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Nonlinear Modification of Positive-Real LQG Compensators for Enhanced Disturbance Rejection and Energy Dissipation

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Abstract

A general framework for the feedback interconnection of passive nonlinear systems is used to investigate the effectiveness of modifying positive-real LQG compensators with nonlinearities to enhance disturbance rejection and energy dissipation. The primary purpose of the nonlinear modifications is to spread the spectrum of the compensator states, and thereby enhance the energy flow from the plant to the compensator. This framework is also used as a basis for the active emulation of vibration absorbers.

1. Introduction

While modern control techniques can produce high performance controllers, the robust stability properties of passive designs are often desirable. This is particularly true for flexible structures that have poorly modeled modes and for which a dissipative controller is adequate for vibration suppression.

To meet this objective, controller synthesis techniques have been developed to yield linear controllers that are positive real. For example, modified LQG and H_∞ controller synthesis techniques for producing positive-real compensators have been investigated in [1], [2]. Additionally, some specialized nonlinear compensator design techniques based on passivity concepts have been investigated in [3], [4].

The passive "chaotic" compensator designs of [3], [4] suggest specific features that may serve to enhance the performance of the control system while maintaining stability robustness. These features include a skew-symmetric term for mixing the compensator modal "energy," as well as an input squaring nonlinearity for spreading the spectrum of the compensator states to higher frequencies, where energy is dissipated more quickly.

In this paper, we investigate the ability of nonlinearities to enhance the performance of positive-real LQG controllers. Throughout this investigation we remain within the framework of passive systems interconnected by feedback in order to guarantee stability [5].

The contents of the paper are as follows. In Section 2 we begin by reviewing the passive compensation framework, which is specialized in Section 3 to linear systems. Section 4 considers the use of the framework for the absorber emulation problem on an example system. In Section 5 we consider the special case of positive-real LQG synthesis, and in Section 6 we introduce nonlinear modifications relating to the approach of [3], [4]. In Section 7 we design a positive-real LQG compensator for an illustrative example and in Section 8 assess the performance enhancements obtained by modifying this compensator with nonlinearities. Some conclusions are made in Section 9.

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2. Feedback Interconnection of Passive Systems

In this section, we describe a general framework for passive controller design. We consider systems of the form

$$\dot{x} = f(x) + G(x)u, \tag{1}$$
$$y = h(x) + J(x)u, \tag{2}$$

where $f(0) = 0$, $h(0) = 0$, $f(\cdot)$, $G(\cdot)$, $h(\cdot)$, and $J(\cdot)$ are smooth functions, and u and y have the same dimension. Furthermore, (1), (2) are assumed to be zero-state detectable and completely reachable. We say that (1), (2) is *zero-state detectable* if $u(t) \equiv 0$, $y(t) \equiv 0$ implies $x(t) \equiv 0$, and it is *completely reachable* if for all finite states x_0, x_1 there exists a finite time t_1 and a square integrable control $u(t)$ defined on $[0,\ t_1]$ such that the state can be driven from $x(0) = x_0$ to $x(t_1) = x_1$.

A system with internal state x, input u, and output y is said to be *passive* if there exists a positive-definite function $V_s(x)$, with $V_s(0) = 0$, called a *storage function* such that for all $T > 0$

$$V_s(x(T)) \le \int_0^T y(t)^{\mathrm{T}} u(t)\mathrm{d}t. \tag{3}$$

The system is called *lossless* if equality holds in (3) for all $T > 0$; otherwise the system is called *dissipative* [6], [7].

The following result, which characterizes a passive system in terms of its realization,is proved in [8].

Lemma 2.1. The system (1), (2) is passive if and only if there exist functions $V_s : \mathbb{R}^n \to \mathbb{R}$, $l : \mathbb{R}^n \to \mathbb{R}^p$, and $W : \mathbb{R}^n \to \mathbb{R}^{p\times m}$ with $V_s(x)$ continuously differentiable and positive definite, $V_s(0) = 0$, such that, for all $x \in \mathbb{R}^n$,

$$V_s'(x)f(x) = -l^{\mathrm{T}}(x)l(x), \tag{4}$$
$$\tfrac{1}{2}V_s'(x)G(x) = h^{\mathrm{T}}(x) - l^{\mathrm{T}}(x)W(x), \tag{5}$$
$$J(x) + J^{\mathrm{T}}(x) = W^{\mathrm{T}}(x)W(x). \tag{6}$$

