Is It Possible to Follow Position Commands Using Velocity Measurements?

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In a simplistic sense, position can be determined from measurements of velocity by integration, and velocity can be determined from measurements of position by differentiation. In the former case, integration requires an initial condition and thus requires at least one position measurement. However, even in the case where the initial position is known, constant-but-unknown velocity-measurement noise, that is, bias, produces a spurious ramp in the computed position. On the other side of the coin, the accuracy of differentiation is limited by sensor noise, and thus approximate differentiation must be used in practice. Hence, low-frequency noise is the bane of integration, whereas high-frequency noise is the nemesis of differentiation.

A more sophisticated approach to obtaining position from velocity measurements and velocity from position measurements is to use a model-based observer. Of course, this can be done only if the state whose estimate is required is observable from the available measurements. In the case of a rigid body, velocity is observable from position measurements, but the opposite is not true, that is, position is not observable from velocity measurements. For an oscillator, however, it is easy to show that position is, in fact, observable from velocity measurements, and thus it is possible to use an observer to estimate position from velocity measurements. In effect, the observer, whose error dynamics are asymptotically stable, can be viewed as an approximate integrator that does not require an initial position measurement.

In addition to not requiring knowledge of the initial position, a position observer is advantageous relative to velocity integration in the presence of measurement bias. In particular, for velocity measurements with bias, position estimates provided by an observer would include an offset but not a spurious ramp component, as in the case of an integrator. In a servo application, the effect of bias in the velocity measurement would need to be assessed within the context of the closed-loop system. With these issues in mind, the purpose of this article is to address the question of whether it is possible to implement a servo feedback loop with either an integrator or an observer so that the position of an oscillator follows position step commands using only velocity measurements?

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It is not immediately obvious whether the answer to this question is yes or no. In particular, a servo loop that can achieve asymptotic following of position step commands requires integral action in the feedback loop. Integral control would not be effective, however, due to pole-zero cancellation at zero. Therefore, a controller that achieves zero asymptotic error to a position step command with velocity measurements must possess a double integrator. In implementing such a controller, the question arises as to whether pole-zero cancellation at zero would lead to a hidden unbounded response due to a constant disturbance injected at some point in the loop [1], [2]. In addition, unknown, arbitrary initial conditions could potentially lead to an unbounded response. The presence of an unbounded response, due to either an injected step or nonzero initial conditions, would mean that the control system could not be successfully implemented in practice. The possible presence of hidden instabilities in a single-input, single-output closed-loop system is addressed in [3] by analyzing the "gang of four" transfer functions.

Although classical in nature, this question does not appear to be addressed in the literature. A related problem involving acceleration measurements is considered in [4], while the effect of sensor and actuator bias on integral control is analyzed in [5].

The problem of using velocity measurements in a position servo loop is not merely academic but has practical implications for static shape control of flexible structures using piezoelectric actuators [6], [7]. There are three types of measurements that can be obtained from piezoelectric materials, namely, charge and voltage, which are associated with strain, and current, which is associated with strain rate [8]. Although charge and voltage measurements provide information about strain, these measurements roll off at low frequency due to capacitance leakage, and thus they provide no direct information about static shape. For the purpose of adding damping for vibration suppression, strain rate is a suitable measurement, and rate-feedback control laws are often used to enhance energy dissipation [9].

If, however, the objective is static shape control, then the structure must be able to follow position step commands [8]. Unfortunately, the rolloff of charge and voltage measurements at dc limits the use of piezoelectric materials for static shape control based on feedback, which suggests that knowledge of static position is impossible without additional

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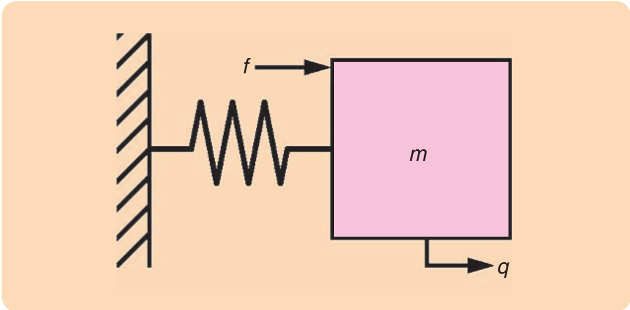

FIGURE 1 An undamped oscillator.

