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Sequential design of decentralized dynamic compensators using the optimal projection equations

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The optimal projection equations for quadratically optimal centralized fixed-order dynamic compensation are generalized to the case in which the dynamic compensator has, in addition, a fixed decentralized structure. Under a stabilizability assumption for the particular feedback configuration, the resulting optimality conditions explicitly characterize each subcontroller in terms of the plant and remaining subcontrollers. This characterization associates an oblique projection with each subcontroller and suggests an iterative sequential design algorithm. The results are applied to an interconnected flexible beam example.

1. Introduction

The purpose of this note is to consider the problem of designing decentralized dynamic feedback controllers using recently obtained results on quadratically optimal fixed-order dynamic compensation (Hyland and Bernstein 1984). As in Bernussou and Titli (1982), Looze *et al.* (1978), and Singh (1981), the overall approach is to fix the structure (information pattern and order) of the linear controller and optimize the steady-state regulation cost with respect to the controller parameters. The underlying philosophy is that the ability to carry out such an optimization procedure permits the evaluation of a particular decentralized configuration which may be dictated by implementation constraints. If there is some flexibility in designing the decentralized architecture, then these results can be used to evaluate the optimal performance of each permissible configuration, and hence to determine preferable structures. Since the present paper is confined to the question of optimal regulation, trade-offs with regard to robustness in the presence of plant variations are not considered. Such trade-offs can be included, however, by utilizing the Stratonovich multiplicative white noise approach developed by Bernstein and Hyland (1985).

To further motivate our approach, consider the problem of controlling an nth-order plant $\mathscr{P}$ by means of a decentralized dynamic compensator consisting of subcontrollers $\mathscr{C}_1$ and $\mathscr{C}_2$. A straightforward design technique that immediately comes to mind is that of *sequential optimization* (Davison and Gesing 1979, Jamshidi 1983). To begin, ignore $\mathscr{C}_2$ and design $\mathscr{C}_1$ as a centralized controller for $\mathscr{P}$. Next, regard the closed-loop system consisting of $\mathscr{P}$ and $\mathscr{C}_1$ as an augmented system $\mathscr{P}'$ and design $\mathscr{C}_2$ as a centralized controller for $\mathscr{P}'$. Now *redesign* $\mathscr{C}_1$ to be a centralized controller for the augmented closed-loop system composed of $\mathscr{P}$ and $\mathscr{C}_2$, and so forth. One difficulty with this scheme, however, is that of *dimension*. If, for example, one were to employ LQG at each step of this algorithm, then on the first iteration $\mathscr{C}_1$ would have dimension n and thus $\mathscr{C}_2$ would have dimension $2n$. On the second iteration, $\mathscr{C}_1$ would require dimension $3n$ and $\mathscr{C}_2$ would have order $4n$, and so forth. Such

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difficulties can be avoided by setting $n=0$, which essentially corresponds to static output feedback. Although easier to implement, static output feedback lacks filtering abilities such as are inherent in LQG controllers, which are purely dynamic (i.e. strictly proper).

As discussed by Sandell *et al.* (1978), p. 119, the explanation for this difficulty is provided by the 'second-guessing' phenomenon: when LQG is used, each subcontroller must consist of linear feedback, not only of estimates of the plant states but also of *estimates of the other subcontrollers' estimates*. Hence the 'optimal' controller is given by an irrational transfer function, i.e. a distributed parameter (infinite-dimensional) system. Such controllers, of course, must be ruled out since their design and implementation (except in special cases) violate physical realizability (see, for example, Bernstein and Hyland 1986).

