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$\mathscr{H}_2$-optimal synthesis of controllers with relative degree two[1,2]

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Abstract

This paper considers fixed-structure $\mathscr{H}_2$-optimal relative degree two controller synthesis. The problem is presented in a decentralized static output feedback framework developed for fixed-order (i.e., full- and reduced-order) dynamic controller synthesis. A quasi-Newton/continuation algorithm is used to compute solutions to the necessary conditions. To demonstrate the approach, a flexible structure example is considered.

Keywords: Relative degree two control; Fixed-order control; $\mathscr{H}_2$-optimal control

1. Introduction

It is well known that modern multivariable control design frameworks such as $\mathscr{H}_2$ and $\mathscr{H}_\infty$ control yield dynamic compensators with relative degree zero or one. Hence, the structure of the dynamic feedback controller is such that the measured system output appears explicitly in the control signal or the measured system output appears explicitly in the control rate signal (MacFarlane and Karcanias, 1976). In the single- input/single-output system case, the resulting controller transfer function is non-strictly proper or strictly proper with relative degree zero or one. In this case, the Bode plot of the controller transfer function at best rolls off at 20 dB per decade. Alternatively, for relative degree r controllers, the Bode plot of the compensator has a high-frequency roll-off of $20r$ dB per decade.

High-frequency roll-off is particularly useful when the system under consideration is a lightly damped flexible structure. Since flexible structure models are by necessity truncated to a finite number of modes, it is desirable for the frequency response to roll-off as quickly as possible after the gain crossover frequency so that unmodeled high-frequency system dynamics are not excited by the controller dynamics.

For single-input/single-output systems, where the $\mathscr{H}_2$ norm corresponds to the area under the Bode plot and the $\mathscr{H}_\infty$ norm corresponds to the maximum magnitude of the Bode plot, roll-off rates cannot be specified by minimization techniques on these norms. Loop shaping weighting functions can be used in the controller design process, but these specify the frequency where the roll-off starts, not the roll-off rate. Furthermore, these techniques also tend to result in high-order controllers when frequency weighting is included in the design process. In this paper we extend the fixed-structure controller design framework of Bernstein et al. (1989) and Erwin et al. (1996) to design $\mathscr{H}_2$-optimal relative degree two controllers for multi-input/multi-output systems. Since we cast the relative degree two design problem within the fixed-structure control framework, fixed-order (i.e., full- and reduced-order) controllers can be designed with increased roll-off rates at the gain crossover frequency. Even though the proposed framework can be easily extended to include desired weighting functions for loop shaping, we do not do so here to facilitate the presentation. Finally, we note that this is the first paper that addresses $\mathscr{H}_2$-optimal control with relative degree two.

The proposed $\mathscr{H}_2$-optimal relative degree two controller design technique is applied to a structural control

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problem, showing that the resulting relative degree two controller incurs minimal increase in $\mathscr{H}_2$ performance over the optimal LQG controller while enforcing a 20 dB per decade increase in the roll-off rate at the gain crossover frequency.

2. $\mathscr{H}_2$-optimal relative degree two control

In this section we state the $\mathscr{H}_2$-optimal relative degree two control problem. Specifically, given the nth-order stabilizable and detectable plant

$$\dot{x}(t) = Ax(t) + Bu(t) + D_1 w(t), \quad t \in [0, \infty) \tag{1}$$

with noisy measurements

$$y(t) = Cx(t) + D_2 w(t) \tag{2}$$

and performance variables

$$z(t) = E_1 x(t) + E_2 u(t), \tag{3}$$

where $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^l$, $z(t) \in \mathbb{R}^p$, and $w(t) \in \mathbb{R}^d$, where $w(t)$ is a unit-intensity, zero-mean, Gaussian white noise signal, and where $E_1^{\mathrm{T}} E_2 = 0$, determine an n_{c}th-order relative degree two dynamic compensator

$$\dot{x}_{\mathrm{c}1}(t) = A_{\mathrm{c}1} x_{\mathrm{c}1}(t) + B_{\mathrm{c}1} y(t), \tag{4}$$