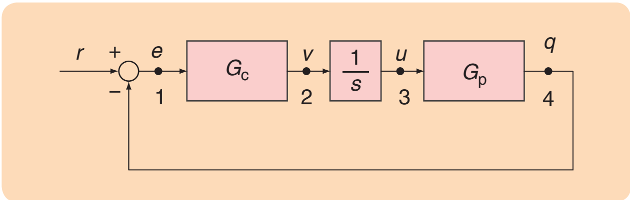

FIGURE 2 Architecture 1.

position sensors. To overcome this difficulty, applications of piezoelectric materials that require shape control employ a position sensor, which is typically based on optics. These sensors, however, are generally expensive and difficult to implement. This limitation is well known, and attempts have been made to overcome this problem, especially within the context of atomic force microscopes [10]–[12].

Despite this limitation, it is easy to see why static shape control using feedback and without position sensing is possible, at least in principle. In particular, a dynamic model of the structure could be used with strain measurements (which unavoidably roll off at low frequency) as the basis of an observer that estimates the *true* shape; this estimate could then be used in a feedback control system. In effect, the observer uses the structural model to replace the low-frequency shape information that is missing from the strain measurements. The observer thus provides the error signal needed by the feedback controller.

Also relevant to the use of charge and voltage measurements is the feedback control law known as positive position feedback (PPF) [13], [14]. As its name suggests, implementing PPF with piezoelectric sensors assumes the availability of a strain measurement. This assumption is incorrect, however, since the strain measurement rolls off at dc. This rolloff gives rise to phase shifts at low frequency, making it difficult to follow low-frequency position commands.

The above discussion shows that the practical construction of controllers that can follow position commands using only velocity measurements depends on knowing whether or not such controllers can be implemented without hidden instabilities due to pole-zero cancellation at zero. The aim of this article is thus to determine the feasibility of using velocity measurements within a servo loop whose goal is to follow position commands.

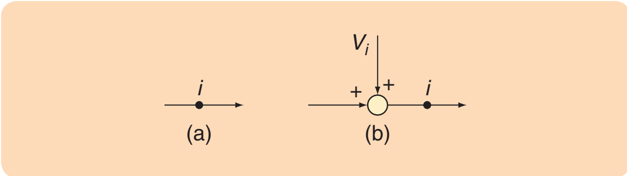

FIGURE 3 For a bias V_i injected at node i in Figures 2, 5, and 8, the node corresponding to (a) is replaced by (b).

PROBLEM SETUP

For the undamped oscillator shown in Figure 1, transfer functions from the force f to the position q and from the force f to the velocity $\dot{q}$ are

$$G_{\mathrm{p}}(s) = \frac{q(s)}{f(s)} = \frac{1/m}{s^2 + k/m}, \tag{1}$$

and

$$G_{\mathrm{v}}(s) = \frac{\dot{q}(s)}{f(s)} = \frac{(1/m)s}{s^2 + k/m}, \tag{2}$$

respectively. Note that G_{v} has a zero at 0. Since no damping is assumed to be present, both transfer functions are Lyapunov stable but neither is asymptotically stable. For all examples, $m = 1$ kg and $k = 1$ N/m.

This article considers the feedback control architectures:

» *Architecture 1:* Position command with linear-quadratic-Guassian (LQG) control, where position is measured.

» *Architecture 2:* Position command with LQG control, where velocity is measured but is integrated to obtain a naïve estimate of the position.

» *Architecture 3:* Position command with LQG control, where velocity is measured and position is estimated using an observer.

Architecture 1 provides an illustrative baseline but violates the standing assumption that only velocity is measured.

ARCHITECTURE 1: POSITION COMMAND WITH POSITION MEASUREMENTS

The case of position command with position measurements is considered first. To ensure zero asymptotic error for position step commands, an integrator is cascaded with the oscillator modeled by G_{p}. For the augmented system, which is Lyapunov stable, a third-order LQG controller is designed. This architecture is shown in Figure 2. To study the effect of bias, steps are injected using the convention shown in Figure 3.

The response to a position unit step command r is shown in Figure 4 for both zero and nonzero initial conditions for the oscillator and the integrator, as well as in the presence of a position-measurement bias injected at node 4. As expected, the position of the mass follows the position step command with zero asymptotic error except for the case of the position-measurement bias, where the asymptotic command-following error is nonzero but bounded.

