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Adaptive State Estimation with Subspace-Constrained State Correction

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***Abstract*—In many applications of state estimation, it is efficient to confine the output-error injection to a prescribed subspace of the state space. This paper considers this problem by applying the unscented Kalman filter and retrospective cost state estimator (RCSE) to linear and nonlinear systems with subspace-constrained state correction. As an application of these techniques, parameter estimation is considered for linear and nonlinear systems with unknown parameters, where the output-error injection is confined to the subspace corresponding to the states representing the unknown parameters.**

I. Introduction

The classical Kalman filter and its variants, such as the extended Kalman filter [1], unscented Kalman filter (UKF) [2], [3], and ensemble Kalman filter (EnKF) [4], construct state estimates by injecting the output error into the system dynamics. To do this, the extended Kalman filter requires the Jacobian of the dynamics along the estimated trajectory, whereas UKF and EnKF require the propagation of an ensemble of estimation models at each time step.

In many applications, however, it may be efficient to confine the output-error injection to a subspace of the state space. For example, in systems that model dynamics occurring over large spatial regions, such as in weather forecasting, it may be known that the measurement data are correlated only with states of model that are associated with a localized region. In this case, it is common practice to confine the data injection to a subspace of the model [5]–[9]. For estimation methods that rely on an ensemble of models, localized subspace injection can potentially reduce the size of the ensemble.

For the case of linear dynamics, a subspace-constrained Kalman filter was derived in [10]. This estimator is based on a modified Riccati difference equation for updating the error covariance, where an additional term involving an oblique projector accounts for the subspace-injection constraint.

The present paper considers the problem of subspace-constrained output-error injection in a much broader context. In particular, we consider the applicability of the unscented Kalman filter (UKF) to this problem as well as retrospective-cost state estimation (RCSE) [11]. In doing so, the goal of the present paper is to assess the feasibility of UKF and RCSE for subspace-constrained state correction. Although UKF is widely used for nonlinear system estimation [2], the application of UKF to subspace-constrained state correction in the present paper appears to be a new application of this technique. RCSE is based on retrospective cost adaptive control (RCAC), which is applicable to stabilization, command-following, and disturbance-rejection [12].

Subspace-constrained state-correction addresses a problem that is entirely different from the constrained state-estimation problem considered in [13]–[17]. In particular, constrained state-estimation constrains the state estimate by either projecting the state estimate to a constraint set based on the physical information about the system, or incorporating the state constraint in the optimization. In contrast, subspace-constrained state-correction constrains the portion of the state to which the correction is applied to reduce the computational cost of the state-correction step. These are distinct problems in state estimation. Consequently, the works [13]–[17] are not relevant to the development in the present paper.

Furthermore, unlike [10], the present paper considers subspace-constrained state correction for nonlinear systems. Although [10] could be applied with the linearized dynamics as in the case of the extended Kalman filter, we focus on UKF and RCSE for subspace-constrained state correction for nonlinear systems. A key distinction between UKF and RCSE is the fact that UKF requires an ensemble of size $2n_1+1$, where n_1 is the dimension of the subspace used for output-error injection. In contrast, RCSE requires propagation of only one copy of the system dynamics.

As a special case of subspace-constrained state correction for nonlinear systems, we consider the problem of estimating unknown parameters. By viewing unknown parameters as constant states, the state correction is confined to the subspace corresponding to the unknown parameters. For the case of linear systems with unknown coefficients, this problem is typically addressed by means of the extended Kalman filter [18].

II. Subspace-Constrained Estimation for Linear Systems

Consider the linear system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w_1(k), \quad (1)$$
$$y(k) = Cx(k) + D_2 w_2(k), \quad (2)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the known input, $y \in \mathbb{R}^p$ is the measured output, and $w_1 \sim \mathcal{N}(0,Q)$ and $w_2 \sim \mathcal{N}(0,R)$ are the process noise and measurement noise.

Consider the estimation model

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + \Gamma v(k), \quad (3)$$
$$\hat{y}(k) = C\hat{x}(k), \quad (4)$$

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where $\hat{x}$ is an estimate of x, v is the subspace-correction signal, and $\Gamma \in \mathbb{R}^{n\times n_1}$ constrains the injection of v to a subspace. Without loss of generality, Γ is assumed to have full column rank. For example,

$$\Gamma \triangleq \begin{bmatrix} I_{n_1} \\ 0_{(n-n_1)\times n_1} \end{bmatrix} \in \mathbb{R}^{n\times n_1} \tag{5}$$

constrains the update of the state estimate to the subspace of $\mathbb{R}^n$ corresponding to the first n_1 components of x.