$$\dot{x}_{\mathrm{c}2}(t) = A_{\mathrm{c}2} x_{\mathrm{c}2}(t) + B_{\mathrm{c}2} v(t), \tag{5}$$

$$v(t) = C_{\mathrm{c}1} x_{\mathrm{c}1}(t), \tag{6}$$

$$u(t) = C_{\mathrm{c}2} x_{\mathrm{c}2}(t), \tag{7}$$

where $x_{\mathrm{c}1}(t) \in \mathbb{R}^{n_{\mathrm{c}1}}$, $x_{\mathrm{c}2}(t) \in \mathbb{R}^{n_{\mathrm{c}2}}$, and $n_{\mathrm{c}} \triangleq n_{\mathrm{c}1} + n_{\mathrm{c}2}$, such that the $\mathscr{H}_2$ performance criterion

$$J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) \triangleq \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \int_0^t [x^{\mathrm{T}}(s) R_1 x(s) + u^{\mathrm{T}}(s) R_2 u(s)] \, \mathrm{d}s, \tag{8}$$

where $\mathbb{E}$ denotes expectation, is minimized.

Note that the dynamic controller (4)–(7) corresponds to a cascade interconnection of the two controllers in the feedback loop (see Fig. 1) so that the controller transfer function realization is given by

$$G_{\mathrm{c}}(s) = G_{\mathrm{c}2} G_{\mathrm{c}1}(s) \sim \left[\begin{array}{c|c} A_{\mathrm{c}} & B_{\mathrm{c}} \\ \hline C_{\mathrm{c}} & 0 \end{array}\right], \tag{9}$$

where

$$A_{\mathrm{c}} = \begin{bmatrix} A_{\mathrm{c}1} & 0 \\ B_{\mathrm{c}2} C_{\mathrm{c}1} & A_{\mathrm{c}2} \end{bmatrix}, \qquad B_{\mathrm{c}} = \begin{bmatrix} B_{\mathrm{c}1} \\ 0 \end{bmatrix},$$
$$C_{\mathrm{c}} = [0 \quad C_{\mathrm{c}2}]. \tag{10}$$

Note that since $B_{\mathrm{c}2}$ is always multiplied with $C_{\mathrm{c}1}$, $B_{\mathrm{c}2} C_{\mathrm{c}1}$ can be considered a single free parameter, thus leaving only five controller gains over which to optimize, instead of six. Finally, we note that this framework can be easily extended to address the design of relative degree r controllers by considering a cascade interconnection of r dynamic controllers in the feedback loop.

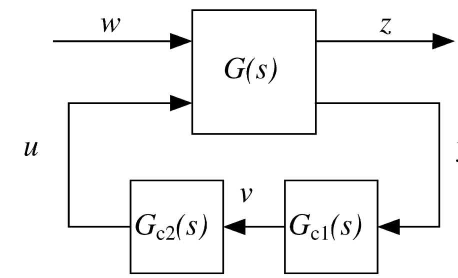

Fig. 1. Relative degree two controller set-up.

3. Decentralized static output feedback formulation

In this section we re-cast the relative degree two dynamic compensation problem as a decentralized static output feedback problem using the fixed-structure controller synthesis framework of Bernstein et al. (1989) and Erwin et al. (1996). Specifically, consider the 6-vector-input, 6-vector-output decentralized system shown in Fig. 2. By treating $A_{\mathrm{c}1}$, $A_{\mathrm{c}2}$, $B_{\mathrm{c}1}$, $B_{\mathrm{c}2} C_{\mathrm{c}1}$, and $C_{\mathrm{c}2}$ as decentralized static output feedback gains, we can rewrite the closed-loop system dynamics (1)–(7) as

$$\dot{\tilde{x}}(t) = \mathscr{A} \tilde{x}(t) + \sum_{i=1}^{5} \mathscr{B}_{ui} u_i(t) + \mathscr{B}_w w(t), \quad t \in [0, \infty), \tag{11}$$