Having thus ruled out zeroth-order and infinite-order decentralized controllers, we focus on the problem of designing purely dynamic decentralized compensators. Moreover, by invoking the constraint of fixed subcontroller order, we overcome the second-guessing phenomenon. Utilizing the parameter optimization approach thus leads to a generalization of the result obtained by Hyland and Bernstein (1984) for centralized control. In brief, it was shown in Hyland and Bernstein (1984) that the unwieldy first-order necessary conditions for fixed-order dynamic compensation can be simplified by exploiting the presence of a previously unrecognized *oblique* projection. The resulting *optimal projection equations*, which consist of a pair of modified Riccati equations and a pair of modified Lyapunov equations coupled by the optimal projection, yield insight into the structure of the optimal dynamic compensator and emphasize the breakdown of the separation principle for reduced-order controller design. For example, the optimal compensator is the projection of a full-order dynamic controller which is generally different from the LQG design. Furthermore, this full-order controller and the oblique projection are intricately related since they are simultaneously determined by the coupled design equations. An immediate consequence is the observation that stepwise schemes employing either model reduction followed by LQG or LQG followed by model reduction are generally suboptimal. For computational purposes, the optimal projection equations permit the development of novel numerical methods which operate through successive iteration of the oblique projection (Hyland and Bernstein 1985). Such algorithms are thus philosophically and operationally distinct from gradient search methods.

The generalization of the optimal projection equations to the decentralized case is straightforward and immediate. In the optimization process each subcontroller is viewed as a centralized controller for an augmented 'plant' consisting of the *actual* plant and all other subcontrollers. It need only be observed that the necessary conditions for optimality for the decentralized problem must consist of the collection of necessary conditions obtained by optimizing over each subcontroller separately while keeping the other subcontrollers fixed. More precisely, this statement corresponds to the fact that setting the Fréchet derivative to zero is equivalent to setting the individual partial derivatives to zero. Hence it is not surprising that the optimal projection equations for the decentralized problem involve multiple oblique projections, one associated with each subcontroller. Furthermore, each subcontroller incorporates an internal model (in the sense of an oblique projection of full-order dynamics) not only of the plant but also of all other subcontrollers. The structure of the equations suggests a sequential design algorithm such as that proposed in this work.

The simplicity with which this result is obtained should not belie its relevance to the decentralized control problem. Specifically, our approach is distinct from subsystem-decomposition techniques (Ikeda and Siljak 1980, 1981, Ikeda *et al.* 1981, 1984, Lindner 1985, Linnemann 1984, Ozguner 1979, Ramakrishna and Viswanadham 1982, Saeks 1979, Sezer and Huseyin 1984, Silkak 1978, 1983) and model-reduction methods since the optimal projection equations retain the full, interconnected plant at all times. For the proposed algorithm, decomposition techniques which exploit subsystem-interconnection data can play a role by providing a starting point for subsequent iterative refinement and optimization. Decomposition methods may also play a role when very high dimensionality precludes direct solution of the optimal projection equations. These are areas for future research.

With regard to the role of the oblique projection, it should be noted that such transformations do not, in general, preserve plant characteristics such as poles, zeros, subspaces, etc. Indeed, since the oblique projection arises as a consequence of optimality, approaches that seek to retain system invariants (e.g. Uskokovic and Medanic 1985) are generally suboptimal. In addition, the complex coupling among the plant and subcontrollers via multiple oblique projections provides an additional measure for evaluating the suboptimality of the methods proposed.

The plan of the paper is as follows. The fixed-structure decentralized dynamic-compensation problem is stated in § 2 along with the generalization of the optimal projection equations. In § 3 we propose a sequential design algorithm for solving these equations and state conditions under which convergence is guaranteed. Finally, in § 4 the algorithm is applied to the 8th-order model of a pair of simply supported beams connected by a spring. For this example, we obtain a two-channel decentralized design which is 4th-order in each channel and compare its performance with the (8th-order) centralized LQG design.

2. Problem statement and main theorem

Given the controlled system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{p} B_i u_i(t) + w_0(t) \tag{2.1}$$

$$y_i(t) = C_i x(t) + w_i(t), \quad i = 1, \dots, p \tag{2.2}$$

design a fixed-structure decentralized dynamic compensator

$$\dot{x}_{ci}(t) = A_{ci} x_{ci}(t) + B_{ci} y_i(t), \quad i = 1, \dots, p \tag{2.3}$$

$$u_i(t) = C_{ci} x_{ci}(t), \quad i = 1, \dots, p \tag{2.4}$$

which minimizes the steady-state performance criterion

$$J(A_{c1}, B_{c1}, C_{c1}, \dots, A_{cp}, B_{cp}, C_{cp}) \triangleq \lim_{t \to \infty} \mathbb{E}\left[x(t)^{\mathrm{T}} R_0 x(t) + \sum_{i=1}^{p} u_i(t)^{\mathrm{T}} R_i u_i(t) \right] \tag{2.5}$$