Next, defining the output error

$$z(k) \triangleq y(k) - \hat{y}(k), \tag{6}$$

the goal is to construct a static or dynamic estimator $G_{\rm e}(k,\mathbf{q})$ such that $v(k)$ given by

$$v(k) = G_{\rm e}(k,\mathbf{q})z(k) \tag{7}$$

minimizes a performance measure involving $\hat{x}(k) - x(k)$, where $\mathbf{q}$ is the forward-shift operator. Note that the first two terms in the RHS of (3) represent the physics update of the state, whereas the last term performs data assimilation. The problem of estimating the state x in (3) using $v(k)$ given by (7) is the *subspace-constrained ouput-error injection problem*.

III. Subspace-Constrained Estimators for Linear Systems

In this section, we consider three estimators for determining the subspace-correction signal $v(k)$. For all three estimators, the matrix Γ constrains the subspace of the estimation model (3) that is corrected by $v(k)$.

Defining the state-estimate error

$$e(k) \triangleq x(k) - \hat{x}(k), \tag{8}$$

which satisfies

$$e(k+1) = Ae(k) - \Gamma v(k) + D_1 w_1(k), \tag{9}$$
$$z(k) = Ce(k) + D_2 w_2(k), \tag{10}$$

it can be seen that the subspace-constrained output-error injection problem is equivalent to output-feedback control of the system (9), (10). Stabilization of this system by means of a static output-feedback gain K requires that the triple (A,Γ,C) be stabilizable, that is, there exists a matrix K such that the spectrum of $A-\Gamma KC$ is contained in the open unit disk. Furthermore, if (A,Γ) is stabilizable and (A,C) is detectable, then stabilization is possible by choosing $G_{\rm e}(k,\mathbf{q})$ to be an asymptotically stabilizing nth-order observer-based compensator constructed by either LQG or estimator/regulator pole placement. Finally, if $\Gamma = I_n$ and $v(k) = K(k)z(k)$, then the optimal gain $K(k)$ is given by the Kalman Filter.

To address subspace-constrained ouput-error injection problem, we consider three techniques for constructing $G_{\rm e}(k,\mathbf{q})$, namely, the subspace-constrained Kalman filter, the unscented Kalman filter, and a subspace-constrained retrospective-cost state estimator.

A. Subspace-Constrained Kalman Filter

In [10], the classical Kalman filter is extended to the subspace-constrained output-error injection problem. In particular, the subspace-correction signal v in [10] is given by

$$v(k) = K(k)z(k), \tag{11}$$

where

$$K(k) = (\Gamma^{\rm T}\Gamma)^{-1}\Gamma S(k)R_1(k)^{-1}, \tag{12}$$
$$R_1(k) = CP(k)C^{\rm T} + R, \tag{13}$$
$$S(k) = AP(k)C^{\rm T}, \tag{14}$$
$$P(k+1) = AP(k)A^{\rm T} + Q - S(k)R_1(k)^{-1}S(k)^{\rm T} + \pi_\perp S(k)R_1(k)^{-1}S(k)^{\rm T}\pi_\perp^{\rm T}, \tag{15}$$

and $\pi_\perp \triangleq I - \Gamma(\Gamma^{\rm T}\Gamma)^{-1}\Gamma^{\rm T}$ is an idempotent matrix, that is, an oblique projector. Note that the estimator $G_{\rm e}(k,\mathbf{q}) = K(k)$.

B. Subspace-Constrained UKF

In [2], [3], UKF is applied to nonlinear estimation using an ensemble of $2n+1$ sigma points, where n is the dimension of the state to be estimated. In this paper, we apply UKF to the constrained subspace defined by Γ. To do so, the estimation model (3), (4) is written as

$$\hat{x}_1(k+1) = A_{11}\hat{x}_1(k) + A_{12}\hat{x}_2(k) + B_1u(k) + K(k)z(k), \tag{16}$$
$$\hat{x}_2(k+1) = A_{21}\hat{x}_1(k) + A_{22}\hat{x}_2(k) + B_2u(k), \tag{17}$$
$$\hat{y}(k) = C_1\hat{x}_1(k) + C_2\hat{x}_2(k), \tag{18}$$

where $\hat{x}_1(k) \in \mathbb{R}^{n_1}$ is the estimated state that is updated using the subspace-correction signal $K(k)z(k)$, $\hat{x}_2(k) \in \mathbb{R}^{n-n_1}$ is the remaining state, and A_{ij}, B_i, and C_i are matrices of appropriate sizes. Note that (7) is satisfied with $G_{\rm e}(k,\mathbf{q}) = K(k)$. Finally, UKF is applied to update $\hat{x}_1$.

C. Subspace-Constrained Retrospective-Cost State Estimator

The coefficient of $G_{\rm e}(k,\mathbf{q})$, written as

$$v(k) = \Phi(k)\theta(k), \tag{19}$$

where $\Phi(k)$ contains the previous data and $\theta(k)$ contains the coefficient of $G_{\rm e}(k,\mathbf{q})$, are obtained by minimizing a retrospective cost function as shown in [12].