If the system (1), (2) is strictly proper, that is, $J(x) \equiv 0$, then $W(x) \equiv 0$ and (4)-(6) are equivalent to

$$V_s'(x)f(x) \le 0, \ \tfrac{1}{2}V_s'(x)G(x) = h^{\mathrm{T}}(x). \tag{7}$$

The framework we consider involves the negative feedback interconnection of a passive nonlinear plant with a passive nonlinear dynamic compensator. Hence, we consider the passive plant

$$\dot{x} = f(x) + G(x)u, \tag{8}$$
$$y = h(x), \tag{9}$$

controlled by the passive compensator

$$\dot{x}_c = f_c(x_c) + G_c(y, x_c)y, \tag{10}$$
$$-u = h_c(y, x_c) + J_c(x_c)y, \tag{11}$$

where $x \in \mathbb{R}^n$, $x_c \in \mathbb{R}^{n_c}$, $u, y \in \mathbb{R}^m$, and $f(0) = 0$, $f_c(0) = 0$, $h(0) = 0$, and $h_c(0,0) = 0$. We assume that

$f(\cdot)$, $G(\cdot)$, $h(\cdot)$, $f_c(\cdot)$, $G_c(\cdot)$, $h_c(\cdot)$, and $J_c(\cdot)$ are smooth functions, and that both the plant and compensator are completely reachable and zero-state detectable. Stability of the closed-loop system (8) - (11) is guaranteed by the following result. Results of this kind are well known, and have been studied in [6] – [14].

Theorem 2.1. Suppose there exist C^1 positive-definite functions $V_s : \mathbb{R}^n \to \mathbb{R}$ and $V_{sc} : \mathbb{R}^{n_c} \to \mathbb{R}$, such that $V(x, x_c) \triangleq V_s(x) + V_{sc}(x_c) > 0$, $(x, x_c) \neq (0,0)$, and functions $l : \mathbb{R}^n \to \mathbb{R}^p$, $l_c : \mathbb{R}^{n_c} \to \mathbb{R}^{p_c}$, $W_c : \mathbb{R}^{n_c} \to \mathbb{R}^{p_c \times m}$, such that

$$V_s'(x)f(x) = -l^T(x)l(x), \qquad x \in \mathbb{R}^n, \quad (12)$$

$$h^T(x) = \tfrac{1}{2}V_s'(x)G(x), \qquad x \in \mathbb{R}^n, \quad (13)$$

$$V_{sc}'(x_c)f_c(x_c) = -l_c^T(x_c)l_c(x_c), \qquad x_c \in \mathbb{R}^{n_c} \quad (14)$$

$$h_c^T(y, x_c) = \tfrac{1}{2}V_{sc}'(x_c)G_c(y, x_c) + l_c^T(x_c)W_c(x_c), \; x_c \in \mathbb{R}^{n_c}, \qquad y \in \mathbb{R}^m \quad (15)$$

$$J(x_c) + J^T(x_c) = W_c^T(x_c)W_c(x_c). \qquad x_c \in \mathbb{R}^{n_c} \quad (16)$$

Then the origin of the closed-loop system (8) – (11) is Lyapunov stable. If, in addition,

$$V_s'(x)f(x) < 0, \quad x \in \mathbb{R}^n,$$

$$\left[l_c(x_c) + \tfrac{1}{2}W_c(x_c)G^T(x)V_s'^T(x)\right]^T \times \left[l_c(x_c) + \tfrac{1}{2}W_c(x_c)G^T(x)V_s'^T(x)\right] > 0, \; x \in \mathbb{R}^n, \; x_c \in \mathbb{R}^{n_c},$$

then the origin of the closed-loop system is asymptotically stable.

3. Passive Framework for Linear Systems

In this section, we specialize Theorem 2.1 to linear systems. The following definitions will be needed. A square transfer function $G(s)$ is called ***positive real*** [15] if 1) all poles of $G(s)$ are in the closed left half plane and poles on the imaginary axis are semisimple, and 2) $G(s) + G^*(s)$ is nonnegative definite for $\mathrm{Re}[s] > 0$, where $(\;)^*$ denotes complex conjugate transpose. Furthermore, $G(s)$ is called ***strictly positive real*** [16] if 1) $G(s)$ is asymptotically stable, and 2) $G(\jmath\omega) + G^*(\jmath\omega)$ is positive definite for all real ω, and 3) if $\det[G(\infty) + G^*(\infty)] = 0$, then $\lim_{\omega\to\infty} \omega^2[G(\jmath\omega) + G^*(\jmath\omega)] > 0$ and $G(\infty) + G^*(\infty) \geq 0$, else $\lim_{\omega\to\infty}[G(\jmath\omega) + G^*(\jmath\omega)] > 0$.