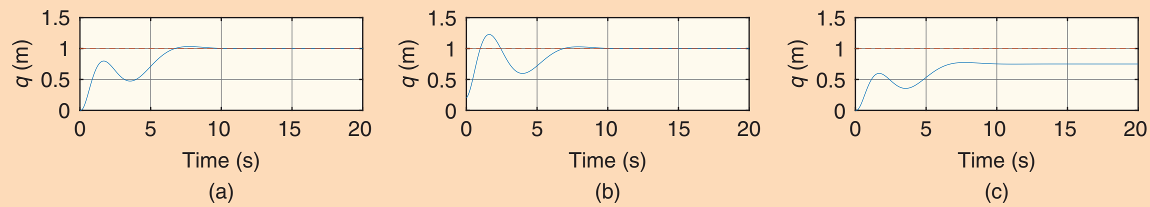

FIGURE 4 Position q of the mass for a position unit step command r in Architecture 1 shown in Figure 2 with (a) zero initial conditions with no position-measurement bias, (b) nonzero initial conditions with no position-measurement bias, and (c) zero initial conditions with position-measurement bias injected at node 4. As expected, with the measurements of true position q available, the command-following error converges to zero in the absence of position-measurement bias. In the case where position-measurement bias is present, the asymptotic command-following error is nonzero but bounded.

ARCHITECTURE 2: POSITION COMMAND WITH VELOCITY MEASUREMENTS, DOUBLE INTEGRATOR, AND MEASUREMENT INTEGRATION

The case where velocity is measured is considered next. In this case, the position of the mass is determined by integrating the velocity measurement, and the integrated measurement is used in a servo loop to follow a position step command. Note that the initial position of the mass is assumed to be unknown, and thus the integrated velocity is not necessarily the true position of the mass. This architecture is shown in Figure 5. Since the output of the plant is velocity, the plant is G_v, whereas G_p is used to determine, as a diagnostic tool, the true position of the mass.

Since the plant has a zero at zero, a double integrator is needed to ensure zero asymptotic error for position step commands. In addition, an integrator is added after the plant to provide a naïve position estimate $\hat{q}$ that can be used to generate an error signal.

To investigate the performance of this architecture, the final-value theorem is applied to all combinations of input and output nodes. The corresponding transfer functions are parameterized as

$$G_c(s) = \frac{a_1 s^3 + a_2 s^2 + a_3 s + a_4}{s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4}, \quad (3)$$

$$G_i(s) = \frac{1}{s^2}, \quad (4)$$

$$G_{ou}(s) = \frac{c_1}{s^2 + d_1 s + d_2}, \quad (5)$$

$$G_{o\dot{q}}(s) = \frac{e_1 s + e_2}{s^2 + d_1 s + d_2}. \quad (6)$$

These transfer functions are used to determine the asymptotic response to unit step disturbances injected at nodes 1–6 shown in Figure 5. At nodes 1, 2, and 5, the step disturbance represents a bias due to the controller, whereas, at nodes 3 and 4, the step disturbance represents an external force disturbance and velocity-measurement bias, respectively. For each pair of input and output nodes, the asymptotic response to a unit step disturbance is shown in Table 1, where

$$L_1 \triangleq \frac{-b_4}{a_4}. \quad (7)$$

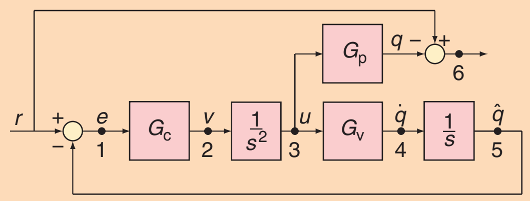

FIGURE 5 Architecture 2. Note that e is not the command-following error since $\hat{q}$ is a naïve position estimate, which is not necessarily the true position. The true command-following error at node 6 is not known in practice but is used as a performance diagnostic.

Note the response at both nodes 3 and 6 with step injection at node 4 is unbounded.

The response to a unit position step command is shown in Figure 6 for zero and nonzero initial conditions as well as the case where velocity-measurement bias is injected at node 4. In all cases, the integral of the velocity measurement is initialized to be zero. Figure 6 shows that, for nonzero initial conditions and no velocity-measurement bias, the asymptotic command-following error in the position q is nonzero but bounded. However, the position q has a ramp component in the case where velocity-measurement bias is injected at node 4. These simulations show that this architecture is unusable in practice for following position step commands.

TABLE 1 Asymptotic response at node "Out" due to a unit step disturbance injected at node "In." The constant L_1 is defined by (3) and (7).