$$y_i(t) = \mathscr{C}_{yi} \tilde{x}(t) + \mathscr{D}_{ywi} w(t), \quad i = 1, \ldots, 5, \tag{12}$$

$$z(t) = \mathscr{C}_z \tilde{x}(t) + \sum_{i=1}^{5} \mathscr{D}_{zui} u_i(t), \tag{13}$$

$$u_1(t) = A_{\mathrm{c}1} y_1(t), \quad u_2(t) = A_{\mathrm{c}2} y_2(t), \quad u_3(t) = B_{\mathrm{c}1} y_3(t),$$
$$u_4(t) = B_{\mathrm{c}2} C_{\mathrm{c}1} y_4(t), \quad u_5(t) = C_{\mathrm{c}2} y_5(t), \tag{14}$$

where

$$\tilde{x}(t) \triangleq \begin{bmatrix} x(t) \\ x_{\mathrm{c}1}(t) \\ x_{\mathrm{c}2}(t) \end{bmatrix}, \qquad \mathscr{A} \triangleq \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathscr{B}_{u1} \triangleq \begin{bmatrix} 0 \\ I_{n_{\mathrm{c}1}} \\ 0 \end{bmatrix}, \qquad \mathscr{B}_{u2} \triangleq \begin{bmatrix} 0 \\ 0 \\ I_{n_{\mathrm{c}2}} \end{bmatrix}, \qquad \mathscr{B}_{u3} \triangleq \begin{bmatrix} 0 \\ I_{n_{\mathrm{c}1}} \\ 0 \end{bmatrix},$$

$$\mathscr{B}_{u4} \triangleq \begin{bmatrix} 0 \\ 0 \\ I_{n_{\mathrm{c}2}} \end{bmatrix}, \qquad \mathscr{B}_{u5} \triangleq \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \qquad \mathscr{B}_w \triangleq \begin{bmatrix} D_1 \\ 0 \\ 0 \end{bmatrix},$$

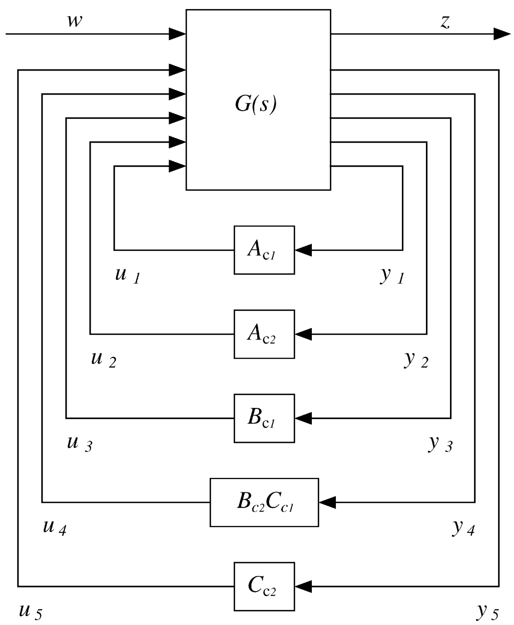

Fig. 2. Decentralized static output feedback framework.

$$\mathscr{C}_{y1} \triangleq [0 \quad I_{n_{c1}} \quad 0], \qquad \mathscr{C}_{y2} \triangleq [0 \quad 0 \quad I_{n_{c2}}],$$

$$\mathscr{C}_{y3} \triangleq [C \quad 0 \quad 0], \qquad \mathscr{C}_{y4} \triangleq [0 \quad I_{n_{c1}} \quad 0],$$

$$\mathscr{C}_{y5} \triangleq [0 \quad 0 \quad I_{n_{c2}}], \qquad \mathscr{D}_{yw1} \triangleq 0, \ \mathscr{D}_{yw2} \triangleq 0,$$

$$\mathscr{D}_{yw3} \triangleq D_2, \qquad \mathscr{D}_{yw4} \triangleq 0, \qquad \mathscr{D}_{yw5} \triangleq 0,$$

$$\mathscr{C}_{z} \triangleq [E_1 \quad 0 \quad 0], \qquad \mathscr{D}_{zu1} \triangleq 0, \ \mathscr{D}_{zu2} \triangleq 0,$$

$$\mathscr{D}_{zu3} \triangleq 0, \qquad \mathscr{D}_{zu4} \triangleq 0, \qquad \mathscr{D}_{zu5} \triangleq E_2.$$