Next we recall the positive-real lemma [15] which relates the positive realness of a transfer function to the KYP conditions, which are algebraic equations involving a minimal realization of the transfer function.

Lemma 3.1. A transfer function $G(s)$ with minimal realization (A, B, C, D) is positive real if and only if there exist matrices P, L, and W, with P positive definite, such that

$$A^TP + PA = -L^TL, \quad (17)$$

$$PB = C^T - L^TW, \quad (18)$$

$$D + D^T = W^TW. \quad (19)$$

It follows that if there exists $\varepsilon > 0$ such that

$$A^TP + PA = -L^TL - \varepsilon P, \quad (20)$$

and (18), (19) are satisfied, then $G(s)$ is strictly positive real.

A minimal realization of $G(s)$ that satisfies conditions (17) and (18) with $P = I$ is called a ***self-dual*** realization [2].

Given P satisfying (17) and (18), a self-dual realization of $G(s)$ can be obtained from the change of coordinates $z = P^{1/2}x$, where x is the internal state of the realization (A, B, C, D) of $G(s)$. Then the realization of $G(s)$ with the internal state z is a self-dual realization.

We now specialize Theorem 2.1 to the feedback interconnection of a strictly proper passive linear plant

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (21)$$

$$y(t) = Cx(t), \quad (22)$$

controlled by a proper passive linear compensator of the form

$$\dot{x}_c(t) = A_cx_c(t) + B_cy(t), \quad (23)$$

$$-u(t) = C_cx_c(t) + D_cy(t), \quad (24)$$

where necessarily $D_c + D_c^T \geq 0$. The following is a corollary of Theorem 2.1.

Corollary 3.1. Suppose there exist positive-definite matrices $P \in \mathbb{R}^{n\times n}$ and $P_c \in \mathbb{R}^{n_c\times n_c}$ and matrices L, L_c, and W_c, such that

$$A^TP + PA = -L^TL, \quad (25)$$

$$C = B^TP, \quad (26)$$

$$A_c^TP_c + P_cA_c = -L_c^TL_c, \quad (27)$$

$$C_c = B_c^TP_c + W_c^TL_c, \quad (28)$$

$$D_c + D_c^T = W_c^TW_c. \quad (29)$$

Then the closed-loop system (21) – (24) is Lyapunov stable. If in addition, $L^TL > 0$, and $[L_c + \frac{1}{2}W_cB_cP]^T[L_c + \frac{1}{2}W_cB_cP] > 0$

The following lemma will be used in the next section to construct a storage function for the inverse of a positive-real transfer function.

Lemma 3.2. Let $G(s)$ with minimal realization (A, B, C, D) be positive real, where $D + D^T > 0$, and let P, L, and W, with $P > 0$ satisfy (17) - (19). Then $G^{-1}(s)$ with realization $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = (A - BD^{-1}C, BD^{-1}, -D^{-1}C, D^{-1})$ is also positive real, and P satisfies

$$\tilde{A}^TP + P\tilde{A} = -\tilde{L}^T\tilde{L}, \quad (30)$$

$$\tilde{B}^TP = \tilde{C} - \tilde{W}^T\tilde{L}, \quad (31)$$

$$\tilde{D} + \tilde{D}^T = \tilde{W}^T\tilde{W}, \quad (32)$$

where $\tilde{L} = L - WD^{-1}C$ and $\tilde{W} = WD^{-1}$.

4. Passive Absorber Emulation: Linear Systems

In this section, we use Theorem 2.1 to design compensators that emulate passive linear absorbers. Related problems have been considered in [17, 18]. For illustrative purposes, we consider the system shown in Figure 1, which consists of a mass-spring-dashpot absorber subsystem, and a primary mass-spring subsystem that is disturbed by a force w. Our goal is to replace the absorber subsystem with a control force generated from a passive linear dynamic compensator. This compensator will then emulate the dynamic vibration absorber.