In \ Out	1	2	3	4	5	6
V_1	0	0	1	0	1	−1
V_2	L_1	0	$-L_1$	0	$-L_1$	L_1
V_3	0	0	0	0	0	0
V_4	0	0	$-\infty$	0	0	∞
V_5	0	0	−1	0	0	1
V_6	0	0	0	0	0	1

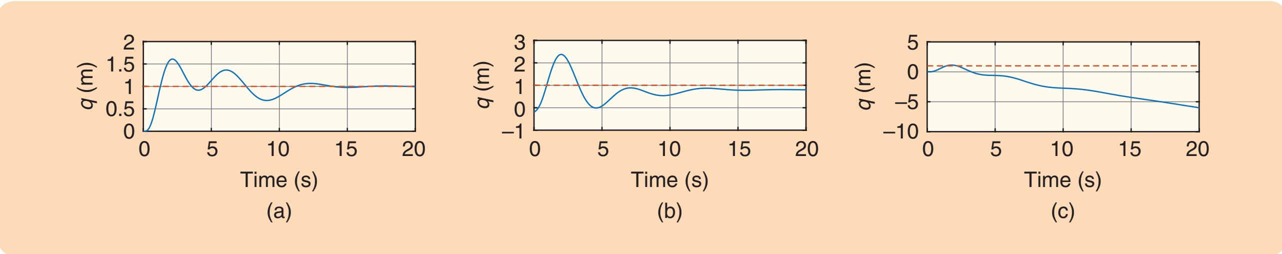

FIGURE 6 Position q of the mass for a position unit step command r in Architecture 2 shown in Figure 5 with (a) zero initial conditions with no velocity-measurement bias, (b) nonzero initial conditions with no velocity-measurement bias, and (c) zero initial conditions with a velocity-measurement bias injected at node 4. In (a), the command-following error converges to zero; in (b), a bounded position offset is present; and in (c), the position of q exhibits a ramp component, and thus the diagnostic error at node 6 in Figure 5 diverges.

The inability of this architecture to follow position step commands stems from the naïve attempt to obtain position by integrating velocity. In the absence of knowledge of the initial position, this approach gives nonzero but bounded asymptotic position error. More seriously, however, velocity-measurement bias leads to a ramp in the position q despite the fact that no ramp appears in the integrated velocity $\hat{q}$.

The sensitivity function in the traditional sense is not applicable to this architecture since the signal e in Figure 5 is not the command-following error but, rather, is the difference between the command r and the naïve position estimate $\hat{q}$. We therefore consider the frequency response of the transfer functions from the command r and an injected velocity-measurement step at node 4 to the true position error at node 6. The rollup at dc in Figure 7(b) indicates that velocity-measurement bias causes an unbounded response in the true position error.

ARCHITECTURE 3: POSITION COMMAND WITH VELOCITY MEASUREMENTS, DOUBLE INTEGRATOR, AND POSITION OBSERVER

To overcome the limitations of Architecture 2, the integrator is replaced with an observer. A realization of G_v is given by

$$\dot{x} = Ax + Bu, \tag{8}$$

$$y = Cx, \tag{9}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad C = [0 \;\; 1], \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}.$$

