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Lyapunov-Based Output-Feedback Adaptive Stabilization of Minimum Phase Second-Order Systems [1]

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Abstract

We consider output feedback adaptive stabilization for second-order systems in the presence of bounded exogenous disturbances. This case is of particular interest as it has been shown that, in the presence of exogenous disturbances, direct adaptive control schemes for minimum phase plants with relative degree 1 exhibit parameter divergence eventually leading to instability. We present controllers that guarantee convergence of the measured output and boundedness of all controller parameters and signals. The controller has the form of a 7th-order dynamic compensator for the relative-degree 1 case. The proof of convergence is based on a variant of Lyapunov's method in which the Lyapunov derivative is shown to be asymptotically nonpositive.

1 Robust Adaptive Control

It has been shown in [9] that several direct adaptive control schemes for minimum phase plants with relative degree 1 (such as those in [1, 2]), exhibit parameter divergence eventually leading to instability. In general, non-identifier based adaptive controllers based on high gain feedback tend to suffer from lack of robustness with respect to bounded disturbances.

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A minimum-phase system with relative degree 1 can be stabilized by output feedback $u = -ky$, provided that $k > k_{\rm s}$, with $k_{\rm s}$ sufficiently large. To ensure that k grows beyond $k_{\rm s}$ the controller takes the form [1, 2, 6]

$$u = -ky, \tag{1}$$
$$\dot{k} = \gamma y^2. \tag{2}$$

With $y \to 0$ as $t \to \infty$ it can be proved [1, 6] that k converges. However, there is a practical limitation in implementing this controller. In the presence of sustained plant disturbance or measurement noise, k diverges eventually leading to instability. There have been several fixes to amend this problem in the literature and some of them are summarized below.

To avoid divergence of the gain k, (2) is replaced by [3]

$$\dot{k} = -\sigma k + \gamma y^2, \tag{3}$$

where $\sigma > 0$ is a small damping term. This modification yields bounded k for bounded y. However, (3) leads to an undesirable response [5] for unstable systems. To see this, suppose $|k|$ grows so that it is large enough to stabilize the unstable plant. As $y \to 0$, the term $-\sigma k$ dominates y^2 leading to a decrease in k which destabilizes the system. This leads to bursting of the plant output. This behavior occurs regardless of the presence of disturbances.

To avoid bursting, [7] proposes the gain update law

$$\dot{k} = -\beta|y|k + \gamma y^2, \tag{4}$$

where σ in (3) is replaced by $\beta|y|$, the rationale being that such a term tends to zero with the measured output. However,(4) introduces new equilibria (p. 312 in [6]) which are functions of the tuning parameter β. The analysis of the resultant adaptive system is considerably difficult.

Another proposed modification to (2) is the *dead-zone modification* [6, 8, 11]. This scheme stops the adaptation process for $|y| \leq \gamma\alpha$, where $\gamma > 0$ and α is a known bound on the disturbance or measurement noise. This modification assures boundedness of all signals, but the disturbance bound α is required to be known. Furthermore, since the modification contains a dead-zone, this method cannot assure asymptotic convergence of the plant output even when the disturbance is not present.

In this paper, we provide an adaptive controller that guarantees convergence of the measured output in the presence of measurement noise as well as plant disturbances. In Theorem 2.1 we provide a controller for minimum-phase second-order systems, which guarantees convergence of the measured plant output and boundedness of all parameters for unknown bounded measurement noise $d_{\rm m}$ and plant disturbance $d_{\rm p}$. As in [6, 10] we assume the plant is minimum phase and that the order of the system, relative degree, and sign of the high frequency gain are known. For this case, the certainty equivalence controller is comprised of three elements, namely, a stable filter, a parameter estimator, and a disturbance rejection controller. Our proof of convergence is Lyapunov based. In particular, we develop a variant of Lyapunov's method in which the Lyapunov derivative is asymptotically nonpositive.