To emulate the absorber subsystem by means of a dynamic compensator, we remove the absorber subsystem from the primary subsystem and analyze the two subsystems separately. The primary subsystem is shown in Figure 2, where u represents the control force. The equations of motion for this subsystem are given by

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \quad (33)$$

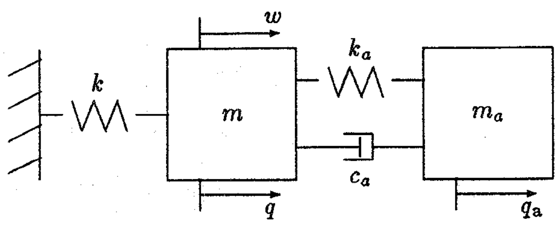

Figure 1: Oscillator with Dynamic Vibration Absorber

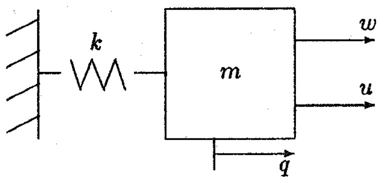

Figure 2: Controlled Oscillator

Choosing as a storage function $V_s(q,\dot{q}) = \frac{1}{2}(m\dot{q}^2 + kq^2)$, condition (12) is satisfied with $l(x) = 0$, and condition (13) is satisfied with the output given by $y = h(q,\dot{q}) = \dot{q}$. With this output, the primary subsystem is passive.

To develop a controller that emulates the effect of the absorber subsystem, we analyze the isolated absorber subsystem shown in Figure 3. Since the transfer function of

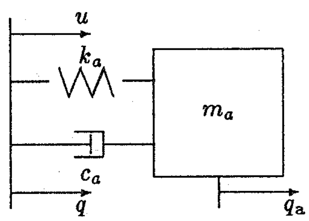

Figure 3: Dynamic Vibration Absorber Subsystem

the primary subsystem is given as an impedance, we write the transfer function of the emulated absorber as an admittance, as shown in Figure 4. The absorber admittance has a realization

$$\dot{x}_c = A_c x_c + B_c y, \tag{34}$$

$$-u = C_c x_c + D_c y, \tag{35}$$

where

$$A_c = \begin{bmatrix} 0 & 1 \\ k_c/m_c & -c_c/m_c \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_c = \begin{bmatrix} -c_c k_c/m_c & k_c - c_c^2/m_c \end{bmatrix}, \ D_c = c_c,$$

with corresponding matrices

$$P_c = \begin{bmatrix} k_c^2/m_c & c_c k_c/m_c \\ c_c k_c/m_c & k_c + c_c^2/m_c \end{bmatrix}, L_c^T = \frac{-\sqrt{2c_c}}{m_c}\begin{bmatrix} k_c \\ c_c \end{bmatrix},$$

$$W_c = \sqrt{2c},$$

to satisfy conditions (17) - (19). The procedure given in [19] was useful in determining the matrices P_c, L_c, and W_c.

Since conditions (12)–(15) of Theorem 2.1 are satisfied, Lyapunov stability is guaranteed. In fact, it follows from the invariant set theorem that for all positive values of m_a, c_a, and k_a the system is asymptotically stable.

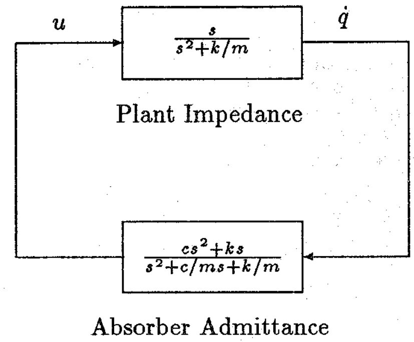

Figure 4: Block Diagram For The Absorber Emulation

5. Positive Real Compensator Synthesis

In this section we review the positive-real LQG synthesis procedure given in [1]. Consider (21), (22) with disturbance and measurement noise. In a self-dual basis we have

$$\dot{x}(t) = Ax(t) + Bu(t) + D_1 w_1(t), \tag{36}$$

$$y(t) = B^T x(t) + D_2 w_2(t), \tag{37}$$

where $A + A^T \leq 0$, w_1 and w_2 are uncorrelated, zero-mean Gaussian noise with normalized intensities, and $R_2 \triangleq D_2 D_2^T > 0$. The following result is given in [1].

