are usually difficult to obtain, are not required and a linear controller is used.

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