An observer for (8) and (9) is

$$\dot{\hat{x}} = A\hat{x} + Bu + F(y - C\hat{x}), \tag{10}$$

$$\hat{y} = C_o\hat{x}, \tag{11}$$

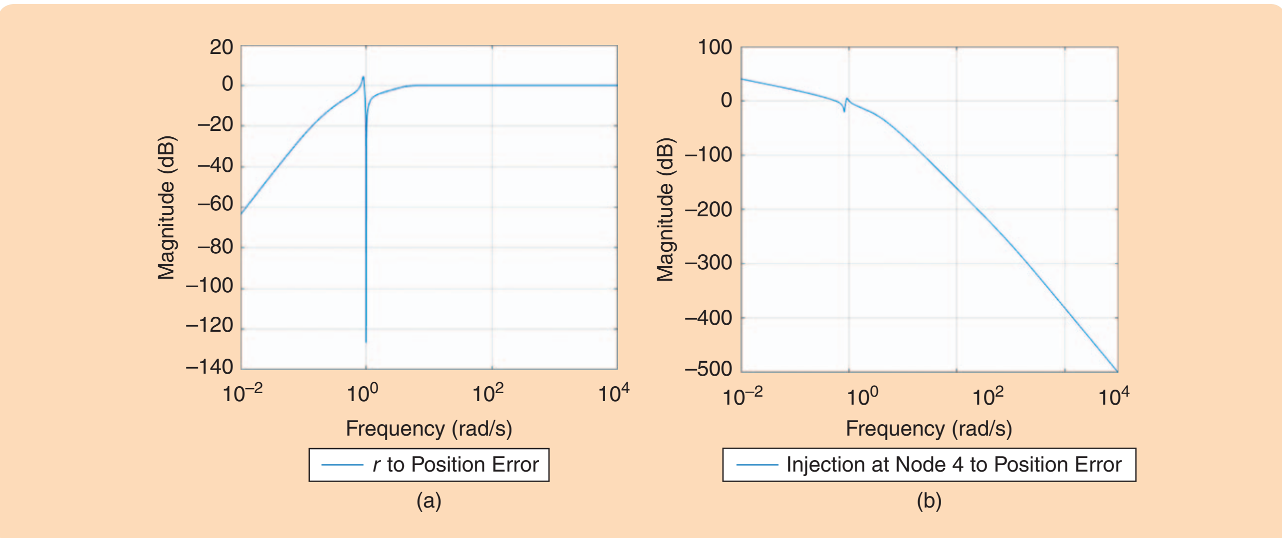

FIGURE 7 Architecture 2. The magnitude frequency response of the transfer functions from (a) the command r and (b) an injection at node 4 to the position command-following error. In (a), the 40-dB/decade rolloff at dc indicates that the asymptotic response at dc due to a step command is bounded, whereas, in (b), the rollup at dc indicates that the asymptotic response at dc due to velocity-measurement bias diverges.

where

$$C_o = [1 \;\; 0],$$

so that the output $\hat{y}$ of the observer (10) and (11) is the position estimate. The observer can be rewritten as

$$\dot{\hat{x}} = (A - FC)\hat{x} + Bu + F\dot{q}. \qquad (12)$$

To ensure zero asymptotic error for a position step command, a double integrator is cascaded with the plant, and an LQG controller of order four is designed for the augmented plant. The architecture for the closed-loop system is shown in Figure 8, where

$$G_o = [G_{ou} \;\; G_{o\dot{q}}], \qquad (13)$$

and thus

$$\hat{q}(s) = G_{ou}(s)u(s) + G_{o\dot{q}}(s)\dot{q}(s). \qquad (14)$$

To investigate the performance of this architecture, the final-value theorem is applied to all combinations of input and output nodes. The corresponding transfer functions are parameterized as in (3)–(6). These transfer functions are used to determine the asymptotic response to unit step disturbances injected at nodes 1–6 shown in Figure 8. At nodes 1, 2, and 5, the step disturbance represents a bias due to the controller, whereas, at nodes 3 and 4, the step disturbance represents an external force disturbance and velocity-measurement bias, respectively. For each pair of input and output nodes, the asymptotic response to a unit step disturbance is shown in Table 2, where

$$L_2 \triangleq \frac{d_2}{c_1}, \quad L_3 \triangleq \frac{b_4 d_2}{a_4 c_1}, \quad L_4 \triangleq \frac{-e_2}{c_1}. \qquad (15)$$

Note that all of the responses are bounded, which shows that the closed-loop system has no hidden instabilities.

Finally, the loop is simulated and the responses shown in Figure 9 are consistent with the corresponding entries in Table 2. In particular, the closed-loop system exhibits zero asymptotic position command-following error for nonzero initial conditions, and, unlike Architecture 2, does not exhibit a ramp in the presence of velocity-measurement bias at node 4, as indicated by Figure 10.

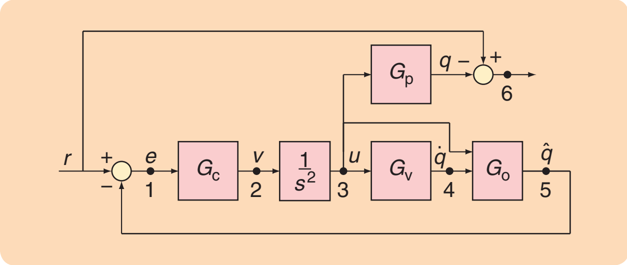

FIGURE 8 Architecture 3. Note that e is not the command-following error since $\hat{q}$ is the output of the observer G_o, and thus $\hat{q}$ is not necessarily the true position. The true command-following error at node 6 is not known in practice but is used as a performance diagnostic in order to assess the feasibility of the architecture.