Theorem 5.1. Let $R_1 \geq 0$ and assume

$$D_1 D_1^T = BR_2^{-1}B^T - A - A^T. \tag{38}$$

Furthermore, let A_c, B_c, C_c be given by

$$A_c = A - BR_2^{-1}B^T(I + P), \tag{39}$$

$$B_c = BR_2^{-1}, \tag{40}$$

$$C_c = R_2^{-1}B^T P, \tag{41}$$

where P is the positive-definite solution of the algebraic Riccati equation

$$A^T P + PA - PBR_2^{-1}B^T P + R_1 + BR_2^{-1}B^T = 0. \tag{42}$$

Then the compensator (23), (24) is positive real, and (A_c, B_c, C_c) minimizes the H_2 cost

$$J(A_c, B_c, C_c) = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T [x^T(t)(R_1 + BR_2^{-1}B^T)x(t) + u^T(t)R_2 u(t)]dt\right]. \tag{43}$$

If, in addition, $R_1 > 0$, the compensator is strictly positive real.

6. Nonlinear Modification of the Linear Design

In this section we modify the linear compensator (39)–(41) to obtain nonlinear dynamic compensators that conform to the framework of Theorem 2.1. As suggested in [3, 4], we are interested in nonlinearities that spread the spectrum of the compensator states in order to enhance energy flow between the plant and compensator and thus increase energy dissipation. One such modification involves squaring the input signal to the compensator, which doubles the frequencies of the harmonic signals. A second modification of the nonlinear compensator involves replacing the compensator dynamics matrix A_c with $A_c + S$, where $S \in \mathbb{R}^{n_c \times n_c}$ is a skew-symmetric matrix. The purpose of this modification is to further spread the spectrum of the compensator states by coupling the compensator modes. For convenience, we assume that the positive-real linear compensator (23), (24) has been transformed into a self-dual realization so that

$$A_c + A_c^T \leq 0, \; C_c = B_c^T, \tag{44}$$

and thus

$$V_{sc}(x_c) = x_c^T x_c, \tag{45}$$

is a storage function for the compensator. Since A_c satisfies (44) and S is skew symmetric, it follows that $(A_c + S) + (A_c + S)^T \leq 0$, so that the modified compensator is also passive.

Next we replace the input y to the compensator by a vector of squared inputs given by

$$\dot{x}_c(t) = (A_c + S)x_c + B_c \mathrm{diag}(y) y, \tag{46}$$

where diag(y) is a diagonal matrix whose entries on the diagonal are the elements of y. Now, to satisfy condition (15) of Theorem 2.1, we replace (24) by

$$u(t) = -\mathrm{diag}(y) B_c^T x_c, \tag{47}$$

so that the resulting compensator (46), (47) with storage function (45) is passive.

Compensators of the form (46), (47) appear in [4] where the skew-symmetric matrix is multiplied by a quadratic term. In [4] the linear dynamics of these compensators are chosen to have modal frequencies near those of the plant, so that energy can be efficiently transferred from the plant to the compensator. The skew-symmetric term couples the modes of the compensator, so that energy in one mode excites all of the other modes. Consequently, the spectral content of the compensator is broadened. By spreading the spectrum of the disturbance, energy dissipation is enhanced through improved energy flow between the mismatched plant and compensator modes, and by faster energy dissipation in the higher frequency modes.

7. Baseline Positive-Real LQG Design for an Illustrative Example

Consider the two-mode, mass-spring system of Figure 7 with disturbance w and control input u. The velocity of the second mass serves as the output signal y which is colocated with the control input. Choosing $m_1 = 2$,

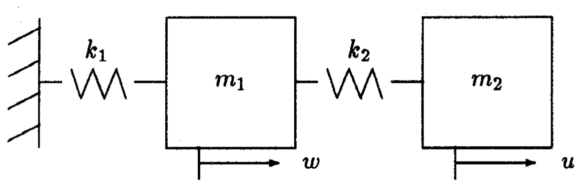

Figure 5: Two-Mode Plant

$m_2 = 1$, $k_1 = \frac{1}{2}$, and $k_2 = 1$, the dynamic equations in a modal basis are given by

$$\dot{x} = Ax + Bu + D_1 w, \tag{48}$$

$$y = B^T x, \tag{49}$$

$$z = E_1 x, \tag{50}$$

where

$$A = \begin{bmatrix} 0 & 0.40 & 0 & 0 \\ -0.40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.26 \\ 0 & 0 & -1.26 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0.64 \\ 0 \\ 0.77 \end{bmatrix}, \; D_1 = \begin{bmatrix} 0 \\ 0.54 \\ 0 \\ -0.45 \end{bmatrix},$$

and $E_1 = D_1^T$, so that the performance variable z represents the velocity of the first mass. Note that the transfer function $G_{yu}(s) = B^T (sI - A)^{-1} B$ is positive real.