where, for $i = 1, \dots, p$: $x \in \mathbb{R}^n$, $u_i \in \mathbb{R}^{m_i}$, $y_i \in \mathbb{R}^{l_i}$, $c_{ci} \in \mathbb{R}^{n_{ci}}$, $n_c \triangleq \sum_{i=1}^{p} n_{ci}$, $n_{ci} \leqslant n + n_c - n_{ci}$, A, B_i, C_i, A_{ci}, B_{ci}, C_{ci}, R_0 and R_i are matrices of appropriate dimension with R_0 (symmetric) non-negative definite and R_i (symmetric) positive definite; w_0 is white disturbance noise with $n \times n$ non-negative-definite intensity V_0, and w_i is white observation noise with $l_i \times l_i$ positive-definite intensity V_i, where $w_0, w_1, \ldots, w_p$ are mutually uncorrelated and have zero mean. $\mathbb{E}$ denotes expectation and superscript T indicates transpose.

To guarantee that J is finite and independent of initial conditions we restrict our attention to the set of admissible stabilizing compensators

$$\mathscr{A} \triangleq \{(A_{c1}, B_{c1}, C_{c1}, \ldots, A_{cp}, B_{cp}, C_{cp}): \tilde{A} \text{ is asymptotically stable}\}$$

where the closed-loop dynamics matrix $\tilde{A}$ is given by

$$\tilde{A} \triangleq \begin{bmatrix} A & \tilde{B}C_c \\ B_c\tilde{C} & A_c \end{bmatrix}$$

where

$$\tilde{B} \triangleq [B_1 \quad \ldots \quad B_p], \quad \tilde{C} \triangleq \begin{bmatrix} C_1 \\ \vdots \\ C_p \end{bmatrix}$$

$$A_c \triangleq \text{block-diagonal}\,(A_{c1}, \ldots, A_{cp})$$

$$B_c \triangleq \text{block-diagonal}\,(B_{c1}, \ldots, B_{cp})$$

$$C_c \triangleq \text{block-diagonal}\,(C_{c1}, \ldots, C_{cp})$$

(For possibly non-square matrices S_1, S_2, block-diagonal (S_1, S_2) denotes the matrix $\begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$.)

It is possible that for certain decentralized structures the system is not stabilizable, i.e. $\mathscr{A}$ is empty (Wang and Davison 1973, Seraji 1982, Sezer and Siljak 1981). Our approach, however, is to assume that $\mathscr{A}$ is not empty and characterize the optimal decentralized controller over the stabilizing class. Since the value of J is independent of the internal realization of each subcompensator, without loss of generality we can further restrict our attention to

$$\mathscr{A}_+ \triangleq \{(A_{c1}, B_{c1}, C_{c1}, \ldots, A_{cp}, B_{cp}, C_{cp}) \in \mathscr{A}: (A_{ci}, B_{ci}) \text{ is controllable and } (C_{ci}, A_{ci}) \text{ is observable}, i = 1, \ldots, p\}$$

The following lemma is an immediate consequence of Theorem 6.2.5, p. 123 of Rao and Mitra (1971). Let I_r denote the $r \times r$ identity matrix.

Lemma 2.1

Suppose $\hat{Q}, \hat{P} \in \mathbb{R}^{q \times q}$ are non-negative definite and rank $\hat{Q}\hat{P} = r$. Then there exist $G, \Gamma \in \mathbb{R}^{r \times q}$ and invertible $M \in \mathbb{R}^{r \times r}$ such that

$$\hat{Q}\hat{P} = G^{\mathrm{T}} M \Gamma \tag{2.6}$$

$$\Gamma G^{\mathrm{T}} = I_r \tag{2.7}$$

For convenience in stating the main theorem, call (G, M, Γ) satisfying (2.6), (2.7) a *projective factorization* of $\hat{Q}\hat{P}$. Such a factorization is unique modulo an arbitrary change in basis in $\mathbb{R}^r$, which corresponds to nothing more than a change of basis for the internal representation of the compensator (or subcompensators in the present context).