It is important to note that the decentralized architecture (12), (13) is used here only as a detail in the derivation of the results and does not limit the applicability of the proposed methodology. In particular, as shown in Bernstein et al. (1989), a fixed-order centralized dynamic compensation problem can always be reduced to a decentralized static output feedback problem by a suitable plant augmentation. Next, defining

$$\hat{u}(t) \triangleq [u_1^{\mathrm{T}}(t) \quad u_2^{\mathrm{T}}(t) \quad u_3^{\mathrm{T}}(t) \quad u_4^{\mathrm{T}}(t) \quad u_5^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$\hat{y}(t) \triangleq [y_1^{\mathrm{T}}(t) \quad y_2^{\mathrm{T}}(t) \quad y_3^{\mathrm{T}}(t) \quad y_4^{\mathrm{T}}(t) \quad y_5^{\mathrm{T}}(t)]^{\mathrm{T}},$$

Eqs. (11)–(13) can be rewritten as

$$\dot{\tilde{x}}(t) = \mathscr{A}\tilde{x}(t) + \mathscr{B}_u\hat{u}(t) + \mathscr{B}_w w(t), \quad t \in [0, \infty), \tag{15}$$

$$\hat{y}(t) = \mathscr{C}_y\tilde{x}(t) + \mathscr{D}_{yw}w(t), \tag{16}$$

$$z(t) = \mathscr{C}_z\tilde{x}(t) + \mathscr{D}_{zu}\hat{u}(t), \tag{17}$$

where

$$\mathscr{B}_u \triangleq [\mathscr{B}_{u1} \quad \mathscr{B}_{u2} \quad \mathscr{B}_{u3} \quad \mathscr{B}_{u4} \quad \mathscr{B}_{u5}],$$

$$\mathscr{D}_{zu} \triangleq [\mathscr{D}_{zu1} \quad \mathscr{D}_{zu2} \quad \mathscr{D}_{zu3} \quad \mathscr{D}_{zu4} \quad \mathscr{D}_{zu5}],$$

$$\mathscr{C}_y \triangleq [\mathscr{C}_{y1}^{\mathrm{T}} \quad \mathscr{C}_{y2}^{\mathrm{T}} \quad \mathscr{C}_{y3}^{\mathrm{T}} \quad \mathscr{C}_{y4}^{\mathrm{T}} \quad \mathscr{C}_{y5}^{\mathrm{T}}]^{\mathrm{T}},$$

$$\mathscr{D}_{yw} \triangleq [\mathscr{D}_{yw1}^{\mathrm{T}} \quad \mathscr{D}_{yw2}^{\mathrm{T}} \quad \mathscr{D}_{yw3}^{\mathrm{T}} \quad \mathscr{D}_{yw4}^{\mathrm{T}} \quad \mathscr{D}_{yw5}^{\mathrm{T}}]^{\mathrm{T}}.$$

Furthermore, by rewriting the decentralized control signals (14) in the compact form $\hat{u}(t) = \mathscr{K}\hat{y}(t)$, where $\mathscr{K} \triangleq \text{block-diag}(A_{c1}, A_{c2}, B_{c1}, B_{c2}C_{c1}, C_{c2})$, the closed-loop system is given by

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}_w w(t), \quad t \in [0, \infty), \tag{18}$$

$$z(t) = \tilde{C}_z\tilde{x}(t), \tag{19}$$

where $\tilde{A} \triangleq \mathscr{A} + \mathscr{B}_u\mathscr{K}\mathscr{C}_y$, $\tilde{B}_w \triangleq \mathscr{B}_w + \mathscr{B}_u\mathscr{K}\mathscr{D}_{yw}$, and $\tilde{C}_z \triangleq \mathscr{C}_z + \mathscr{D}_{zu}\mathscr{K}\mathscr{C}_y$.