Example III.1. *State estimation in a linear system*. Consider (1), (2) with the Lyapunov-stable dynamics

$$A = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix}, \tag{20}$$
$$C = \begin{bmatrix} 1.1 & 0.5 \end{bmatrix}, \quad D_1 = I_2, \quad D_2 = 1, \tag{21}$$

and with $\Gamma = [1\ 0]^{\rm T}$ so that only the first component of the state estimate in the estimation model is updated by output-error injection. Let $Q = 10^{-4}I_2$, $R = 10^{-3}$, $x(0) = [1\ 1]^{\rm T}$, and $u(k) \sim \mathcal{N}(0,1)$.

First, we apply subspace-constrained Kalman filter to correct the state $\hat{x}_1(k)$. The estimation-error covariance $P(0) = I_2$. For the subspace-constrained Kalman filter, Figure 1 shows the state estimates, the state-estimate error, the subspace-correction signal, and the Kalman gain.

Next, we apply subspace-constrained UKF to update the subspace state $\hat{x}_1(k)$. For subspace-constrained UKF with $\alpha = 1.2$ and $P(0) = 1$, Figure 2 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the UKF gain. Note that, since the state update is subspace-constrained, the ensemble constructed by UKF uses three sigma points instead of five sigma points, which are needed in the case where the update is unconstrained.

Finally, we apply subspace-constrained RCSE to correct the state $\hat{x}_1(k)$. We set $l_{\rm w} = 2$, $G_{\rm f}(\mathbf{q}) = \frac{-1}{\mathbf{q}}$, and $P(0) = I_{l_\theta}$. For RCSE, Figure 3 shows the state estimates, the state-estimate error, the subspace-correction signal, and the optimized estimator coefficients $\theta(k)$.

Note that the kalman filter requires the system model in order to compute the estimator gain $K(k)$ and the unscented kalman filter used the system model multiple times at each step to compute the estimator gain $K(k)$, whereas RCSE does not need require the system model or the state ensemble to compute the subspace-correction signal $v(k)$ and instead uses the past $z(k)$ and $v(k)$ data to optimize the dynamic estimator $G_{\rm e}(k,\mathbf{q})$. ◇

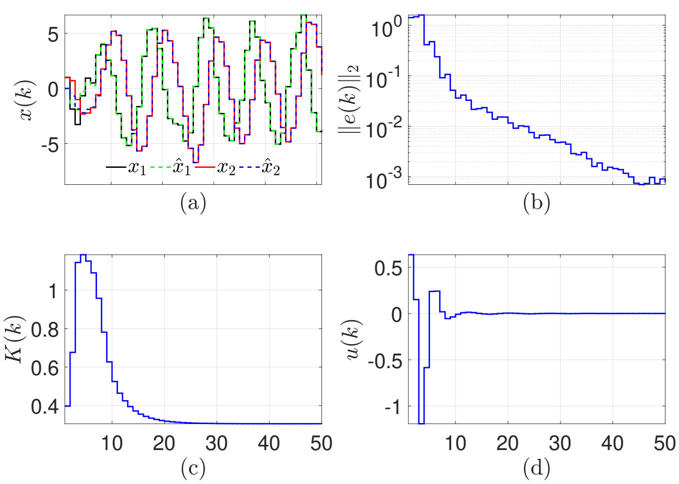

Fig. 1: Example III.1. Subspace-constrained KF state estimation. (a) shows the states and their estimates, (b) shows the norm of the state-estimation error, (c) shows the estimator coefficient $K(k)$, and (d) shows the subspace-correction signal $v(k)$.

IV. Subspace-Constrained Estimation for Nonlinear Systems

Consider the nonlinear system

$$x(k+1) = f(x(k),u(k)) + D_1 w_1(k), \tag{22}$$

$$y(k) = g(x(k),u(k)) + D_2 w_2(k), \tag{23}$$

where $x \in \mathbb{R}^n$ is the state of the system, $u \in \mathbb{R}^m$ is the known input to the system, $y \in \mathbb{R}^p$ is the measured output of the system, $w_1 \sim \mathcal{N}(0,Q)$ is the process noise, and $w_2 \sim \mathcal{N}(0,R)$ is the measurement noise.