We shall also require the following notation. Let $\tilde{A}_i$ denote $\tilde{A}$ with the rows and columns containing A_{ci} deleted. Similarly, let $\tilde{R}_i$ be obtained by deleting the rows and columns corresponding to $C_{ci}^T R_i C_{ci}$ in the matrix

$$\tilde{R} \triangleq \text{block-diagonal}\,(R_0, C_{c1}^T R_1 C_{c1}, \ldots, C_{cp}^T R_p C_{cp})$$

And furthermore, $\tilde{V}_i$ is obtained by deleting the rows and columns containing $B_{ci} V_i B_{ci}^T$ in

$$\tilde{V} \triangleq \text{block-diagonal}\,(V_0, B_{c1} V_1 B_{c1}^T, \ldots, B_{cp} V_p B_{cp}^T)$$

Also define

$$\tilde{B}_i \triangleq \begin{bmatrix} B_i \\ 0_{(n_c - n_{ci}) \times m_i} \end{bmatrix}, \quad \tilde{C}_i \triangleq [C_i \quad 0_{l_i \times (n_c - n_{ci})}]$$

where $0_{r \times s}$ denotes the $r \times s$ zero matrix. Note that $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$, $\tilde{R}_i$ and $\tilde{V}_i$ essentially represent the closed-loop system minus the ith subcontroller as controlled by the latter. Finally, define

$$\Sigma_i \triangleq \tilde{B}_i R_i^{-1} \tilde{B}_i^T, \quad \bar{\Sigma}_i \triangleq \tilde{C}_i^T V_i^{-1} \tilde{C}_i$$

and, for $\tau \in \mathbb{R}^{r \times r}$, let

$$\tau_\perp \triangleq I_r - \tau$$

Main theorem

Suppose $(A_{c1}, B_{c1}, C_{c1}, \ldots, A_{cp}, B_{cp}, C_{cp}) \in \mathscr{A}_+$ solves the steady-state fixed-structure decentralized dynamic-compensation problem. Then for $i = 1, \ldots, p$ there exist $(n + n_c - n_{ci}) \times (n + n_c - n_{ci})$ non-negative-definite matrices Q_i, P_i, $\hat{Q}_i$ and $\hat{P}_i$ such that A_{ci}, B_{ci} and C_{ci} are given by

$$A_{ci} = \Gamma_i(\tilde{A}_i - Q_i \bar{\Sigma}_i - \Sigma_i P_i) G_i^T \tag{2.8}$$

$$B_{ci} = \Gamma_i Q_i \tilde{C}_i^T V_i^{-1} \tag{2.9}$$

$$C_{ci} = -R_i^{-1} \tilde{B}_i^T P_i G_i^T \tag{2.10}$$

for some projective factorization G_i, M_i, Γ_i of $\hat{Q}_i \hat{P}_i$, and such that, with $\tau_i = G_i^T \Gamma_i$, the following conditions are satisfied:

$$0 = \tilde{A}_i Q_i + Q_i \tilde{A}_i^T + \tilde{V}_i - Q_i \bar{\Sigma}_i Q_i + \tau_{i\perp} Q_i \bar{\Sigma}_i Q_i \tau_{i\perp}^T \tag{2.11}$$

$$0 = \tilde{A}_i^T P_i + P_i \tilde{A}_i + \tilde{R}_i - P_i \Sigma_i P_i + \tau_{i\perp}^T P_i \Sigma_i P_i \tau_{i\perp} \tag{2.12}$$

$$0 = (\tilde{A}_i - \Sigma_i P_i)\hat{Q}_i + \hat{Q}_i(\tilde{A}_i - \Sigma_i P_i)^T + Q_i \bar{\Sigma}_i Q_i - \tau_{i\perp} Q_i \bar{\Sigma}_i Q_i \tau_{i\perp}^T \tag{2.13}$$

$$0 = (\tilde{A}_i - Q_i \bar{\Sigma}_i)^T \hat{P}_i + \hat{P}_i(\tilde{A}_i - Q_i \bar{\Sigma}_i) + P_i \Sigma_i P_i - \tau_{i\perp}^T P_i \Sigma_i P_i \tau_{i\perp} \tag{2.14}$$

$$\text{rank}\, \hat{Q}_i = \text{rank}\, \hat{P}_i = \text{rank}\, \hat{Q}_i \hat{P}_i = n_{ci} \tag{2.15}$$