2 Adaptive Disturbance Rejection

Consider the minimum-phase second-order system

$$\ddot{q} + a_1\dot{q} + a_2 q = \frac{1}{m}\dot{u} - \frac{z}{m}u + d_{\rm p}. \tag{5}$$

We assume that only q is available for feedback via the measurement

$$y = q + d_{\rm m}, \tag{6}$$

and we assume that

(A1) a_1 and a_2 are constant but otherwise unknown;

(A2) m is nonzero, constant and $\mu \stackrel{\Delta}{=} \operatorname{sign}(m)$ is known but otherwise unknown;

(A3) $z < 0$ and constant but otherwise unknown;

(A4) The plant disturbance $d_{\rm p}$ is bounded but otherwise unknown;

(A5) The measurement noise $d_{\rm m}$ is differentiable and bounded with bounded derivative but otherwise unknown.

The disturbance rejection problem constitutes finding a control input u such that $y(t) \to 0$ as $t \to \infty$.

Theorem 2.1 Let λ, r_1, r_2, f_1, f_2 and k be positive constants, and let $p_1 = r_1/(2f_2)$ and $p_2 = (r_1 + f_2 r_2)/(2f_1 f_2)$. Consider the 7th-order dynamic compensator

$$\begin{aligned}
\dot{q}_{\rm f} &= -\lambda q_{\rm f} + y, & (7)\\
\dot{u}_{\rm f} &= -\hat{z}u_{\rm f} + \hat{m}v, & (8)\\
\dot{\hat{a}}_1 &= -\tilde{x}_{\rm f}\dot{q}_{\rm f}, & (9)\\
\dot{\hat{a}}_2 &= -\tilde{x}_{\rm f}q_{\rm f}, & (10)\\
\dot{\hat{m}} &= -\mu\tilde{x}_{\rm f}v, & (11)\\
\dot{\hat{z}} &= \mu\tilde{x}_{\rm f}u_{\rm f}\hat{z}^{3/2},\ \hat{z}(0) > 0, & (12)\\
\dot{\hat{\alpha}} &= -k\hat{\alpha}^{3/2}|\tilde{x}_{\rm f}|, & (13)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{x}_{\rm f} &\stackrel{\Delta}{=} p_1 q_{\rm f} + p_2\dot{q}_{\rm f}, & (14)\\
v &\stackrel{\Delta}{=} (\hat{a}_1 - f_1)\dot{q}_{\rm f} + (\hat{a}_2 - f_2)q_{\rm f} - \hat{\alpha}\operatorname{sign}(\tilde{x}_{\rm f}) & (15)
\end{aligned}$$

Let the control input u be given by

$$u = (\lambda - \hat{z})u_{\rm f} + \hat{m}v. \tag{16}$$

Then y, $q_{\rm f}$, and $\dot{q}_{\rm f} \to 0$ as $t \to \infty$. Furthermore, $\hat{a}_1$, $\hat{a}_2$, $\hat{m}$, $\hat{z}$ and $\hat{\alpha}$ are bounded, $\inf_{t\geq 0}\hat{z}(t) > 0$, and $\inf_{t\geq 0}\hat{\alpha}(t) > 0$.

Proof: Define the noise filters with filter states $d_{\rm fm}$ and $d_{\rm fp}$ satisfying

$$\begin{aligned}
\dot{d}_{\rm fm} &= -\lambda d_{\rm f1} + d_{\rm m}, & (17)\\
\dot{d}_{\rm fp} &= -\lambda d_{\rm f2} + d_{\rm p}, & (18)
\end{aligned}$$

with $d_{\rm fm}(0) = d_{\rm fp}(0) = 0$. It follows from (8) and (16) that $u_{\rm f}$ satisfies

$$\dot{u}_{\rm f} = -\lambda u_{\rm f} + u. \tag{19}$$

Let s be the Laplace variable and let $q_{\rm f}(s)$, $d_{\rm fm}(s)$, $d_{\rm fp}(s)$ and $u_{\rm f}(s)$ denote the Laplace transforms of $q_{\rm f}(t)$, $d_{\rm fm}(t)$, $d_{\rm fp}(t)$ and $u_{\rm f}(t)$, respectively. Next, define

$$\xi(s) \triangleq (s^2+a_1 s+a_2)(q_{\rm f}(s)-d_{\rm fm}(s)) - \left(\frac{1}{m}s-\frac{z}{m}\right)u_{\rm f}(s)-d_{\rm fp}(s). \quad (20)$$

Using (5), (7), (17), (18) and (19), it follows that $\xi(s)$ satisfies $(s+\lambda)\xi(s)=0$. Hence $\xi(t)=\xi(0)e^{-\lambda t}$. Rewrite (20) as

$$(s^2+a_1 s+a_2)q_{\rm f}(s) = (s^2+a_1 s+a_2)d_{\rm fm}(s) + \left(\frac{1}{m}s-\frac{z}{m}\right)u_{\rm f}(s)+d_{\rm fp}(s)+\xi(s). (21)$$

Next, define $\psi(s) \triangleq (s^2+a_1 s+a_2)d_{\rm fm}(s)+d_{\rm fp}(s)$. Since $\lambda>0$, $\frac{(a_1-\lambda)s+a_2}{s+\lambda}$ and $\frac{1}{s+\lambda}$ are stable transfer functions, which yield bounded outputs for bounded inputs.