Consider the estimation model

$$\hat{x}(k+1) = f(\hat{x}(k),u(k)) + \Gamma v(k) \tag{24}$$

$$\hat{y}(k) = g(\hat{x}(k),u(k)). \tag{25}$$

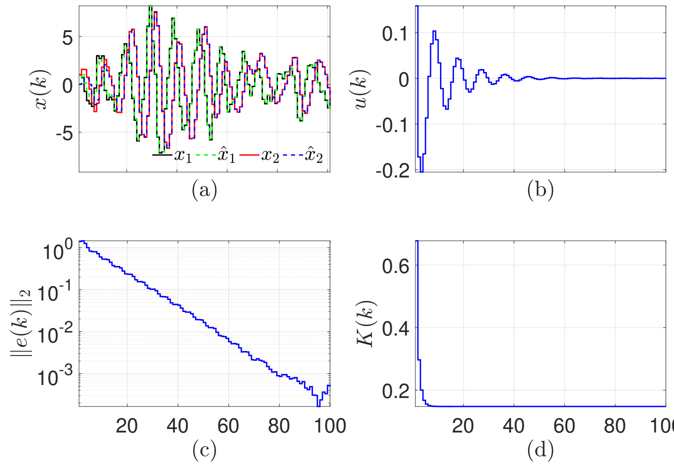

Fig. 2: Example III.1. Subspace-constrained UKF estimation. (a) shows the states and their estimates, (b) shows the norm of the state-estimation error, (c) shows the estimator coefficient $K(k)$, and (d) shows the subspace-correction signal $v(k)$.

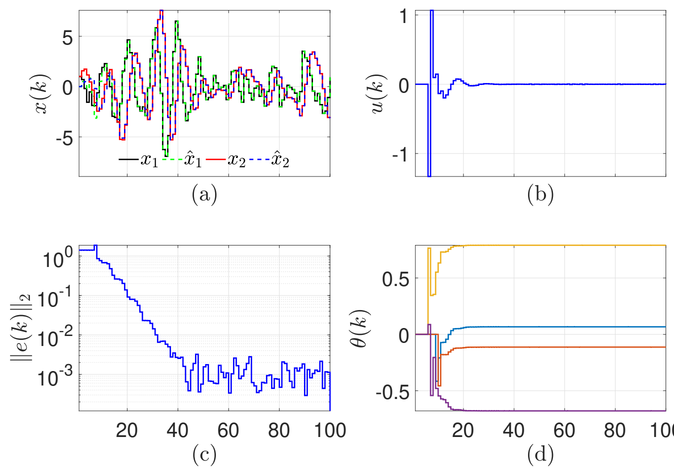

Fig. 3: Example III.1. Subspace-constrained RCSE estimation. (a) shows the states and their estimates, (b) shows the norm of the state-estimation error, (c) shows the estimator coefficients $\theta(k)$, and (d) shows the subspace-correction signal $v(k)$.

where $\hat{x}$ is an estimate of x, v is the subspace-correction signal, and $\Gamma \in \mathbb{R}^{n \times n_1}$ constrains the injection of v to a subspace.

Example IV.1. *State estimation in a chaotic system*. Consider the discrete-time Lorenz system

$$x_1(k+1) = T_{\rm s}(x_1(k) + \sigma(x_2(k) - x_1(k))), \tag{26}$$

$$x_2(k+1) = T_{\rm s}(x_2(k) + x_1(k)(\rho - x_3(k)) - x_2(k)), \tag{27}$$

$$x_3(k+1) = T_{\rm s}(x_3(k) + (x_1(k)x_2(k) - \beta x_3(k))), \tag{28}$$

$$y(k) = x_2(k), \tag{29}$$

where $T_{\rm s} = 0.01$ seconds is the sampling time, $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$. The initial state $x(0) = [10\ 10\ 10]^{\rm T}$. Note that the Lorenz system exhibits chaotic behavior for the chosen values of the constants σ, ρ and β.

Let $\Gamma = [1\ 0\ 0]^{\rm T}$ so that only the first component of the state estimate in the estimation model is updated by output-error injection.

First, we apply subspace-constrained UKF to update the subspace $\hat{x}_1$. For subspace-constrained UKF with $\alpha = 0.2$ and $P(0) = 1$, Figure 4 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the UKF gain. Note that, since the state update is subspace-constrained, the ensemble constructed by UKF uses three sigma points instead of seven sigma points, which are needed in the case where the update is unconstrained.

Finally, we apply subspace-constrained RCSE to correct the state $\hat{x}_1(k)$. We set $l_{\rm w} = 2$, $G_{\rm f}(\mathbf{q}) = \frac{-100}{\mathbf{q}}$, and $P(0) = 10^5 I_{l_\theta}$. For RCSE, Figure 5 shows the state estimates, the state-estimate error, the subspace-correction signal, and the optimized estimator coefficients $\theta(k)$. ◇