Remark 2.1

Because of (2.7) the matrix τ_i is idempotent, i.e. $\tau_i^2 = \tau_i$. This projection corresponding to the ith subcontroller is an *oblique* projection (as opposed to an orthogonal projection) since it is not necessarily symmetric. Furthermore, τ_i is given in closed form by

$$\tau_i = \hat{Q}_i \hat{P}_i (\hat{Q}_i \hat{P}_i)^{\#}$$

where $(\ )^{\#}$ denotes the (Drazin) group generalized inverse (see, for example, Campbell and Meyer, 1979, p. 124).

3. Proposed algorithm

Sequential design algorithm

Step 1. Choose a starting point consisting of initial subcontroller designs;

Step 2. For a sequence $\{i_k\}_{k=1}^{\infty}$, where $i_k \in \{1, \ldots, p\}$, $k = 1, 2, \ldots$, redesign subcontroller i_k as an optimal fixed-order centralized controller for the plant and remaining subcontrollers;

Step 3. Compute the cost J_k of the current design and check $J_k - J_{k-1}$ for convergence.

Note that the first two steps of the algorithm consist of (i) bringing suboptimal subcontrollers 'on line' and (ii) iteratively refining each subcontroller. As discussed in § 1, the choice of a starting design for Step 1 can be obtained by a variety of existing methods such as subsystem decomposition. As for subcontroller refinement, note that each subcontroller redesign procedure is equivalent to replacing a suboptimal subcontroller with a subcontroller which is optimal with respect to the plant and remaining subcontrollers.

Proposition 3.1

For a given starting design and redesign sequence $\{i_k\}_{k=1}^{\infty}$ suppose that the optimal projection equations can be solved for each k to yield the global minimum. Then $\{J_k\}_{k=1}^{\infty}$ is monotonically non-increasing and hence convergent.

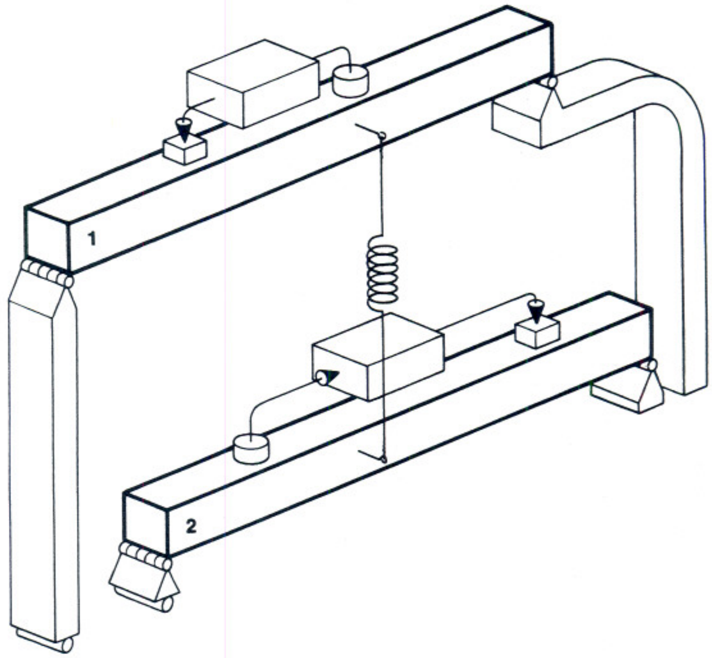
Determining both a suitable starting point and redesign sequence for solvability and attaining the decentralized global minimum remain areas for future research. With regard to algorithms for solving the optimal projection equations for each subcontroller redesign procedure, details of proposed algorithms can be found in the works of Hyland (1983, 1984) and Hyland and Bernstein (1985).