Using (17), (18) and noting that $d_{\rm m}(t)$, $d_{\rm p}(t)$ and $\dot d_{\rm m}(t)$ are bounded, it follows that there exists $\alpha>0$ such that

$$|\psi(t)| < |\dot d_{\rm m}(t)| + \left|\mathcal{L}^{-1}\left(\frac{(a_1-\lambda)s+a_2}{s+\lambda}\mathcal{L}(d_{\rm m}(t))\right)\right| + |d_{\rm p}(t)| < \alpha,\ t\geq 0. \quad (22)$$

Now equations (8), (15), and (21) yield

$$\ddot q_{\rm f}+f_1\dot q_{\rm f}+f_2 q_{\rm f} = \tilde a_1\dot q_{\rm f}+\tilde a_2 q_{\rm f}+\frac{\tilde m}{m}v-\frac{z+\hat z}{m}u_{\rm f} + (\psi-\hat\alpha\,{\rm sign}(\tilde x_{\rm f}))+\xi, \quad (23)$$

where $\tilde a_1 \triangleq \hat a_1-a_1$, $\tilde a_2 \triangleq \hat a_2-a_2$, and $\tilde m \triangleq \hat m-m$.

Next, let $\tilde x \triangleq [q_{\rm f},\dot q_{\rm f},\tilde a_1,\tilde a_2,\tilde m,\hat z,\hat\alpha]^{\rm T}$. Note that since a_1, a_2 and m are constant, $\dot{\tilde a}_1=\dot{\hat a}_1$, $\dot{\tilde a}_2=\dot{\hat a}_2$, and $\dot{\tilde m}=\dot{\hat m}$. Therefore the equations defining the state $\tilde x$ are given by (23) and (9)-(13) in the order of the components of $\tilde x$.

Now, define the positive-definite function $V:\mathbb{R}^5\times(0,\infty)^2\to\mathbb{R}$ by

$$V(\tilde x) \triangleq \frac{1}{2}x_{\rm f}^{\rm T}Px_{\rm f}+\frac{1}{2}\tilde a_1^2+\frac{1}{2}\tilde a_2^2+\frac{1}{2|m|}\tilde m^2 + \frac{2}{|m|}\left(\sqrt{\hat z}-\frac{z}{\sqrt{\hat z}}\right)+\frac{2}{k}\left(\sqrt{\hat\alpha}+\frac{\alpha}{\sqrt{\hat\alpha}}\right) - \frac{4\sqrt{-z}}{|m|}-\frac{4\sqrt{\alpha}}{k}. \quad (24)$$

Here $x_{\rm f} \triangleq [q_{\rm f}\quad \dot q_{\rm f}]^{\rm T}$ and matrix P is the solution of the Lyapunov equation $F^{\rm T}P+PF=-R$, where $F \triangleq \begin{bmatrix}0 & 1\\ -f_2 & -f_1\end{bmatrix}$ and $R=\begin{bmatrix}r_1 & 0\\ 0 & r_2\end{bmatrix}$. Then $P=\begin{bmatrix}p_0 & p_1\\ p_1 & p_2\end{bmatrix}$, where $p_0=(f_1 r_1+f_1 f_2 r_2+f_1^2 r_2)/(2f_1 f_2)$. Note that V is unbounded as $\hat z,\hat\alpha\to 0$ and radially unbounded as $x_{\rm f},\tilde a_1,\tilde a_2,m,\hat z,\hat\alpha\to\infty$. Furthermore, V has its minimum at $\tilde x_0=[0,0,0,0,0,-z,\alpha]^{\rm T}$ at which $V(\tilde x_0)=0$.