TABLE 2 Asymptotic response at node "Out" due to a unit step disturbance injected at node "In." The constants L_1, L_2, L_3, and L_4 are defined by (3)–(7) and (15).

In \ Out	1	2	3	4	5	6
V_1	0	0	L_2	0	1	L_2
V_2	L_1	0	L_3	0	$-L_1$	L_3
V_3	0	0	0	0	0	0
V_4	0	0	L_4	1	0	L_4
V_5	0	0	$-L_2$	0	0	$-L_2$
V_6	0	0	0	0	0	1

CONCLUSIONS

The analysis of Architecture 3 shows that position step commands for an oscillator can be followed with zero asymptotic command-following error in the absence of velocity-measurement bias and step disturbances and with bounded asymptotic command-following error in the presence of velocity-measurement bias and step disturbances. Most importantly, by considering the effect of a step disturbance

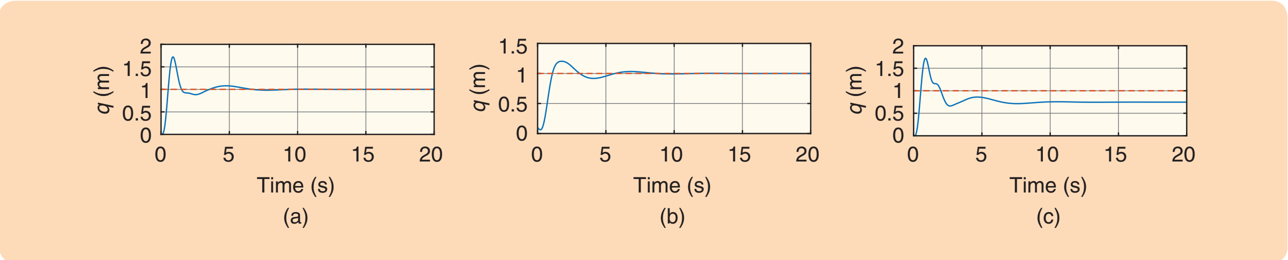

FIGURE 9 Position q of the mass for a position unit-step command r in Architecture 3 shown in Figure 8 with (a) zero initial conditions with no velocity-measurement bias, (b) nonzero initial conditions with no velocity-measurement bias, and (c) zero initial conditions with velocity-measurement bias injected at node 4. The command-following error converges to zero for arbitrary initial conditions and is bounded in the presence of velocity-measurement bias.