4. Application to interconnected flexible beams

To demonstrate the applicability of the main theorem and the sequential design algorithm, we consider a pair of simply supported Euler–Bernoulli flexible beams interconnected by a spring (see the Figure). Each beam possesses one rate sensor and one force actuator. Retaining two vibrational modes in each beam, we obtain the 8th-order interconnected model

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B_1 = \begin{bmatrix} B_{11} \\ 0_{4\times 1} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0_{4\times 1} \\ B_{22} \end{bmatrix}$$

$$C_1 = [C_{11} \quad 0_{1\times 4}], \quad C_2 = [0_{1\times 4} \quad C_{22}]$$



where

$$A_{ii} = \begin{bmatrix} 0 & \omega_{1i} & 0 & 0 \\ -\omega_{1i} - (k/\omega_{1i})(\sin \pi c_i)^2 & -2\zeta_i\omega_{1i} & -(k/\omega_{2i})(\sin \pi c_i)(\sin 2\pi c_i) & 0 \\ 0 & 0 & 0 & \omega_{2i} \\ -(k/\omega_{1i})(\sin \pi c_i)(\sin 2\pi c_i) & 0 & -\omega_{2i} - (k/\omega_{2i})(\sin 2\pi c_i)^2 & -2\zeta_i\omega_{2i} \end{bmatrix}$$

$$A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ (k/\omega_{1j})(\sin \pi c_i)(\sin \pi c_j) & 0 & (k/\omega_{2j})(\sin \pi c_i)(\sin 2\pi c_j) & 0 \\ 0 & 0 & 0 & 0 \\ (k/\omega_{1j})(\sin \pi c_j)(\sin 2\pi c_i) & 0 & (k/\omega_{2j})(\sin 2\pi c_i)(\sin 2\pi c_j) & 0 \end{bmatrix}$$

$$i \neq j$$

$$B_{ii} = \begin{bmatrix} 0 \\ -\sin \pi a_i \\ 0 \\ -\sin 2\pi a_i \end{bmatrix}, \quad C_{ii} = [0 \quad \sin \pi s_i \quad 0 \quad \sin 2\pi s_i]$$

$$a_i = \hat{a}_i/L_i, \quad s_i = \hat{s}_i/L_i, \quad c_i = \hat{c}_i/L_i$$

In the above definitions, k is the spring constant, ω_{ji} is the jth modal frequency of the ith beam, ζ_i is the damping ratio of the ith beam, L_i is the length of the ith beam, and $\hat{a}_i$, $\hat{s}_i$ and $\hat{c}_i$ are, respectively, the actuator, sensor and spring-connection coordinates as measured from the left in the Figure. The chosen values are

$$k = 10$$

$$\omega_{1i} = 1, \quad \omega_{2i} = 4, \quad \zeta_i = 0{\cdot}005, \quad L_i = 1, \quad i = 1, 2$$

$$\hat{a}_1 = 0{\cdot}3, \quad \hat{s}_1 = 0{\cdot}65, \quad \hat{c}_1 = 0{\cdot}6$$

$$\hat{a}_2 = 0{\cdot}8, \quad \hat{s}_2 = 0{\cdot}2, \quad \hat{c}_2 = 0{\cdot}4$$

In addition, weighting and intensity matrices are chosen to be

$$R_1 = \text{block-diagonal}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_{11} \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_{21} \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_{12} \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1/\omega_{22} \end{bmatrix}\right)$$

$$R_2 = R_3 = 0{\cdot}1 I_2$$

$$V_0 = \text{block-diagonal}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$V_1 = V_2 = 0{\cdot}1 I_2$$

For this problem the open-loop cost was evaluated and the centralized 8th-order LQG design was obtained to provide a baseline. To provide a starting point for the sequential design algorithm, a pair of 4th-order LQG controllers were designed for each beam separately ignoring the interconnection, i.e. setting $k = 0$. The optimal projection equations were then utilized to iteratively refine each subcontroller. The results are summarized in the Table.

Design	Cost
Open loop	163·5
Centralized LQG	
$n_c = 8$	19·99
Suboptimal decentralized	
$n_{c1} = n_{c2} = 4$	59·43
Redesign subcontroller 2	28·19
Redesign subcontroller 1	23·29
Redesign subcontroller 2	23·04
Redesign subcontroller 1	22·25
Redesign subcontroller 2	21·94
Redesign subcontroller 1	21·86
Redesign subcontroller 2	21·81
Redesign subcontroller 1	21·79

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