Suppose that either $\hat z$ or $\hat\alpha$ vanishes on $(0,\infty)$ and let $t_1=\min\{t>0:\hat z(t)=1 \text{ or } \hat\alpha(t)=1\}$. Using (9)-(12), (22) and (23), we obtain for $t\in[0,t_1)$

$$\dot V(\tilde x(t)) \leq -\frac{1}{4}r_1 q_{\rm f}^2-\frac{1}{4}r_2\dot q_{\rm f}^2+\left(\frac{p_1^2}{r_1}+\frac{p_2^2}{r_2}\right)\xi^2 \leq \left(\frac{p_1^2}{r_1}+\frac{p_2^2}{r_2}\right)\xi^2(0)e^{-2\lambda t}. \quad (25)$$

Let $V_0(t) \triangleq V(\tilde x(t))+\frac{\beta}{2\lambda}e^{-2\lambda t}$, where $\beta=(\frac{p_1^2}{r_1}+\frac{p_2^2}{r_2})\xi^2(0)$. Then $\dot V_0(t)=\dot V(\tilde x(t))-\beta e^{-2\lambda t}\leq \beta e^{-2\lambda t}-\beta e^{-2\lambda t}=0$. Therefore, V_0 is non-increasing along the closed-loop state trajectories. Hence, $V(\tilde x(t))=V_0(t)-\frac{\beta}{2\lambda}e^{-2\lambda t}$ is bounded for $t\in[0,t_1)$, which implies that $z/\sqrt{\hat z(t)}$ and $\alpha/\sqrt{\hat\alpha(t)}$ remain bounded for all $t\in[0,t_1)$. Hence, there exist $\delta_1,\delta_2>0$ such that $\hat z(t)>\delta_1$ and $\hat\alpha(t)>\delta_2$ for all $t\in[0,t_1)$, respectively. Since $\hat z(t)$ and $\hat\alpha(t)$ are continuous, this contradicts $\hat z(t_1)=0$ and $\hat\alpha(t_1)=0$, respectively. Therefore the inequality (25) is valid for all $t\in[0,\infty)$ and $V(\tilde x(t))$ is bounded for all $t\in[0,\infty)$, which implies $\inf_{t\geq 0}\hat z(t)>0$ and $\inf_{t\geq 0}\hat\alpha(t)>0$. Since $V_0(t)>0$ and is non-increasing for all $t\geq 0$, $\lim_{t\to\infty}V_0(t)$ exists. Noting that $\lim_{t\to\infty}\frac{\beta}{2\lambda}e^{-2\lambda t}=0$, it follows that $V(\tilde x(t))$ has a limit as $t\to\infty$. Hence all components of $\tilde x(t)$ are bounded for $t\in[0,\infty)$.

Next, note that

$$\int_0^t \frac{1}{4}r_1 q_{\rm f}^2 d\tau+\int_0^t \frac{1}{4}r_2\dot q_{\rm f}^2 d\tau \leq V(\tilde x(0))-V(\tilde x(t)) + \alpha\int_0^t \xi^2(\tau)d\tau \quad (26)$$

along the trajectories of the closed-loop system. Since the right-hand-side of (26) is bounded as $t\to\infty$, it follows that $q_{\rm f}\in\mathcal{L}_2$ and $\dot q_{\rm f}\in\mathcal{L}_2$. Since $\dot q_{\rm f}$ is bounded it follows from Barbalat's lemma [12] that $q_{\rm f}\to 0$ as $t\to\infty$.

Next, using (8) it follows that

$$u_{\rm f}(t) = \Phi(\hat{z},t)u_{\rm f}(0) + \int_0^t \Phi(\hat{z},t-\tau)\hat{m}(\tau)v(\tau)d\tau. \quad (27)$$

where $\Phi(\hat{z},t) = \exp\left(-\int_0^t \hat{z}(\sigma)d\sigma\right)$. Let $\delta > 0$ be such that $\hat{z}(t) \geq \delta$ for all $t \geq 0$. Then it can be shown that

$$|u_{\rm f}(t)| \quad \leq \quad |u_{\rm f}(0)| + \frac{1}{\delta}\sup_{0\leq\tau\leq t}|\hat{m}(\tau)v(\tau)|.$$