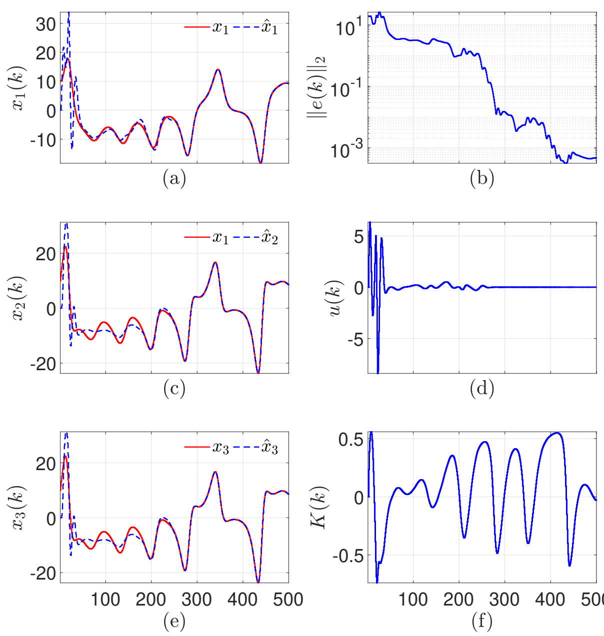

Fig. 4: Example IV.1. Subspace-constrained UKF. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficient $K(k)$, and (f) shows the subspace-correction signal $v(k)$.

V. Application to Parameter Estimation

This section applies subspace-constrained state correction to the problem of parameter estimation. Consider the system

$$x(k+1) = f(x(k), \mu, u(k)) + D_1 w_1(k), \quad (30)$$

$$y(k) = g(x(k), \mu, u(k)) + D_2 w_2(k), \quad (31)$$

where $\mu \in \mathbb{R}^{l_\mu}$ is an unknown parameter to be estimated. This problem can be formulated as a subspace-constrained estimation problem by defining the augmented dynamics

$$\begin{bmatrix} \mu(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} \mu(k) \\ f(x(k), \mu(k), u(k)) + D_1 w_1(k) \end{bmatrix}, \quad (32)$$

$$y(k) = g(x(k), \mu(k), u(k)) + D_2 w_2(k). \quad (33)$$

For parameter estimation, we constrain the state correction to the subspace corresponding to the unknown parameters μ.

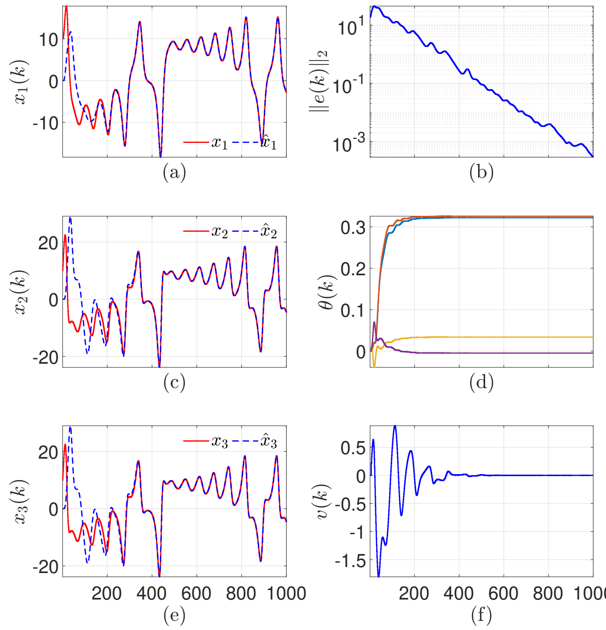

Fig. 5: Example IV.1. Subspace-constrained RCSE. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficients $\theta(k)$, and (f) shows the subspace-correction signal $v(k)$.

Therefore, $\Gamma = [I_{l_\mu} \ 0_{l_\mu \times n}]^{\rm T}$, and the estimation model is given by

$$\begin{bmatrix} \hat{\mu}(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} \hat{\mu}(k) \\ f(\hat{x}(k), \hat{\mu}(k), u(k)) \end{bmatrix} + \Gamma v(k), \quad (34)$$

$$(35)$$

$$\hat{y}(k) = g(\hat{x}(k), \hat{\mu}(k), u(k)). \quad (36)$$

Note that only the subspace corresponding to the dynamics of $\hat{\mu}(k)$ is updated by the subspace-correction signal $v(k)$.

Example V.1. *Parameter estimation in a linear system.* Consider the problem of estimating the unknown parameter μ in the linear system (1), (2), where

$$A(\mu) = \begin{bmatrix} \mu & 0.2 \\ 0.1 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix}, \quad (37)$$

$$C = \begin{bmatrix} 1.1 & 0.5 \end{bmatrix}, \quad D_1 = I_2, \quad D_2 = 1, \quad (38)$$

and $\mu = 0.3$. Let $Q = 10^{-4} I_2$, $R = 10^{-3}$, $x(0) = [10 \ 10]^{\rm T}$, and

$$u(k) = 2 + \sum_{i=1}^{5} \sin \frac{2\pi i}{100} k, \quad (39)$$

which is chosen to be persistently exciting to estimate μ. The estimation model is given by