Now, if $\tilde{A}$ is asymptotically stable for a given feedback gain $\mathscr{K} \in \mathbb{R}^{(2n_c+m)\times(2n_c+l)}$ it follows that the $\mathscr{H}_2$ performance criterion (8) is given by

$$J(A_c, B_c, C_c) = \|\tilde{G}_{zw}(s)\|_2^2 = \operatorname{tr} \tilde{P}\tilde{B}_w\tilde{B}_w^{\mathrm{T}}, \tag{20}$$

where $\tilde{P}$ is the unique, $\tilde{n} \times \tilde{n}$ nonnegative-definite solution to the algebraic Lyapunov equation

$$0 = \tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + \tilde{C}_z^{\mathrm{T}}\tilde{C}_z, \tag{21}$$

where $\tilde{n} \triangleq n + n_{c1} + n_{c2}$. Now, the necessary conditions for optimality can be derived by forming the Lagrangian

$$\mathscr{L}(\tilde{P}, \tilde{Q}, \mathscr{K}) = \operatorname{tr}\{\tilde{P}\tilde{B}_w\tilde{B}_w^{\mathrm{T}} + \tilde{Q}[\tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + \tilde{C}_z{}^{\mathrm{T}}\tilde{C}_z]\}, \tag{22}$$

where $\tilde{Q} \in \mathbb{R}^{\tilde{n}\times\tilde{n}}$ is a Lagrange multiplier. The gradient expressions with respect to the free parameters in Eq. (22) are given by

$$\frac{\partial\mathscr{L}}{\partial\tilde{P}} = \tilde{A}\tilde{Q} + \tilde{Q}\tilde{A}^{\mathrm{T}} + \tilde{B}_w\tilde{B}_w^{\mathrm{T}},$$

$$\frac{\partial\mathscr{L}}{\partial\tilde{Q}} = \tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + \tilde{C}_z^{\mathrm{T}}\tilde{C}_z, \tag{23}$$

$$\frac{\partial\mathscr{L}}{\partial A_{c1}} = \mathscr{B}_{u1}^{\mathrm{T}}\tilde{P}\tilde{Q}\mathscr{C}_{y1}^{\mathrm{T}}, \quad \frac{\partial\mathscr{L}}{\partial A_{c2}} = \mathscr{B}_{u2}^{\mathrm{T}}\tilde{P}\tilde{Q}\mathscr{C}_{y2}^{\mathrm{T}},$$

$$\frac{\partial\mathscr{L}}{\partial B_{c1}} = \mathscr{B}_{u3}^{\mathrm{T}}\tilde{P}\tilde{B}_w\mathscr{D}_{yw3}^{\mathrm{T}} + \mathscr{B}_{u3}^{\mathrm{T}}\tilde{P}\tilde{Q}\mathscr{C}_{y3}^{\mathrm{T}}, \tag{24}$$

$$\frac{\partial\mathscr{L}}{\partial C_{c2}} = \mathscr{B}_{u5}^{\mathrm{T}}\tilde{P}\tilde{Q}\mathscr{C}_{y5}^{\mathrm{T}} + \mathscr{D}_{zu5}^{\mathrm{T}}\tilde{C}_z\tilde{Q}\mathscr{C}_{y5}^{\mathrm{T}},$$

$$\frac{\partial\mathscr{L}}{\partial B_{c2}C_{c1}} = \mathscr{B}_{u4}^{\mathrm{T}}\tilde{P}\tilde{Q}\mathscr{C}_{y4}^{\mathrm{T}}. \tag{25}$$

4. Quasi-Newton/continuation algorithm

To solve the nonlinear optimization problem posed in Section 3, a general-purpose BFGS quasi-Newton algorithm is used. The line-search portions of the algorithm were modified to include a constraint-checking subroutine which decreases the length of the search direction vector until it lies entirely within the set of parameters that yield a stable closed-loop system. This modification ensures that the cost function J remains defined at every point in the line-search process. Numerical experience indicates that this subroutine is usually invoked only during the first few iterations of a synthesis problem.