Since $\hat{m}v$ is bounded it follows that $u_{\rm f}$ is bounded. Hence, it follows that the right-hand-side of (23) is bounded and therefore $\ddot{q}_{\rm f}$ is bounded. Therefore Barbalat's lemma [12] implies that $\dot{q}_{\rm f} \to 0$ as $t \to \infty$. Therefore, (7) implies that $y \to 0$ as $t \to \infty$. ■

Remark 2.1 The variable $\hat{\alpha}$ given by (13) attempts to estimate the overall disturbance bound characterized by α in (22). The variable $\hat{\alpha}$ in conjunction with the switching term in (15) achieves disturbance rejection. However, if $d_{\rm m} = d_{\rm p} = 0$, then it suffices to omit (13) and set $\hat{\alpha} = 0$ in (15). The resulting simplified controller can be shown to guarantee q, $\dot{q}$, $q_{\rm f}$, and $\dot{q}_{\rm f} \to 0$ as $t \to \infty$. Therefore the controller presented in Theorem 2.1 achieves two purposes simultaneously, namely, adaptive stabilization and adaptive disturbance rejection.

Remark 2.2 Since q is available for feedback and $q_{\rm f}$ is obtained from the filter (7), $\dot{q}_{\rm f}$ can be computed using (7). Therefore, the parameter update laws (9)-(12) and equation (15) are implementable. Finally, the controller implementation (7)-(12) requires only q for feedback and does not require knowledge of a_1, a_2, z or $|m|$. However, as required by (A2), m must be nonzero and μ must be known.

Remark 2.3 Consider the case $d_{\rm m} = d_{\rm p} = 0$ so that we omit (13) and set $\hat{\alpha} = 0$ in (15). Then (21) simplifies to

$$\ddot{q}_{\rm f} + a_1\dot{q}_{\rm f} + a_2 q_{\rm f} = \frac{1}{m}\dot{u}_{\rm f} - \frac{z}{m}u_{\rm f} + \xi, \quad (28)$$

where $\xi = \xi(0)e^{-\lambda t}$. The controller (7)-(16) can be viewed as the combination of three elements. First, a stable filter represented by (7) generates the filter state $q_{\rm f}$ which mimics the state q as indicated by (28). Next, equations (9)-(12) constitute the parameter update laws. Finally, the controller can be regarded as a certainty-equivalence controller which attempts to place the closed-loop poles at the eigenvalues of the Hurwitz matrix $F = \begin{bmatrix} 0 & 1 \\ -f_2 & -f_1 \end{bmatrix}$. To see this, assume that the parameters $\hat{a}_1$, $\hat{a}_2$, $\hat{m}$, and $\hat{z}$ converge to the parameters a_1, a_2, m and $-z$, respectively. In this case, the controller simplifies to the linear time invariant form

$$u(s) = -G_{\rm c}(s)q(s), \quad (29)$$

where $G_{\rm c}(s) = m\frac{(f_1-a_1)s+(f_2-a_2)}{s-z}$. Defining the plant transfer function $G(s) = \frac{s-z}{m(s^2+a_1 s+a_2)}$ we obtain the stable sensitivity function

$$\frac{1}{1+G(s)G_{\rm c}(s)} = \frac{s^2+a_1 s+a_2}{s^2+f_1 s+f_2},$$

whose poles are given by the eigenvalues of F.

3 Numerical Examples

Consider the minimum-phase unstable second-order system

$$\ddot{x} \;=\; 4\dot{x} - 10.5x + 4.5\dot{u} + 13.5u + d_{\rm p}, \quad (30)$$

$$y \;=\; x + d_{\rm m}. \quad (31)$$

To apply Theorem 2.1, let $\lambda = 10$, $f_1 = 11$ and $f_2 = 36$. In the following simulations we compare Theorem 2.1 to the controllers presented in Section 1. For convenience we refer to the controller in (1), (2) by 'controller1', the controller in (1), (3) by 'controller2', and the controller in (1), (4) by 'controller3', respectively.

3.1 No exogenous disturbances

Let $d_{\rm p} = d_{\rm m} = 0$. The closed-loop responses, shown in Figures 1, 2, indicate that Theorem 2.1, 'controller1', and 'controller3' successfully stabilize the system in (30), (31). Although 'controller2' guarantees boundedness, it does not yield convergence of the output even in the absence of disturbances. It is further observed (not shown) that the gain k corresponding to 'controller1', 'controller2' and 'controller3' remains bounded. The parameter estimates $\hat{a}_1$, $\hat{a}_2$, $\hat{m}$ and $\hat{z}$ (see Figure 2) converge to constant values that are not the true values of a_1, a_2, m or z. This is consistent with Theorem 2.1.