$$\begin{bmatrix} \hat{\mu}(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{bmatrix} \hat{\mu}(k) \\ A(\hat{\mu}(k))\hat{x}(k) + Bu(k) \end{bmatrix} + \Gamma v(k), \quad (40)$$

$$(41)$$

$$\hat{y}(k) = C\hat{x}(k). \quad (42)$$

First, we apply subspace-constrained UKF to estimate μ and x, where $\Gamma = [1 \ 0 \ 0]^{\rm T}$ constrains the state correction to the space corresponding to the dynamics of the unknown parameter. For subspace-constrained UKF with $\alpha = 1.2$ and $P(0) = 10^{-5}$, Figure 6 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the UKF gain. Note that, since the state update is subspace-constrained, the ensemble constructed by UKF uses three sigma points instead of seven sigma points, which are needed in the case where the update is unconstrained.

Finally, we apply subspace-constrained RCSE to estimate the parameter μ and the state x. We set $l_{\rm w} = 1$, $G_{\rm f}(\mathbf{q}) = \frac{-1}{\mathbf{q}}$, and $P(0) = 10^4 I_{l_\theta}$. For RCSE, Figure 7 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the optimized estimator coefficients $\theta(k)$. ◇

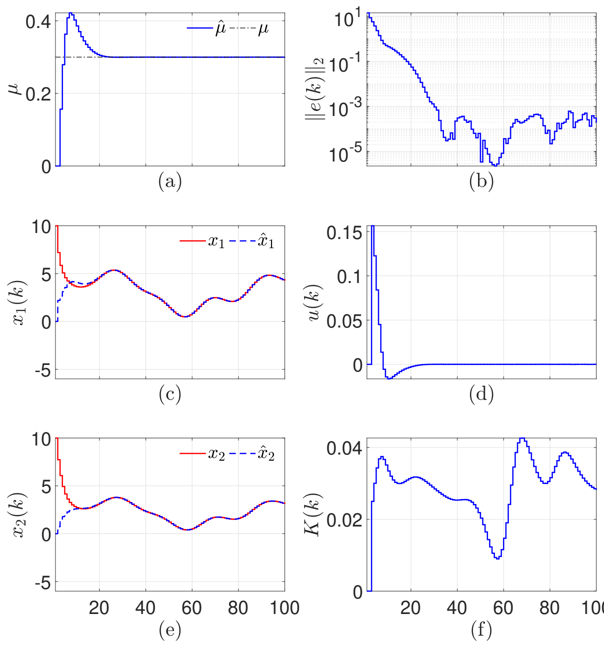

Fig. 6: Example V.1. Subspace-constrained UKF estimation. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficient $K(k)$, and (f) shows the subspace-correction signal $v(k)$.

Example V.2. *Parameter estimation in a nonlinear system.* Consider the problem of estimating the unknown parameter μ in the nonlinear system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} x_2(k) \\ \dfrac{\sin\mu + \frac{e^{\mu}}{3}x_2(k) + x_1(k)}{1+0.6x_2(k)+1.1x_1(k)} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + w_1(k), \tag{43}$$

$$y(k) = x_1(k) + x_2(k) + w_2(k), \tag{44}$$

where $\mu = 0.8$. Let $Q = 10^{-6} I_2$, $R = 10^{-3}$, $x(0) = [10\ \ 10]^{\rm T}$,

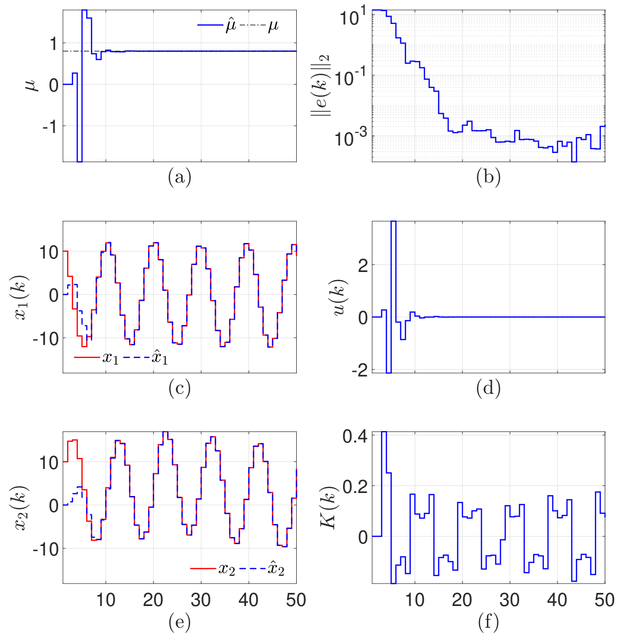

Fig. 7: Example V.1. Subspace-constrained RCSE estimation. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficients $\theta(k)$, and (f) shows the subspace-correction signal $v(k)$.