The positive-real compensator was synthesized using Theorem 5.1 with $R_1 = \frac{1}{2}I$ and $R_2 = \frac{1}{2}$. The resulting positive-real compensator in its self-dual basis is given by

$$\dot{x}_c = A_c x_c + B_c y, \tag{51}$$

$$u = -B_c^T x_c, \tag{52}$$

where

$$A_c = \begin{bmatrix} -0.1366 & 0.3461 & -0.0395 & -0.1223 \\ -0.3917 & -2.3322 & -0.0497 & -2.4111 \\ -0.0440 & -0.2865 & -0.2592 & 0.8861 \\ -0.0693 & -2.4555 & -1.4074 & -2.8731 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.1691 \\ 1.7837 \\ 0.6357 \\ 1.8941 \end{bmatrix}.$$

Notice that while the compensator is realized in a self-dual basis, it is not in a modal basis. The basis for the realization of the plant, however, is both self dual and modal.

8. Nonlinear Modification of the Positive-Real LQG Compensator

In this section we consider nonlinear modifications to the linear compensator given in Section 7. We then compare the disturbance rejection capability of the resulting nonlinear compensators with that of the baseline linear design.

8.1. Squaring the Input Signal

Consider the SISO dynamic compensator

$$\dot{x}_c = A_c x_c + B_c y^2, \tag{53}$$

$$u = -B_c^T y x_c, \tag{54}$$

where A_c and B_c are given in the previous section. Note that this compensator is passive, with the storage function $V_{sc} = x_c^T x_c$ as before.

To examine the disturbance rejection properties of the closed-loop system, we inject sinusoidal disturbance signals at fixed amplitude. For each disturbance, we analyze the spectrum of the performance output, and determine the steady-state amplitude of the portion of the signal that has the same frequency as the disturbance. This amplitude is compared to the frequency response of the closed-loop system involving the linear positive-real LQG design of Section 7.

As can be seen in Figure 6, the nonlinear compensator outperforms the positive-real design at some frequencies, and reduces the peak amplification of the linear design considerably. Spectral analysis of the performance output indicates that energy is found at multiple frequencies, most often at the disturbance frequency, at three times the disturbance frequency, and for higher frequency disturbances, at the open-loop modal frequencies of the plant.

8.2. Skew Symmetric Mixing

Since the positive-real LQG design is in a self-dual realization, we can add a skew-symmetric matrix to the dynamics matrix A_c, without losing the positive-real property, as discussed in the previous section. We consider this skew-symmetric augmentation of the dynamics matrix while also squaring the input signal. In particular, we augment the dynamics matrix A_c (53) with the matrix

$$S = \begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{bmatrix} = -S^T. \tag{55}$$

As in the previous subsection, we probe the nonlinear control system with sinusoidal disturbance signals of fixed amplitude and analyze the spectrum of the performance output. The results, which are plotted in Figure 6, show

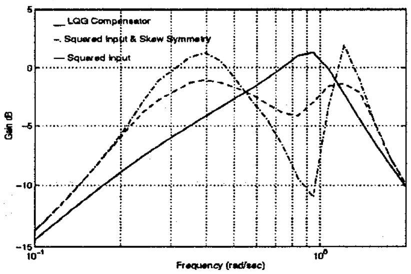

Figure 6: Fixed Amplitude Gain Plot for Positive Real LQG with Squared Input and Skew-Symmetric Mixing

that while this compensator does not improve the disturbance rejection at every frequency, it does much better than the linear design over a significant range of frequencies. In particular it has roughly 14 dB better suppression of the disturbance frequency content at 0.9 rad/sec. Similar frequency spreading is observed for these cases as was observed for the squared input case above. A typical plot of the frequency spreading effect is given in Figure 7. This figure shows the spectral content of the output when the closed-loop system is forced at $w = \sin 0.3t$.