3.2 Measurement Noise

Next we investigate the performance of the controllers in the presence of measurement noise. Specifically, we consider dual-tone measurement noise given by $d_{\rm m}(t) = 0.1\sin(t) + 0.2\sin(3.33t)$ and $d_{\rm p} = 0$. The closed-loop responses, shown in Figures 3-4, indicate that the algorithm in Theorem 2.1 successfully stabilizes the system (30), (31). Figure 4 indicates that although the output remains bounded for 'controller1', 'controller2', and 'controller3', the gain k for 'controller1' diverges in the presence of measurement noise. The gains for 'controller2' and 'controller3' remain bounded as predicted. The time histories of $\hat{a}_1$, $\hat{a}_2$, $\hat{m}$ or $\hat{z}$ are similar to the ones shown in Figure 2.

3.3 Plant Disturbance

Next we investigate the performance of the controllers in the presence of plant disturbance. Specifically, we consider a single-tone plant disturbance given by $d_{\rm p}(t) = 10\sin(3t)$ and $d_{\rm m} = 0$. The closed-loop responses, shown in Figure 5 and Figure 6, indicate that the algorithm in Theorem 2.1 successfully stabilizes the system (30), (31) and achieves complete regulation. In contrast, 'controller1', 'controller2', and 'controller3' yield only boundedness. Figure 5 and Figure 6 indicate that the gain k for 'controller1' diverges without bound in the presence of measurement noise and the gain k for 'controller2' and 'controller3' remain bounded as predicted.

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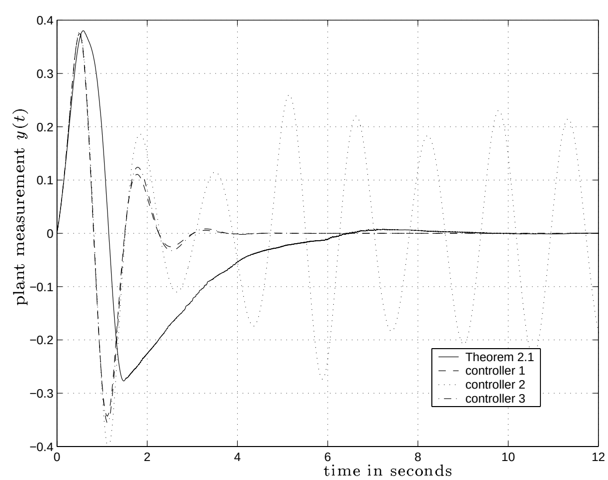

Figure 1: Adaptive stabilization small of the unstable second-order minimum-phase plant (30), (31) with no exogenous disturbances. Comparison of performance between controller presented in Theorem 2.1 depicted by 'Theorem 2.1', controller (1), (2) depicted by 'controller1', controller (1), (3) depicted by 'controller2' and controller (1), (4) depicted by 'controller3'.

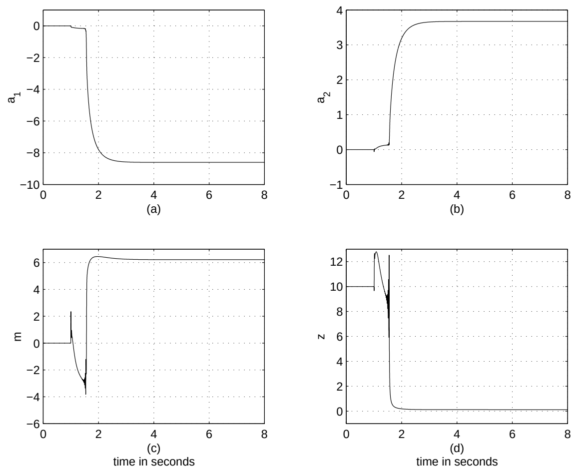

Figure 2: Parameter estimates (a) $\hat{a}_1$, (b) $\hat{a}_2$, (c) $\hat{m}$ and (d) $\hat{z}$ for Theorem 2.1 controller corresponding to Figure 1.