and let $u(k)$ be given by (39). The estimation model is thus

$$\begin{bmatrix} \hat{\mu}(k+1) \\ \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \hat{\mu}(k) \\ \hat{x}_2(k) \\ \dfrac{\sin\hat{\mu}(k) + \frac{e^{\hat{\mu}(k)}}{3}\hat{x}_2(k) + \hat{x}_1(k)}{1+0.6\hat{x}_2(k)+1.1\hat{x}_1(k)} + u(k) \end{bmatrix} + \Gamma v(k), \tag{45}$$

$$\hat{y}(k) = \hat{x}_1(k) + \hat{x}_2(k). \tag{46}$$

First, we apply subspace-constrained UKF to estimate μ and x, where $\Gamma = [1\ \ 0\ \ 0]^{\rm T}$ constrains the state correction to the subspace corresponding to the dynamics of the unknown parameter. For subspace-constrained UKF with $\alpha = 1.2$ and $P(0) = 10^{-5}$, Figure 8 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the UKF gain. Note that, since the state update is subspace-constrained, the ensemble constructed by UKF uses three sigma points instead of seven sigma points, which are needed in the case where the update is unconstrained.

Finally, we apply subspace-constrained RCSE to estimate the parameter μ and the state x. We set $l_{\rm w} = 1$, $G_{\rm f}(\mathbf{q}) = \frac{-1}{\mathbf{q}}$, and $P(0) = I_{l_\theta}$. For RCSE, Figure 9 shows the parameter estimate, the state estimates, the state-estimate error, the subspace-correction signal, and the optimized estimator coefficients $\theta(k)$. ◇

VI. Conclusions

Three estimation algorithms were applied to state estimation with subspace-constrained output-error correction, namely, subspace-constrained Kalman filter, subspace-constrained UKF, and subspace-constrained RCSE. The accuracy of the algorithms was numerically demonstrated in the state estimation problem and in the parameter estimation

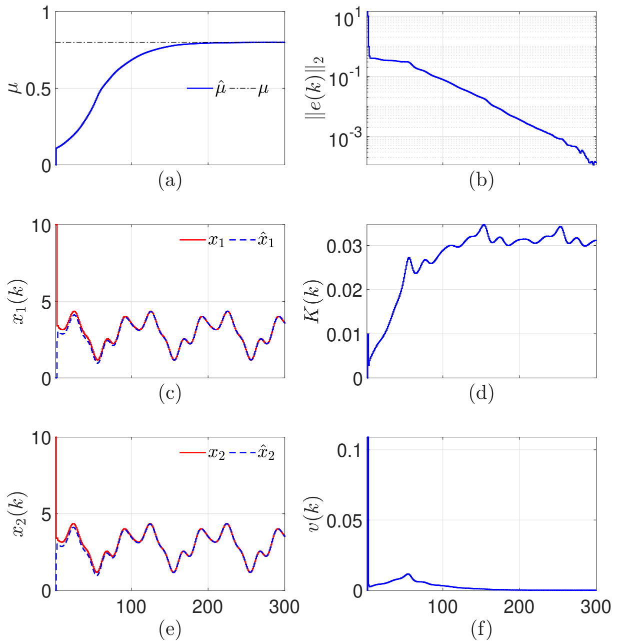

Fig. 8: Example V.2. Subspace-constrained UKF estimation. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficient $K(k)$, and (f) shows the subspace-correction signal $v(k)$.

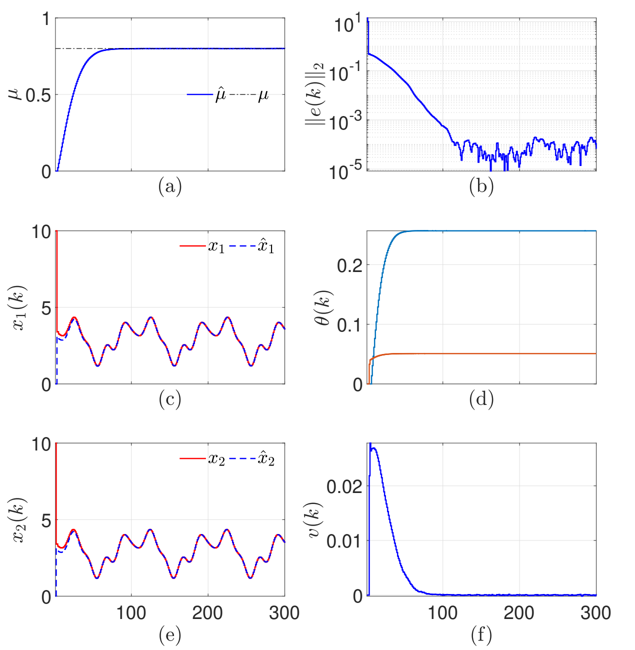

Fig. 9: Example V.2. Subspace-constrained RCSE estimation. (a), (c), (e) show the states and their estimates, (b) shows the norm of the state-estimation error, (d) shows the estimator coefficients $\theta(k)$, and (f) shows the subspace-correction signal $v(k)$.

problem in linear and nonlinear systems. These examples suggest that it suffices to constrain output-error injection to a prescribed subspace. Future research will focus on quantifying the achievable accuracy for a given subspace constraint versus the computational savings relative to full-state output-error injection, especially for high-dimensional systems.