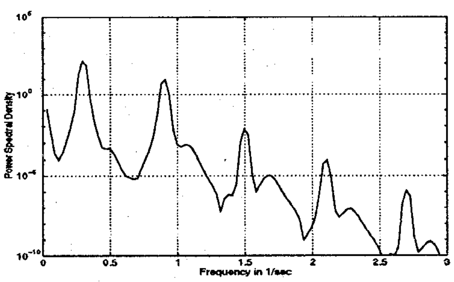

Figure 7: FFT of Performance Output Signal

9. Conclusions

We have investigated the ability of certain nonlinear modifications to enhance disturbance rejection and energy dissipation compared with linear positive-real compensator designs. In particular, we considered squaring the input to the compensator, and mixing the modal states with a skew-symmetric addition to the compensator dynamics matrix. These passive nonlinear designs enhanced the disturbance rejection through a range of frequencies by distributing the energy among multiple frequencies. The extent to which nonlinear modifications improve the performance robustness in the presence of plant uncertainties will be the subject of later work.

References

[1] R. Lozano-Leal and S. M. Joshi. On the design of dissipative LQG-type controllers. In *Proc. IEEE Conf. Dec. Contr.*, pp. 1645–1646, Austin, TX, 1988.

[2] W. M. Haddad, D. S. Bernstein, and Y. W. Wang. Dissipative H_2/H_∞ controller synthesis. *IEEE Trans. Autom. Contr.*, Vol. 39, pp. 827–831, 1994.

[3] D. C. Hyland. A nonlinear vibration control design with a neural network realization. In *Proc. IEEE Conf. Dec. Contr.*, pp. 2569–2574, 1990.

[4] D. C. Hyland. Chaotic controllers. unpublished.

[5] R. Lozano, B. Brogliato, and I. D. Landau. Passivity and global stabilization of cascaded nonlinear systems. *IEEE Trans. Autom. Contr.*, Vol. 37, No. 9, pp. 1386–1388, 1992.

[6] J. C. Willems. Dissipative dynamical systems Part I: General theory. *Arch. Rational Mech. Anal.*, Vol. 45, pp. 321 – 351, 1972.

[7] J. C. Willems. Dissipative dynamical systems Part II: Linear systems with quadratic supply rates. *Arch. Rational Mech. Anal.*, Vol. 45, pp. 352 – 393, 1972.

[8] P. J. Moylan. Implications of passivity in a class of nonlinear systems. *IEEE Trans. Autom. Contr.*, Vol. 19, No. 4, pp. 373–381, 1974.

[9] D. Hill and P. Moylan. The stability of nonlinear dissipative systems. *IEEE Trans. Autom. Contr.*, Vol. 21, No. 5, pp. 708–711, 1976.

[10] D. J. Hill and P. J. Moylan. Stability results for nonlinear feedback systems. *Automatica*, Vol. 13, pp. 377–382, 1977.

[11] D. J. Hill and P. J. Moylan. Dissipative dynamical systems: Basic input-output and state properties. *J. Franklin Inst.*, Vol. 309, No. 5, pp. 327–357, 1980.

[12] R. Ortega. Passivity properties for stabilization of cascaded nonlinear systems. *Automatica*, Vol. 27, No. 2, pp. 423–424, 1991.

[13] C. I. Byrnes, A. Isidori, and J. C. Willems. Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Trans. Autom. Contr.*, Vol. 36, No. 11, pp. 1228–1240, 1991.

[14] D. S. Bernstein and W. M. Haddad. Nonlinear controllers for positive real systems with arbitrary input nonlinearities. *IEEE Trans. Autom. Contr.*, Vol. 39, pp. 1513 – 1517, 1994.

[15] B. D. O. Anderson and S. Vongpanitlerd. *Network Analysis and Synthesis: A Modern Systems Theory Approach.* Prentice Hall, 1973.

[16] G. Tao and P. A. Ioannou. Strictly positive real matrices and the Lefschetz-Kalman-Yakubovich lemma. *IEEE Trans. Autom. Contr.*, Vol. 33, No. 12, pp. 1183 – 1185, 1988.

[17] J.-N. Juang and M. Phan. Robust controller designs for second-order dynamic systems: A virtual passive approach. *Journal of Guidance, Control and Dynamics*, Vol. 15, No. 5, pp. 1192–1198, 1992.

[18] J.-N. Juang, S.-C. Wu, M. Phan, and R. W. Longman. Passive dynamic controllers for nonlinear mechanical systems. *Journal of Guidance, Control and Dynamics*, Vol. 16, No. 5, pp. 845–851, 1993.

[19] N. Sadegh, J. D. Finney, and B. Heck. An explicit method for computing the positive real lemma matrices. In *Proc. IEEE Conf. Dec. Contr.*, pp. 1464 – 1469, Orlando, FL, 1994.