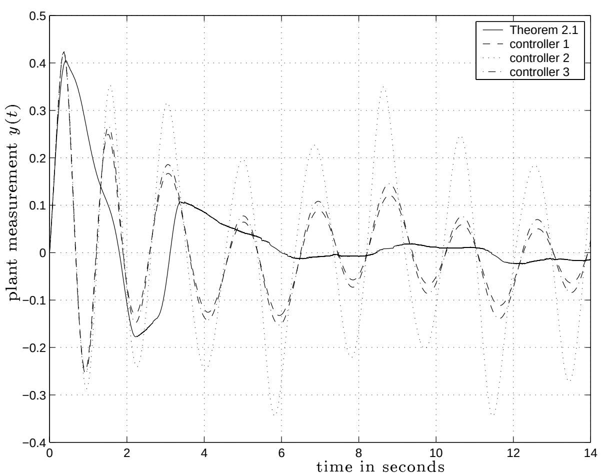

Figure 3: Adaptive stabilization of the unstable second-order minimum-phase plant (30), (31) with dual-tone measurement noise $d_{\mathrm{m}}(t) = 0.1\sin(t) + 0.2\sin(3.33t)$.

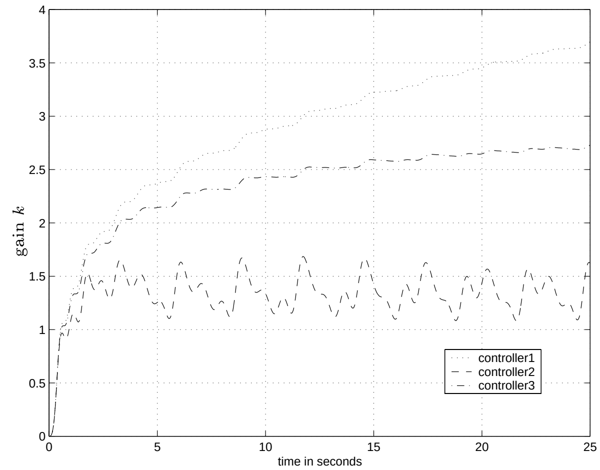

Figure 4: Time-history of gain k for various schemes corresponding to results in Figure 3.

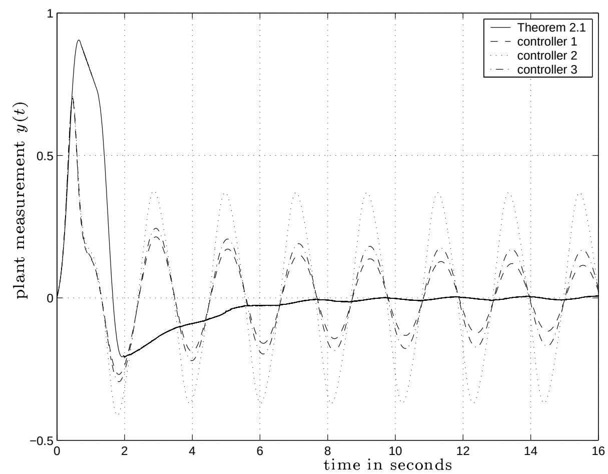

Figure 5: Adaptive stabilization of the unstable second-order minimum-phase plant (30), (31) with plant disturbance $d_{\mathrm{p}}(t) = 10\sin(3t)$.

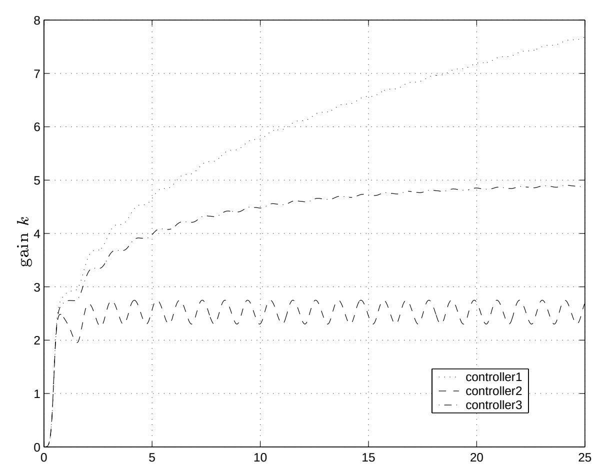

Figure 6: Time-history of gain k for various schemes corresponding to results in Figure 5.