Acknowledgments

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References

[1] D. Simon, *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons, 2006.

[2] E. A. Wan and R. Van Der Merwe, "The unscented Kalman filter for nonlinear estimation," in *Adaptive Sys. Sig. Proc. Comm. Contr. Symp.*, 2000, pp. 153–158.

[3] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," in *Proc. IEEE*, vol. 92, no. 3, 2004, pp. 401–422. [Online]. Available: http://ieeexplore.ieee.org/document/1271397/

[4] G. Evensen, *Data Assimilation: The Ensemble Kalman filter*. Springer, 2009.

[5] D. Simon, "Reduced order kalman filtering without model reduction." *Control & Intelligent Systems*, vol. 35, no. 2, 2007.

[6] E. Ott, B. R. Hunt, I. Szunyogh, A. V. Zimin, E. J. Kostelich, M. Corazza, E. Kalnay, D. Patil, and J. A. Yorke, "A local ensemble kalman filter for atmospheric data assimilation," *Tellus A: Dynamic Meteorology and Oceanography*, vol. 56, no. 5, pp. 415–428, 2004.

[7] R. Furrer, M. G. Genton, and D. Nychka, "Covariance tapering for interpolation of large spatial datasets," *J. Comp. Graph. Stat.*, vol. 15, no. 3, pp. 502–523, 2006.

[8] N. Cressie and G. Johannesson, "Fixed rank kriging for very large spatial data sets," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 70, no. 1, pp. 209–226, 2008.

[9] C. G. Kaufman, M. J. Schervish, and D. W. Nychka, "Covariance tapering for likelihood-based estimation in large spatial data sets," *J. Amer. Stat. Assn.*, vol. 103, no. 484, pp. 1545–1555, 2008.

[10] J. Chandrasekar, D. S. Bernstein, O. O. Barrero, and B. L. R. De Moor, "Kalman filtering with constrained output injection," *Int. J. Contr.*, vol. 80, no. 12, pp. 1863–1879, 2007. [Online]. Available: http://www.informaworld.com/smpp/title~content=t713393989URL:http://dx.doi.org/10.1080/00207170701373633http://www.informaworld.com/

[11] A. A. Ali, A. Goel, A. J. Ridley, and D. S. Bernstein, "Retrospective-Cost-Based Adaptive Input and State Estimation for the Ionosphere-Thermosphere," *J. Aero. Infor. Sys.*, vol. 12, no. 12, pp. 767–783, 2015. [Online]. Available: http://arc.aiaa.org/doi/10.2514/1.I010286

[12] Y. Rahman, A. Xie, and D. S. Bernstein, "Retrospective Cost Adaptive Control: Pole Placement, Frequency Response, and Connections with LQG Control," *IEEE Contr. Sys. Mag.*, vol. 37, no. 5, pp. 28–69, 10 2017. [Online]. Available: http://ieeexplore.ieee.org/document/8039008/

[13] C. Yang and E. Blasch, "Kalman filtering with nonlinear state constraints," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 45, no. 1, pp. 70–84, 2009.

[14] R. Kandepu, L. Imsland, and B. A. Foss, "Constrained state estimation using the unscented kalman filter," in *2008 16th Mediterranean Conference on Control and Automation*. IEEE, 2008, pp. 1453–1458.

[15] C. V. Rao, J. B. Rawlings, and D. Q. Mayne, "Constrained state estimation for nonlinear discrete-time systems: Stability and moving horizon approximations," *IEEE transactions on automatic control*, vol. 48, no. 2, pp. 246–258, 2003.

[16] D. Simon and T. L. Chia, "Kalman filtering with state equality constraints," *IEEE transactions on Aerospace and Electronic Systems*, vol. 38, no. 1, pp. 128–136, 2002.

[17] D. Simon, "Kalman filtering with state constraints: a survey of linear and nonlinear algorithms," *IET Control Theory & Applications*, vol. 4, no. 8, pp. 1303–1318, 2010.

[18] L. Ljung, "Asymptotic behavior of the extended kalman filter as a parameter estimator for linear systems," *IEEE Trans. Autom. Contr.*, vol. 24, pp. 36–50, 1979.