One requirement of gradient-based optimization algorithms is an initial stabilizing design. This was accomplished by using a balanced truncation to obtain a reduced-order LQG controller corresponding to the controller in the feedback loop with the lower order. Note that since two controllers are being synthesized, it is not necessary for this first truncated controller to stabilize the system. This controller was augmented to the plant. If G_{c1} is designed first, the augmented realization is

$$\hat{G}(s) \sim \left[\begin{array}{cc|c:c} A & 0 & B & D_1 \\ B_{c1}C & A_{c1} & B_{c1}D & B_{c1}D_2 \\ \hline 0 & C & 0 & 0 \\ \hdashline E_1 & 0 & E_2 & E_0 \end{array}\right], \tag{26}$$

whereas if G_{c2} is designed first, the augmented realization is

$$\tilde{G}(s) \sim \left[\begin{array}{cc|c:c} A & BC_{c2} & 0 & D_1 \\ 0 & A_{c2} & B_{c2} & 0 \\ \hline C & DC_{c2} & 0 & D_2 \\ \hdashline E_1 & E_2C_{c2} & 0 & E_0 \end{array}\right]. \tag{27}$$

Note that in the first case, the $\hat{D}_2$ term is identically zero, whereas in the second case, the $\hat{E}_2$ term is zero. Thus these augmented matrices result in a singular control problem. This was overcome by replacing these terms with nonzero matrices structured such that $\hat{D}_1\hat{D}_2^{\mathrm{T}} = 0$ or $\hat{E}_1^{\mathrm{T}}\hat{E}_2 = 0$, as appropriate. Once an LQG controller was designed on this "artificial" system, the dynamics of the original system were tested, and if the closed loop was asymptotically stable, these designed controllers were used as the initial controllers for the gradient search algorithm. For details of the algorithm, see Erwin et al. (1996).

5. Illustrative numerical example

Consider a two-mass–spring–damper system with a colocated sensor/actuator pair and state space realization in real normal coordinates given by

$$\dot{x}(t) = \begin{bmatrix} -0.0002 & 0.2208 & 0 & 0 \\ -0.2208 & -0.0002 & 0 & 0 \\ 0 & 0 & -0.0103 & 1.4320 \\ 0 & 0 & -1.4320 & -0.0103 \end{bmatrix}$$

$$\cdot\, x(t) + \begin{bmatrix} -0.1439 \\ 0.2168 \\ -0.0426 \\ 1.1890 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} -0.0545 & 0.0819 & -0.0352 & 0.8181 \end{bmatrix} x(t).$$

The weighting matrices D_1, D_2, E_1, and E_2 were chosen so that LQG synthesis would place a notch at the second mode. This is accomplished when (Friedman and Bernstein, 1993)

$$D_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_2 = [0 \quad 1],$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For this system, the G_{c1} and G_{c2} controllers in the feedback loop were chosen to be of order two. Initializing reduced-order LQG controllers were designed, and the gradient search algorithm was initiated. The Bode plots of the loop gain of the full-order LQG controller and of the relative degree two controller, along with dotted lines representing the respective high-frequency asymptotes, are shown in Fig. 3. The $\mathscr{H}_2$-optimal LQG cost is 3.9734 and the $\mathscr{H}_2$-optimal relative degree two controller is 4.0743 which corresponds to only a 2.5% increase over the $\mathscr{H}_2$-optimal LQG controller cost. This marginal increase in the $\mathscr{H}_2$ cost is not surprising since *$\mathscr{H}_2$-optimal* relative degree two controllers are sought.