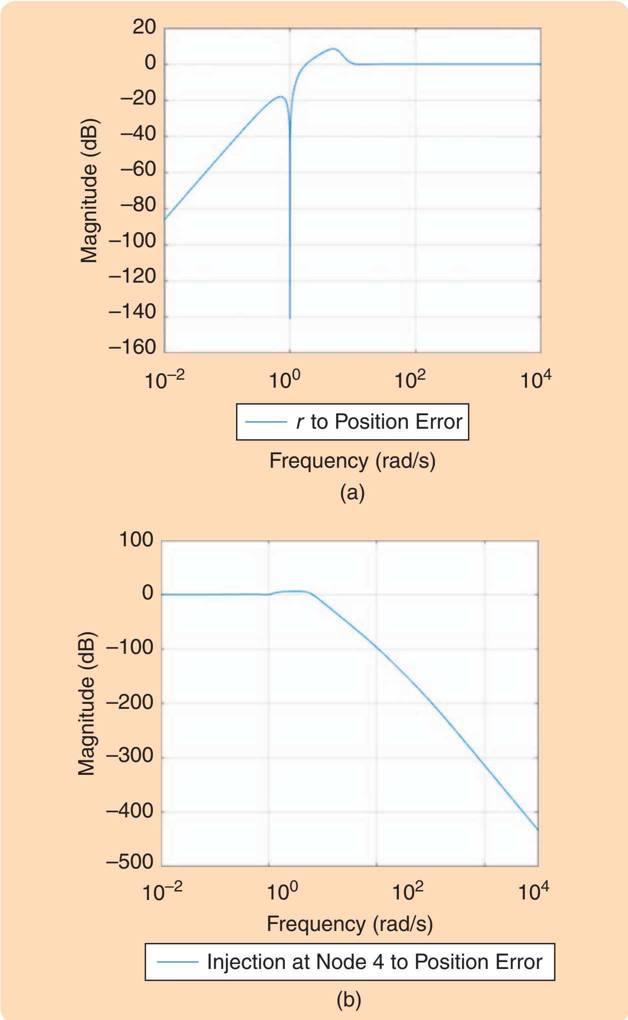

FIGURE 10 Architecture 3. The magnitude frequency response of the transfer functions from (a) the command r and (b) an injection at node 4 to the position command-following error. In (a), the 40-dB/decade rolloff at dc indicates that the asymptotic response at dc to a step command is bounded, whereas, in (b), the flat frequency response at dc indicates that the asymptotic response at dc due to velocity-measurement bias is bounded.

injected at all key points in the loop, it was shown by means of the final-value theorem and verified numerically that no hidden instabilities are present in the loop due to pole-zero cancellation at zero. This shows that, for an oscillator, it is possible to follow position step commands using velocity measurements. This result has practical implications for the problem of constant-shape control using velocity (for example, strain-rate) measurements, which may be the only available measurements in cases where it is not feasible to measure position without rolloff and phase shift down to dc.

Finally, a practically relevant extension of the problem considered in this article is to consider the case where the oscillator dynamics are uncertain. This problem is challenging due to the fact that both the observer and LQG controller must be robustified to the plant uncertainty. This problem may also present a nontrivial challenge to adaptive-control methods.

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REFERENCES

[1] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Englewood Cliffs, NJ: Prentice Hall, 1996.

[2] J. B. Hoagg and D. S. Bernstein, "Nonminimum-phase zeros: Much to do about nothing," *IEEE Contr. Syst. Mag.*, vol. 27, no. 3, pp. 45–57, June 2007.

[3] K. J. Astrom and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton, NJ: Princeton Univ. Press, 2008.

[4] J. Chandrasekar and D. S. Bernstein, "Position control using acceleration-based identification and feedback with unknown measurement bias," *ASME Trans. Dyn. Sys. Meas. Contr.*, vol. 130, no. 1, pp. 014501–1–014501–9, 2008.

[5] J. Chandrasekar and D. S. Bernstein, "Setpoint tracking with actuator offset and sensor bias: Probing the limits of integral control," *IEEE Contr. Syst. Mag.*, vol. 27, pp. 61–68, Feb. 2007.

[6] S. Devasia, T. Meressi, B. Paden, and E. Bayo, "Piezoelectric actuator design for vibration suppression: Placement and sizing," *J. Guidance Contr. Dyn.*, vol. 16, no. 5, pp. 859–864, 1993.

[7] J. J. Dosch, D. J. Inman, and E. Garcia, "A self-sensing piezoelectric actuator for collocated control," *J. Intell. Mat. Syst. Struct.*, vol. 3, no. 1, pp. 166–185, 1992.

[8] H. Irschik, "A review on static and dynamic shape control of structures by piezoelectric actuation," *Eng. Struct.*, vol. 24, no. 1, pp. 5–11, 2002.

[9] M. J. Balas, "Direct velocity feedback control of large space structures," *J. Guidance Contr. Dyn.*, vol. 2, no. 3, pp. 252–253, 1979.

[10] I. A. Ivan, M. Rakotondrabe, P. Lutz, and N. Chaillet, "Current integration force and displacement self-sensing method for cantilevered piezoelectric actuators," *Rev. Sci. Instrum.*, vol. 80, no. 12, pp. 12603–1–4, 2009.

[11] I. A. Ivan, M. Rakotondrabe, P. Lutz, and N. Chaillet, "Quasistatic displacement self-sensing method for cantilevered piezoelectric actuators," *Rev. Sci. Instrum.*, vol. 80, no. 5, pp. 065102–1–8, 2009.

[12] M. G. Ruppert and S. O. R. Moheimani, "A novel self-sensing technique for tapping-mode atomic force microscopy," *Rev. Sci. Instrum.*, vol. 84, no. 12, pp. 125006–1–7, 2013.

[13] J. L. Fanson and T. K. Caughey, "Positive position feedback control for large space structures," *AIAA J.*, vol. 28, no. 4, pp. 717–724, 1990.

[14] G. Song, S. P. Schmidt, and B. N. Agrawal, "Experimental robustness study of positive position feedback control for active vibration suppression," *J. Guidance Contr. Dyn.*, vol. 25, no. 1, pp. 179–182, 2002.