The transfer function for the LQG controller is

$$G_{c(\mathrm{LQG})}(s) = \frac{-0.9090s^3 - 0.1073s^2 - 1.8660s - 0.1817}{s^4 + 0.3629s^3 + 1.2425s^2 + 0.6170s + 0.1443},$$

which has natural frequencies at 0.345 and 1.10 rad/s. The relative degree two controller transfer function is given by

$$G_{c(\mathrm{Rel.Deg.2})}(s) = \frac{-99.9827s^2 - 2.0960s - 205.2481}{s^4 + 64.6193s^3 + 46.1170s^2 + 34.9138s + 77.6907}.$$

This transfer function has a natural frequency at 1.02 rad/s and break frequencies at 0.862 and 0.015 rad/s. Thus it is seen in this case that constraining the controller

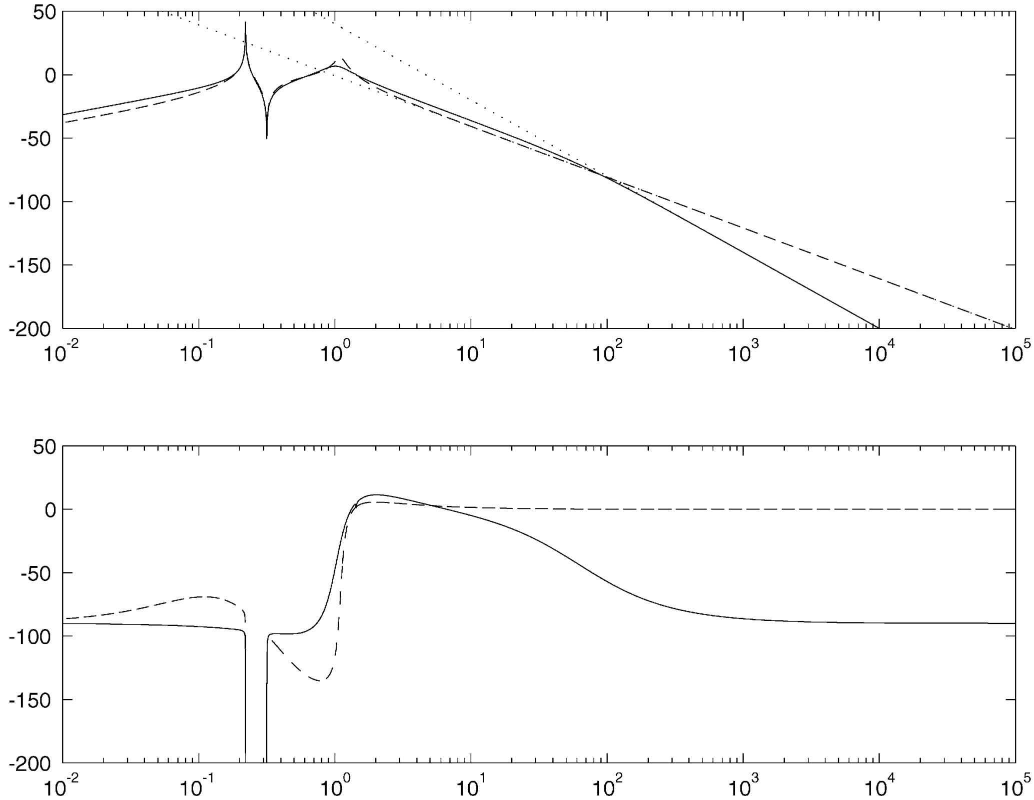

Fig. 3. Bode plots of LQG and relative degree two controllers [LQG: Dashed, Relative Degree Two: Solid].

to be of relative degree two does not push the controller poles out to such high frequencies that the extra 20 dB/decade roll-off is not useful.

6. Conclusions

In this paper we proposed a scheme to synthesize $\mathscr{H}_2$-optimal relative degree two controllers by cascading two controllers in the feedback loop and optimizing over the five free controller parameters. The problem was formulated in a decentralized static output feedback framework, which facilitated the use of a quasi-Newton optimization algorithm. This technique was applied to a flexible structure example. It was shown that constraining the controller to have a relative degree of at least two only marginally increased the $\mathscr{H}_2$ cost of the closed-loop system.

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